12. The rate of change of the volume, V, of water in a tank with respect to time, t, is directly proportional to the square root of the volume. Which of the following is a differential equation that describes this relationship?

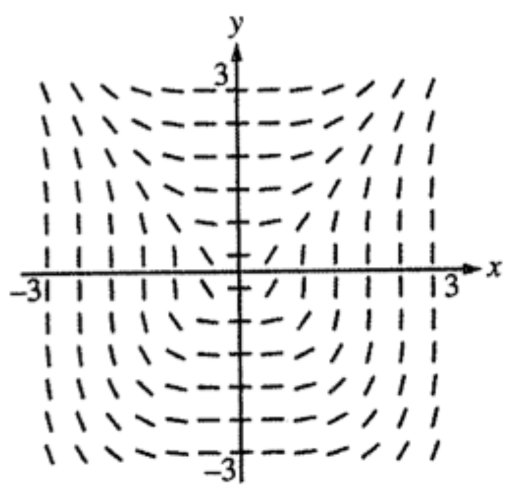
(A) $V(t) = k\sqrt{t}$

(B) $V(t) = k\sqrt{V}$

(C) $\frac{dV}{dt} = k\sqrt{t}$

(D) $\frac{dV}{dt} = \frac{k}{\sqrt{V}}$

(E) $\frac{dV}{dt} = k\sqrt{V}$



14. Shown above is a slope field for which of the following differential equations?

(A) $\frac{dy}{dx} = \frac{x}{y}$ (B) $\frac{dy}{dx} = \frac{x^2}{y^2}$ (C) $\frac{dy}{dx} = \frac{x^3}{y}$ (D) $\frac{dy}{dx} = \frac{x^2}{y}$ (E) $\frac{dy}{dx} = \frac{x^3}{y^2}$

12. The situation as described is that the rate of change of the volume, V, with respect to time t is directly proportional to the square root of the volume. Recall that when we say the quantity

A is directly proportional to the quantity B

we mean that there is a constant k so that $A = kB$.

So, since the rate of change of the volume, V, with respect to time t is $\frac{dV}{dt}$, we have (with $\frac{dV}{dt}$ playing the role of A and $\sqrt{V}$ playing the role of B in the above statement)

$$\frac{dV}{dt} = k\sqrt{V}.$$

Thus, the correct equation is (E).

14. Notice that in each quadrant of the plane the sign of he slopes in the slope field are constant. More specifically the signs of the slope field break down as in the table below

Slope	QI	QII	QIII	QIV
$\frac{dy}{dx}$	$+$	$-$	$-$	$+$

So to determine which differential equation has the given slope field, we look at the sign of $\frac{dy}{dx}$ for each quadrant for each equation. To summarize all of the results we create a table.

Equation	QI	QII	QIII	QIV
(A) $\frac{dy}{dx} =$	$\frac{x}{y} = \frac{+}{+} = +$	$\frac{x}{y} = \frac{-}{+} = -$	$\frac{x}{y} = \frac{-}{-} = +$	$\frac{x}{y} = \frac{+}{-} = -$
(B) $\frac{dy}{dx} =$	$\frac{x^2}{y^2} = \frac{+}{+} = +$	$\frac{x^2}{y^2} = \frac{+}{+} = +$	$\frac{x^2}{y^2} = \frac{+}{+} = +$	$\frac{x^2}{y^2} = \frac{+}{+} = +$
(C) $\frac{dy}{dx} =$	$\frac{x^3}{y} = \frac{+}{+} = +$	$\frac{x^3}{y} = \frac{-}{+} = -$	$\frac{x^3}{y} = \frac{-}{-} = +$	$\frac{x^3}{y} = \frac{+}{-} = -$
(D) $\frac{dy}{dx} =$	$\frac{x^2}{y} = \frac{+}{+} = +$	$\frac{x^2}{y} = \frac{+}{+} = +$	$\frac{x^2}{y} = \frac{+}{-} = -$	$\frac{x^2}{y} = \frac{+}{-} = -$
(E) $\frac{dy}{dx} =$	$\frac{x^3}{y^2} = \frac{+}{+} = +$	$\frac{x^3}{y^2} = \frac{-}{+} = -$	$\frac{x^3}{y^2} = \frac{-}{+} = -$	$\frac{x^3}{y^2} = \frac{+}{+} = +$

From the table we see that the only equation that gives the correct sign for the slope in all four quadrants is equation (E).

