3.4a

1. $f(x) = 3x^2 - 2\cos(x)$

$f'(x) = 6x - 2(-\sin(x)) = 6x + 2\sin(x)$

3. $f(x) = \sin(x) + \frac{1}{2}\cot(x)$

$f'(x) = \cos(x) + \frac{1}{2}[-\csc^2(x)] = \cos(x) - \frac{1}{2}\csc^2(x)$

5. $g(t) = t^3\cos(t)$

$g'(t) = t^3[-\sin(t)] + \cos(t) \cdot 3t^2$
$= -t^3\sin(t) + 3t^2\cos(t)$

7. $h(\theta) = \theta\csc(\theta) - \cot(\theta)$

$h'(\theta) = [\theta \cdot (-\csc(\theta)\cot(\theta)) + \csc(\theta) \cdot 1] - (-\csc^2(\theta))$
$= -\theta\csc(\theta)\cot(\theta) + \csc(\theta) + \csc^2(\theta)$

9. $y = \dfrac{x}{2 - \tan(x)}$

$$y' = \frac{[2 - \tan(x)] \cdot 1 - x[0 - \sec^2(x)]}{[2 - \tan(x)]^2}$$
$$= \frac{2 - \tan(x) + x\sec^2(x)}{[2 - \tan(x)]^2}$$

11. $f(\theta) = \dfrac{\sec(\theta)}{1+\sec(\theta)}$

$$f'(\theta) = \frac{[1+\sec(\theta)][\sec(\theta)\tan(\theta)] - \sec(\theta)[0+\sec(\theta)\tan(\theta)]}{[1+\sec(\theta)]^2}$$

$$= \frac{\sec(\theta)\tan(\theta) + \sec^2(\theta)\tan(\theta) - \sec^2(\theta)\tan(\theta)}{[1+\sec(\theta)]^2}$$

$$= \frac{\sec(\theta)\tan(\theta)}{[1+\sec(\theta)]^2}$$

13. $y = \dfrac{\sin(x)}{x^2}$

$$y' = \frac{x^2 \cdot \cos(x) - \sin(x) \cdot 2x}{(x^2)^2} = \frac{x\cos(x) - 2\sin(x)}{x^3}$$

15. $y = \sec(\theta)\tan(\theta)$

$$y' = \sec(\theta)[\sec^2(\theta)] + \tan(\theta)[\sec(\theta)\tan(\theta)]$$

$$= \sec^3(\theta) + \sec(\theta)\tan^2(\theta)$$

$$= \sec(\theta)[\sec^2(\theta) + \tan^2(\theta)]$$

Note: $1 + \tan^2(\theta) = \sec^2(\theta)$

$$= \sec(\theta)[1 + 2\tan^2(\theta)]$$

or

$$= \sec(\theta)[2\sec^2(\theta) - 1]$$

17. prove $\frac{d}{dx}(\csc(x)) = -\csc(x)\cot(x)$

$$\frac{d}{dx}(\csc(x)) = \frac{d}{dx}\left(\frac{1}{\sin(x)}\right)$$
$$= \frac{\sin(x)\cdot 0 - 1\cdot\cos(x)}{\sin^2(x)}$$
$$= \frac{-1}{\sin(x)} \cdot \frac{\cos(x)}{\sin(x)}$$
$$= -\csc(x)\cot(x)$$

19. prove $\frac{d}{dx}(\cot(x)) = -\csc^2(x)$

$$\frac{d}{dx}(\cot(x)) = \frac{d}{dx}\left(\frac{\cos(x)}{\sin(x)}\right)$$
$$= \frac{\sin(x)[-\sin(x)] - \cos(x)[\cos(x)]}{\sin^2(x)}$$
$$= \frac{-\sin^2(x) - \cos^2(x)}{\sin^2(x)}$$
$$= \frac{-[\sin^2(x) + \cos^2(x)]}{\sin^2(x)}$$
$$= \frac{-1}{\sin^2(x)}$$
$$= -\csc^2(x)$$

