Algebra 2H Extra Notes: Binomial Theorem

April 30

The coefficients of the expansion of $(x+y)^n$ are the numbers in Pascal's triangle. The binomial coefficients are also combinations.

Pascal's Triangle	Combinations	Binomial Expansion
1	${}_0C_0$ $\frac{0!}{0!0!}=\frac{1}{1}$ 1	$(x+y)^0 = 1$
1 1	${}_1C_0$ ${}_1C_1$	$(x+y)^1 = 1x + 1y$
1 + 2 + 1	${}_2C_0$ ${}_2C_1$ ${}_2C_2$	$(x+y)^2 = 1x^2 + 2xy + 1y^2$
1 3 3 1	${}_3C_0$ ${}_3C_1$ ${}_3C_2$ ${}_3C_3$	$(x+y)^3 = 1x^3 + 3x^2y^1 + 3x^1y^2 + 1y^3$

- The number of terms in the expansion of $(x+y)^n$ is $n + 1$.
- The exponents of x decrease while the exponents of y increase.
- The sum of the exponents of the variables in any term is n.
- The Pascal numbers or combinations are the coefficients of each term.

The **Binomial Theorem** can be used to find a single term of a binomial expansion or instead of Pascal's Triangle to expand $(x+y)^n$.

Binomial Theorem
For any whole number n,

$$\frac{n!}{r!(n-r)!}$$

$$(x+y)^n = \sum_{r=0}^{n} {}_nC_r\, x^{n-r} y^r = {}_nC_0\, x^n y^0 + {}_nC_1\, x^{n-1} y^1 + {}_nC_2\, x^{n-2} y^2 + \ldots + {}_nC_n\, x^0 y^n$$

↑Pascal #s

Example 1. Find the term containing x^7 in $(\boxed{2} - \boxed{3x})^{10}$.

$$\frac{10\cdot 9\cdot 8}{3\cdot 2\cdot 1} \quad \frac{10!}{7!\,3!}(2)^3(-3x)^7 = -2{,}099{,}520x^7$$

120

${}_{10}C_7$

Example 2. Find the middle term of $(2 - 3x)^{10}$.

11 terms 6th term = middle

r = 0 1 2 3 4 5

$$\frac{10\cdot 9\cdot 8\cdot 7\cdot 6}{5\cdot 4\cdot 3\cdot 2} \quad \frac{10!}{5!\,5!}(2)^5(-3x)^5$$

${}_{10}C_5$

$252(32)(-243x^5)$