22. A rumor spreads among a population of N people at a rate proportional to the product of the number of people who have heard the rumor and the number of people who have not heard the rumor. If p denotes the number of people who have heard the rumor, which of the following differential equations could be used to model this situation with respect to time t, where k is a positive constant?

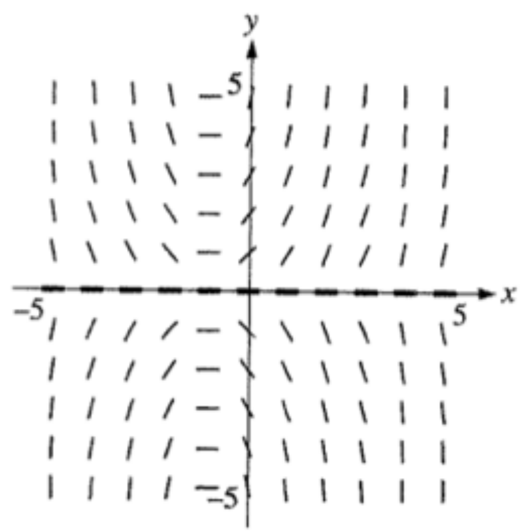
(A) $\frac{dp}{dt} = kp$

(B) $\frac{dp}{dt} = kp(N - p)$

(C) $\frac{dp}{dt} = kp(p - N)$

(D) $\frac{dp}{dt} = kt(N - t)$

(E) $\frac{dp}{dt} = kt(t - N)$



27. Shown above is a slope field for which of the following differential equations?

(A) $\frac{dy}{dx} = xy$

(B) $\frac{dy}{dx} = xy - y$

(C) $\frac{dy}{dx} = xy + y$

(D) $\frac{dy}{dx} = xy + x$

(E) $\frac{dy}{dx} = (x + 1)^3$

22. The situation as described is that the rate at which a rumor spreads in a population of size N is proportional to product of the number of people p who have heard the rumor and the number of people in the population who have not heard the rumor. Recall that when we say the quantity

A is directly proportional to the quantity B

we mean that there is a constant k so that $A = kB$.

The rate at which the rumor spreads is the rate of change of the number of people in the population who have heard the rumor. Note that the rate of change of the number of people in the population who have heard the rumor with respect to time t is $\frac{dp}{dt}$ and the number of people in the population who have NOT heard the rumor is the total number of people in the population N minus the number of people, p, who have heard the rumor, i.e. $N - p$. So the product of the number of people who have heard the rumor and the number who have not is $p(N - p)$. So we have (with $\frac{dp}{dt}$ playing the role of A and $p(N - p)$ playing the role of B in the above statement)

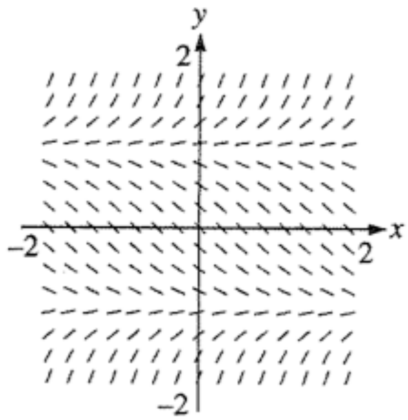
$$\frac{dp}{dt} = kp(N - p).$$

Thus, the correct equation is (B).

