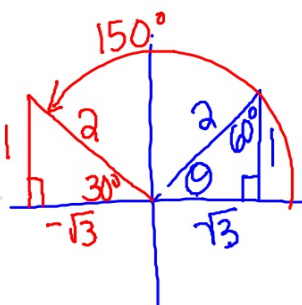


If $\sin \theta = \frac{1}{2}$, what is the value of θ ?

..., $30^\circ, 150^\circ, 390^\circ, 510^\circ, \dots$



Definitions of Inverse Functions (see also page 792)

Inverse sine:

$\sin^{-1} a = \theta$, if $\sin \theta = a$ and $-90^\circ \leq \theta \leq 90^\circ$ $\left(-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2} \right)$

Inverse cosine:

$\cos^{-1} a = \theta$, if $\cos \theta = a$ and $0^\circ \leq \theta \leq 180^\circ$ $\left(0 \leq \theta \leq \pi \right)$

Inverse tangent:

$\tan^{-1} a = \theta$, if $\tan \theta = a$ and $-90^\circ < \theta < 90^\circ$ $\left(-\frac{\pi}{2} < \theta < \frac{\pi}{2} \right)$

Inverse cotangent:

$\cot^{-1} a = \theta$, if $\cot \theta = a$ and $0^\circ < \theta < 180^\circ$ $\left(0 < \theta < \pi \right)$

(not in our textbook)

Summary:

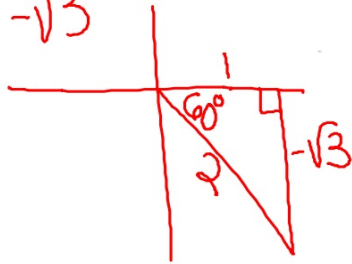
$\begin{matrix} + & - \\ Q1, Q4 \\ + & - \\ Q1, Q2 \\ + & - \\ Q1, Q4 \\ + & - \\ Q1, Q2 \\ + & - \end{matrix}$

summary:

Example 1: Evaluate.

a) $\tan^{-1}(-\sqrt{3}) = -60^\circ = -\pi/3$

$$\tan \theta = -\sqrt{3}$$



c)

b) $\cos^{-1}\left(\frac{-\sqrt{2}}{2}\right) = 135^\circ$

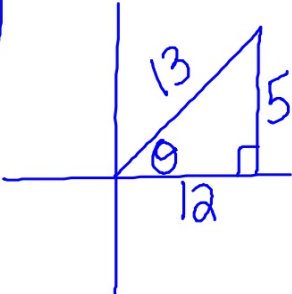
$$\cos 135^\circ = -\frac{\sqrt{2}}{2}$$

c) $\cos^{-1}\left(\sin\frac{-\pi}{6}\right)$

$$\cos^{-1}\left(-\frac{1}{2}\right) = 120^\circ = 2\pi/3$$

d) $\cot\left(\sin^{-1}\frac{5}{13}\right)$ $\sin\theta = \frac{5}{13}$

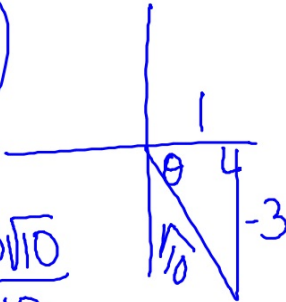
$\cot\theta = \frac{12}{5}$



e) $\sin\left(\tan^{-1}(-3)\right)$

$= \sin\theta$

$= -\frac{3}{\sqrt{10}} = -\frac{3\sqrt{10}}{10}$



Example 2: Use a calculator to find θ to the nearest tenth of a degree

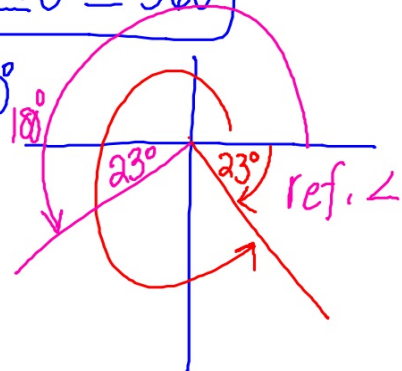
Given $\sin\theta = -0.3907$ & $180^\circ \leq \theta \leq 360^\circ$

find θ

$\theta \approx -23.0^\circ$

$\theta_4 \approx 337.0^\circ$

$\theta_3 \approx 203.0^\circ$



Definitions of Inverse Functions (not in our textbook)

Inverse cosecant: $\csc^{-1} a = \sin^{-1} \left(\frac{1}{a} \right)$, if $a \neq 0$

Inverse secant: $\sec^{-1} a = \cos^{-1} \left(\frac{1}{a} \right)$, if $a \neq 0$

Example 3. Find exact values. No decimals.

a) $\csc^{-1}(2) = \sin^{-1}\left(\frac{1}{2}\right)$
 $= 30^\circ = \frac{\pi}{6}$

b) $\sin(\sec^{-1} 3)$
 $\sin(\cos^{-1}(\frac{1}{3}))$
 $= \sin \theta = \frac{2\sqrt{2}}{3}$

$\cos \theta = \frac{1}{3}$