21. $y = \sec(x)$ point $\left(\frac{\pi}{3}, 2\right)$

$y' = \sec(x)\tan(x)$

at $x = \frac{\pi}{3}$ $y' = \sec\left(\frac{\pi}{3}\right)\tan\left(\frac{\pi}{3}\right) = (2)(\sqrt{3}) = 2\sqrt{3}$

$$y - 2 = 2\sqrt{3}\left(x - \frac{\pi}{3}\right)$$

23. $y = x + \cos(x)$ point $(0,1)$

$y' = 1 - \sin(x)$

at $x = 0$ $y' = 1 - \sin(0) = 1$

$y - 1 = 1(x - 0)$

$y = x + 1$

25. (a) $y = 2x\sin(x)$ point $(\frac{\pi}{2}, \pi)$

$y' = 2[x \cdot \cos(x) + \sin(x) \cdot 1] = 2[x\cos(x) + \sin(x)]$

at $x = \frac{\pi}{2}$ $y' = 2[\frac{\pi}{2}\cos(\frac{\pi}{2}) + \sin(\frac{\pi}{2})] = 2(0+1) = 2$

$y - \pi = 2(x - \frac{\pi}{2})$

$y = 2x$

(b)

26 ⓐ $y = \sec(x) - 2\cos(x)$ point $(\frac{\pi}{3}, 1)$

$y' = \sec(x)\tan(x) + 2\sin(x)$

at $x = \frac{\pi}{3}$ $y' = \sec(\frac{\pi}{3})\tan(\frac{\pi}{3}) + 2\sin(\frac{\pi}{3})$

$= 2(\sqrt{3}) + 2(\frac{\sqrt{3}}{2})$

$= 3\sqrt{3}$

$y - 1 = 3\sqrt{3}(x - \frac{\pi}{3})$

30. $f(x) = \sec(x)$

$f'(x) = \sec(x)\tan(x)$

$f''(x) = \sec(x)[\sec^2(x)] + \tan(x)[\sec(x)\tan(x)]$

$= \sec^3(x) + \sec(x)\tan^2(x)$

$= \sec(x)[\sec^2(x) + \tan^2(x)]$

$f''(\frac{\pi}{4}) = \sec(\frac{\pi}{4})[\sec^2(\frac{\pi}{4}) + \tan^2(\frac{\pi}{4})]$

$\frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2}}{2}$ $= \frac{2}{\sqrt{2}}[(\frac{2}{\sqrt{2}})^2 + 1^2]$

$= \sqrt{2}$ $= \sqrt{2}(2+1)$

$= 3\sqrt{2}$

32. $f(\frac{\pi}{3}) = 4 \qquad f'(\frac{\pi}{3}) = -2$

$g(x) = f(x)\sin(x) \qquad\qquad h(x) = \dfrac{\cos(x)}{f(x)}$

(a) find $g'(\frac{\pi}{3})$

$g'(x) = f(x)\cos(x) + \sin(x) f'(x)$

$$g'(\tfrac{\pi}{3}) = f(\tfrac{\pi}{3})\cos(\tfrac{\pi}{3}) + \sin(\tfrac{\pi}{3}) f'(\tfrac{\pi}{3})$$
$$= 4(\tfrac{1}{2}) + (\tfrac{\sqrt{3}}{2})(-2)$$
$$= 2 - \sqrt{3}$$

(b) find $h'(\frac{\pi}{3})$

$$h'(x) = \frac{f(x)[-\sin(x)] - \cos(x) f'(x)}{[f(x)]^2}$$
$$h'(\tfrac{\pi}{3}) = \frac{-f(\frac{\pi}{3})\sin(\frac{\pi}{3}) - \cos(\frac{\pi}{3}) f'(\frac{\pi}{3})}{[f(\frac{\pi}{3})]^2}$$
$$= \frac{-4(\frac{\sqrt{3}}{2}) - \frac{1}{2}(-2)}{(4)^2}$$
$$= \frac{-2\sqrt{3} + 1}{16}$$

33. $f(x) = x + 2\sin(x)$

$f'(x) = 1 + 2\cos(x)$

$$1 + 2\cos(x) = 0$$
$$2\cos(x) = -1$$
$$\cos(x) = \tfrac{-1}{2}$$
$$\left.\begin{aligned} x &= \tfrac{2\pi}{3} + 2\pi n \\ x &= \tfrac{4\pi}{3} + 2\pi n \end{aligned}\right\} \text{n is an integer}$$