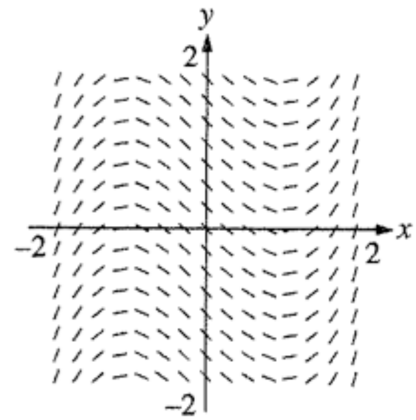
27.-1 Notice that the slopes when $y = 0$ are all 0 and the slopes are all 0 when $x = -1$. So y and $x + 1$ should BOTH be factors of $\frac{dy}{dx}$. The only one of the five given equations for which this is true is equation (C). Thus, the answer is (C).

27. Which of the following could be the slope field for the differential equation $\frac{dy}{dx} = y^2 - 1$?

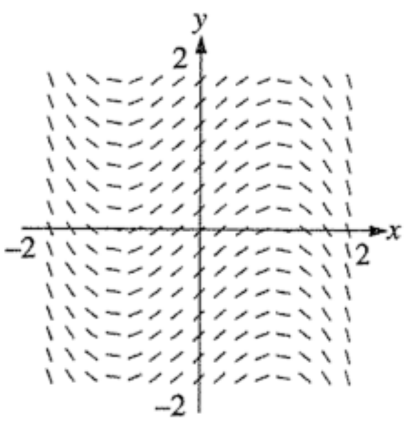
(A)



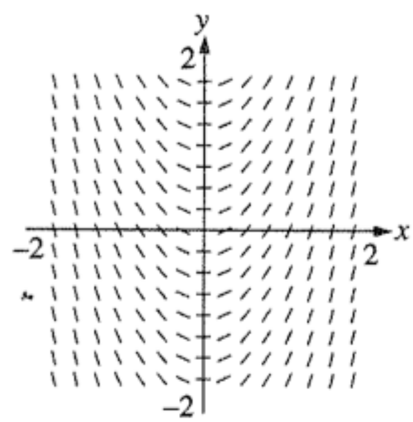
(B)



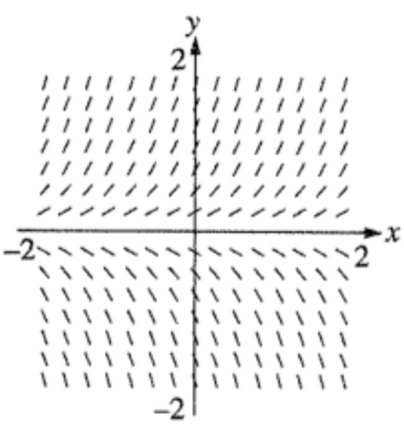
(C)



(D)



(E)



27.-2 Since $\frac{dy}{dx} = y^2 - 1$ the slopes will be negative when $y^2 - 1 < 0$ which only occurs for values of y with $-1 < y < 1$. So we look for a slope field with negative slopes ONLY between the horizontal lines $y = -1$ and $y = 1$. the only one of the given fields for which that is true is (A).

83. If $\frac{dy}{dx} = (1 + \ln x)\, y$ and if $y = 1$ when $x = 1$, then $y =$

(A) $e^{\frac{x^2-1}{x^2}}$

(B) $1 + \ln x$

(C) $\ln x$

(D) $e^{2x + x\ln x - 2}$

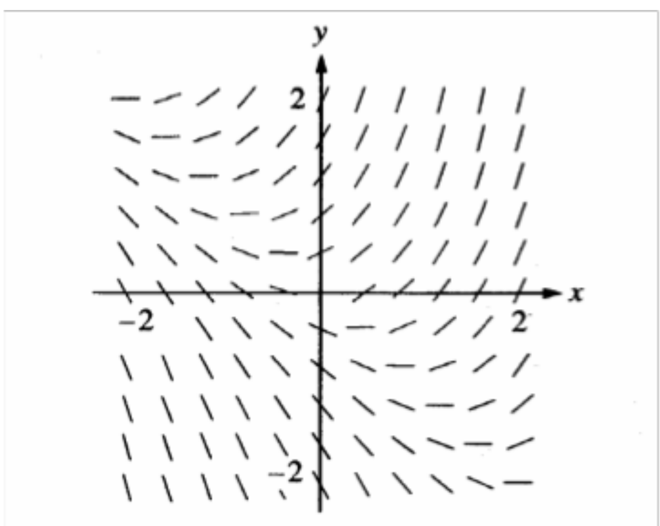
(E) $e^{x \ln x}$

21. If $\frac{dy}{dt} = ky$ and k is a nonzero constant, then y could be

(A) $2e^{kty}$ (B) $2e^{kt}$ (C) $e^{kt} + 3$ (D) $kty + 5$ (E) $\frac{1}{2}ky^2 + \frac{1}{2}$

8. If $\frac{dy}{dx} = \sin x \cos^2 x$ and if $y = 0$ when $x = \frac{\pi}{2}$, what is the value of y when $x = 0$?

(A) -1 (B) $-\frac{1}{3}$ (C) 0 (D) $\frac{1}{3}$ (E) 1



24. Shown above is a slope field for which of the following differential equations?

(A) $\frac{dy}{dx} = 1 + x$ (B) $\frac{dy}{dx} = x^2$ (C) $\frac{dy}{dx} = x + y$ (D) $\frac{dy}{dx} = \frac{x}{y}$ (E) $\frac{dy}{dx} = \ln y$

83. E $\frac{dy}{y} = (1+\ln x)\,dx$; $\ln|y| = x + x\ln x - x + k = x\ln x + k$; $|y| = e^k e^{x\ln x} \Rightarrow y = Ce^{x\ln x}$. Since $y = 1$ when $x = 1$, $C = 1$. Hence $y = e^{x\ln x}$.

21. B The solution to this differential equation is known to be of the form $y = y(0)\cdot e^{kt}$. Option (B) is the only one of this form. If you do not remember the form of the solution, then separate the variables and antidifferentiate.

$\frac{dy}{y} = k\,dt$; $\ln|y| = kt + c_1$; $|y| = e^{kt+c_1} = e^{kt}e^{c_1}$; $y = ce^{kt}$.

8. B $y(x) = -\frac{1}{3}(\cos x)^3 + C$; Let $x = \frac{\pi}{2}$, $0 = -\frac{1}{3}\left(\cos\frac{\pi}{2}\right)^3 + C \Rightarrow C = 0$. $y(0) = -\frac{1}{3}(\cos 0)^3 = -\frac{1}{3}$

24. C All slopes along the diagonal $y = -x$ appear to be 0. This is consistent only with option (C). For each of the others you can see why they do not work. Option (A) does not work because all slopes at points with the same x coordinate would have to be equal. Option (B) does not work because all slopes would have to be positive. Option (D) does not work because all slopes in the third quadrant would have to be positive. Option (E) does not work because there would only be slopes for $y > 0$.

33. If $\frac{dy}{dx}=2y^2$ and if $y=-1$ when $x=1$, then when $x=2$, $y=$

(A) $-\frac{2}{3}$ (B) $-\frac{1}{3}$ (C) 0 (D) $\frac{1}{3}$ (E) $\frac{2}{3}$

38. If the second derivative of f is given by $f''(x)=2x-\cos x$, which of the following could be $f(x)$?

(A) $\frac{x^3}{3}+\cos x-x+1$

(B) $\frac{x^3}{3}-\cos x-x+1$

(C) $x^3+\cos x-x+1$

(D) $x^2-\sin x+1$

(E) $x^2+\sin x+1$

13. If $\frac{dy}{dx}=x^2y$, then y could be

(A) $3\ln\left(\frac{x}{3}\right)$ (B) $e^{\frac{x^3}{3}}+7$ (C) $2e^{\frac{x^3}{3}}$ (D) $3e^{2x}$ (E) $\frac{x^3}{3}+1$

20. A particle moves along the x-axis so that at any time $t\geq 0$ the acceleration of the particle is $a(t)=e^{-2t}$. If at $t=0$ the velocity of the particle is $\frac{5}{2}$ and its position is $\frac{17}{4}$, then its position at any time $t>0$ is $x(t)=$

(A) $-\frac{e^{-2t}}{2}+3$

(B) $\frac{e^{-2t}}{4}+4$

(C) $4e^{-2t}+\frac{9}{2}t+\frac{1}{4}$

(D) $\frac{e^{-2t}}{2}+3t+\frac{15}{4}$

(E) $\frac{e^{-2t}}{4}+3t+4$

33. B Separate the variables. $y^{-2}dy = 2dx$; $-\frac{1}{y} = 2x + C$; $y = \frac{-1}{2x+C}$. Substitute the point $(1,-1)$ to find the value of C. Then $-1 = \frac{-1}{2+C} \Rightarrow C = -1$, so $y = \frac{1}{1-2x}$. When $x = 2$, $y = -\frac{1}{3}$.

38. A $f'(x) = x^2 - \sin x + C$, $f(x) = \frac{1}{3}x^3 + \cos x + Cx + K$. Option A is the only one with this form.

13. C $\frac{dy}{y} = x^2 dx$, $\ln|y| = \frac{1}{3}x^3 + C_1$, $y = Ce^{\frac{1}{3}x^3}$. Only C is of this form.

20. E $v(t) = -\frac{1}{2}e^{-2t} + 3$ and $x(t) = \frac{1}{4}e^{-2t} + 3t + 4$

29. If $y'' = 2y'$ and if $y = y' = e$ when $x = 0$, then when $x = 1$, $y =$

(A) $\frac{e}{2}\left(e^2+1\right)$ (B) e (C) $\frac{e^3}{2}$ (D) $\frac{e}{2}$ (E) $\frac{\left(e^3-e\right)}{2}$

4. If $\frac{dy}{dx} = \cos(2x)$, then $y =$

(A) $-\frac{1}{2}\cos(2x)+C$ (B) $-\frac{1}{2}\cos^2(2x)+C$ (C) $\frac{1}{2}\sin(2x)+C$

(D) $\frac{1}{2}\sin^2(2x)+C$ (E) $-\frac{1}{2}\sin(2x)+C$

33. If $\frac{dy}{dt} = -2y$ and if $y = 1$ when $t = 0$, what is the value of t for which $y = \frac{1}{2}$?

(A) $-\frac{\ln 2}{2}$ (B) $-\frac{1}{4}$ (C) $\frac{\ln 2}{2}$ (D) $\frac{\sqrt{2}}{2}$ (E) $\ln 2$

39. If $\frac{dy}{dx} = y\sec^2 x$ and $y = 5$ when $x = 0$, then $y =$

(A) $e^{\tan x}+4$ (B) $e^{\tan x}+5$ (C) $5e^{\tan x}$

(D) $\tan x+5$ (E) $\tan x+5e^x$

29. A Let $z = y'$. Then $z = e$ when $x = 0$. Thus $y'' = 2y' \Rightarrow z' = 2z$. Solve this differential equation.

$z = Ce^{2x}$; $e = Ce^0 \Rightarrow C = e \Rightarrow y' = z = e^{2x+1}$. Solve this differential equation.

$y = \frac{1}{2}e^{2x+1} + K$; $e = \frac{1}{2}e^1 + K \Rightarrow K = \frac{1}{2}e$; $y = \frac{1}{2}e^{2x+1} + \frac{1}{2}e$, $y(1) = \frac{1}{2}e^3 + \frac{1}{2}e = \frac{1}{2}e\left(e^2+1\right)$

Alternative Solution: $y'' = 2y' \Rightarrow y' = Ce^{2x} = e \cdot e^{2x}$. Therefore $y'(1) = e^3$.
$y'(1) - y'(0) = \int_0^1 y''(x)dx = \int_0^1 2y'(x)dx = 2y(1) - 2y(0)$ and so
$y(1) = \frac{y'(1) - y'(0) + 2y(0)}{2} = \frac{e^3 + e}{2}$.

4. C $\int \cos(2x)\,dx = \frac{1}{2}\int \cos(2x)(2\,dx) = \frac{1}{2}\sin(2x) + C$

33. C This is the differential equation for exponential growth.
$y = y(0)e^{-2t} = e^{-2t}$; $\frac{1}{2} = e^{-2t}$; $-2t = \ln\left(\frac{1}{2}\right) \Rightarrow t = -\frac{1}{2}\ln\left(\frac{1}{2}\right) = \frac{1}{2}\ln 2$

39. C $\frac{dy}{y} = \sec^2 x\,dx \Rightarrow \ln|y| = \tan x + k \Rightarrow y = Ce^{\tan x}$. $y(0) = 5 \Rightarrow y = 5e^{\tan x}$

27. If $\frac{dy}{dx} = \tan x$, then $y =$

(A) $\frac{1}{2}\tan^2 x + C$ (B) $\sec^2 x + C$ (C) $\ln|\sec x| + C$

(D) $\ln|\cos x| + C$ (E) $\sec x \tan x + C$

23. If the graph of $y = f(x)$ contains the point $(0, 2)$, $\frac{dy}{dx} = \frac{-x}{ye^{x^2}}$ and $f(x) > 0$ for all x, then $f(x) =$

(A) $3+e^{-x^2}$ (B) $\sqrt{3} + e^{-x}$ (C) $1 + e^{-x}$

(D) $\sqrt{3+e^{-x^2}}$ (E) $\sqrt{3+e^{x^2}}$

37. If $\frac{dy}{dx} = 4y$ and if $y = 4$ when $x = 0$, then $y =$

(A) $4e^{4x}$ (B) e^{4x} (C) $3 + e^{4x}$ (D) $4 + e^{4x}$ (E) $2x^2 + 4$

22. A particle moves on the curve $y = \ln x$ so that the x-component has velocity $x'(t) = t + 1$ for $t \geq 0$. At time $t = 0$, the particle is at the point $(1,0)$. At time $t = 1$, the particle is at the point

(A) $(2, \ln 2)$ (B) $(e^2, 2)$ (C) $\left(\frac{5}{2}, \ln\frac{5}{2}\right)$

(D) $(3, \ln 3)$ (E) $\left(\frac{3}{2}, \ln\frac{3}{2}\right)$

27. C $\int \tan x\,dx = \int \frac{\sin x}{\cos x}\,dx = -\ln|\cos x| + C = \ln|\sec x| + C$

23. D $\frac{dy}{dx} = \frac{-xe^{-x^2}}{y} \Rightarrow 2y\,dy = -2xe^{-x^2}\,dx \Rightarrow y^2 = e^{-x^2} + C$

$4 = 1 + C \Rightarrow C = 3;\;\; y^2 = e^{-x^2} + 3 \Rightarrow y = \sqrt{e^{-x^2} + 3}$

37. A $\frac{dy}{dx} = 4y$, $y(0) = 4$. This is exponential growth. The general solution is $y = Ce^{4x}$. Since $y(0) = 4$, $C = 4$ and so the solution is $y = 4e^{4x}$.

22. C $x'(t) = t + 1 \Rightarrow x(t) = \frac{1}{2}(t+1)^2 + C$ and $x(0) = 1 \Rightarrow C = \frac{1}{2} \Rightarrow x(t) = \frac{1}{2}(t+1)^2 + \frac{1}{2}$

$x(1) = \frac{5}{2},\; y(1) = \ln\frac{5}{2};\qquad \left(\frac{5}{2}, \ln\frac{5}{2}\right)$