Station 2

Stewart, 6th E

Find the Derivative

19. $f(x)=\left(\frac{1}{2}x\right)^5$

20. $f(t)=\sqrt{t}-\frac{1}{\sqrt{t}}$

21. $y=ax^2+bx+c$

22. $\sqrt{x}(x-1)$

23. $y=\frac{x^2+4x+3}{\sqrt{x}}$

24. $\frac{x^2-2\sqrt{x}}{x}$

25. $y=4\pi^2$

26. $g(u)=\sqrt{2u}+\sqrt{3u}$

27. $H(x)=\left(x+x^{-1}\right)^3$

28. $y(v)=ae^v+\frac{b}{v}+\frac{c}{v^2}$

29. $u=\sqrt[5]{t}+4\sqrt{t^5}$

30. $v=\left(\sqrt{x}+\frac{1}{\sqrt[3]{x}}\right)^2$

31. $x=\frac{A}{y^{10}}+Be^y$

32. $y=e^{x+1}+1$

Choose from #19-32

Find the equation of the tangent line to the curve at the given point.

33. $y=\sqrt[4]{x},\quad (1,1)$

34. $y=x^4+2x^2-x\quad (1,2)$

Find the equations of the tangent line and normal line to the curve at the given point.

35. $y=x^4+2e^x,\quad (0,2)$

36. $y=(1+2x)^2,\quad (1,9)$

Choose at least 2

In Problems 37–40 find the point(s) on the graph of the given function at which the tangent is horizontal. **

37. $y=(x^2-4)(x^2-6)$

38. $y=x(x-1)^2$

39. $y=\frac{x^2}{x^4+1}$

40. $y=\frac{1}{x^2-6x}$

Choose 2

In Problems 41–44 find the point(s) on the graph of the given function at which the tangent has the indicated slope.

41. $y=4x^{-1};\ m_{\tan}=-64$

42. $y=x-1/x^2;\ m_{\tan}=55$

43. $y=\frac{x+3}{x+1};\ m_{\tan}=-\frac{1}{8}$

44. $y=(x+1)(2x+5);\ m_{\tan}=-3$

**

Choose 2

17. $G(x) = \sqrt{x} - 2e^x = x^{1/2} - 2e^x \quad \Rightarrow \quad G'(x) = \frac{1}{2}x^{-1/2} - 2e^x = \dfrac{1}{2\sqrt{x}} - 2e^x$

Key Stewart

~~23~~ 17 – 25

18. $y = \sqrt[3]{x} = x^{1/3} \quad \Rightarrow \quad y' = \frac{1}{3}x^{-2/3} = \dfrac{1}{3x^{2/3}}$

19. $F(x) = (\frac{1}{2}x)^5 = \left(\frac{1}{2}\right)^5 x^5 = \frac{1}{32}x^5 \quad \Rightarrow \quad F'(x) = \frac{1}{32}(5x^4) = \frac{5}{32}x^4$

20. $f(t) = \sqrt{t} - \dfrac{1}{\sqrt{t}} = t^{1/2} - t^{-1/2} \quad \Rightarrow \quad f'(t) = \frac{1}{2}t^{-1/2} - \left(-\frac{1}{2}t^{-3/2}\right) = \dfrac{1}{2\sqrt{t}} + \dfrac{1}{2t\sqrt{t}} \quad \left[\text{or } \dfrac{t+1}{2t\sqrt{t}}\right]$

21. $y = ax^2 + bx + c \quad \Rightarrow \quad y' = 2ax + b$

22. $y = \sqrt{x}\,(x-1) = x^{3/2} - x^{1/2} \quad \Rightarrow \quad y' = \frac{3}{2}x^{1/2} - \frac{1}{2}x^{-1/2} = \frac{1}{2}x^{-1/2}(3x-1)$ [factor out $\frac{1}{2}x^{-1/2}$]

or $y' = \dfrac{3x-1}{2\sqrt{x}}$.

23. $y = \dfrac{x^2 + 4x + 3}{\sqrt{x}} = x^{3/2} + 4x^{1/2} + 3x^{-1/2} \quad \Rightarrow$

$y' = \frac{3}{2}x^{1/2} + 4\left(\frac{1}{2}\right)x^{-1/2} + 3\left(-\frac{1}{2}\right)x^{-3/2} = \frac{3}{2}\sqrt{x} + \dfrac{2}{\sqrt{x}} - \dfrac{3}{2x\sqrt{x}}$ [note that $x^{3/2} = x^{2/2} \cdot x^{1/2} = x\sqrt{x}$]

The last expression can be written as $\dfrac{3x^2}{2x\sqrt{x}} + \dfrac{4x}{2x\sqrt{x}} - \dfrac{3}{2x\sqrt{x}} = \dfrac{3x^2 + 4x - 3}{2x\sqrt{x}}$.

24. $y = \dfrac{x^2 - 2\sqrt{x}}{x} = x - 2x^{-1/2} \quad \Rightarrow \quad y' = 1 - 2\left(-\frac{1}{2}\right)x^{-3/2} = 1 + \dfrac{1}{x\sqrt{x}}$

25. $y = 4\pi^2 \quad \Rightarrow \quad y' = 0$ since $4\pi^2$ is a constant.

$$y' = \tfrac{3}{2}x^{1/2} + 4\left(\tfrac{1}{2}\right)x^{-1/2} + 3\left(-\tfrac{1}{2}\right)x^{-3/2} = \tfrac{3}{2}\sqrt{x} + \frac{2}{\sqrt{x}} - \frac{3}{2x\sqrt{x}}$$ [note that ...

Key Stewart # 24 - 29

The last expression can be written as $\dfrac{3x^2}{2x\sqrt{x}} + \dfrac{4x}{2x\sqrt{x}} - \dfrac{3}{2x\sqrt{x}} = \dfrac{3x^2 + 4x - 3}{2x\sqrt{x}}$.

24. $y = \dfrac{x^2 - 2\sqrt{x}}{x} = x - 2x^{-1/2} \Rightarrow y' = 1 - 2\left(-\tfrac{1}{2}\right)x^{-3/2} = 1 + \dfrac{1}{x\sqrt{x}}$

25. $y = 4\pi^2 \Rightarrow y' = 0$ since $4\pi^2$ is a constant.

26. $g(u) = \sqrt{2}\,u + \sqrt{3u} = \sqrt{2}\,u + \sqrt{3}\,\sqrt{u} \Rightarrow g'(u) = \sqrt{2}\,(1) + \sqrt{3}\left(\tfrac{1}{2}u^{-1/2}\right) = \sqrt{2} + \dfrac{\sqrt{3}}{2\sqrt{u}}$

27. We first expand using the Binomial Theorem (see Reference Page 1).

$$H(x) = (x + x^{-1})^3 = x^3 + 3x^2x^{-1} + 3x(x^{-1})^2 + (x^{-1})^3 = x^3 + 3x + 3x^{-1} + x^{-3} \Rightarrow$$

$$H'(x) = 3x^2 + 3 + 3(-1x^{-2}) + (-3x^{-4}) = 3x^2 + 3 - 3x^{-2} - 3x^{-4}$$

28. $y = ae^v + \dfrac{b}{v} + \dfrac{c}{v^2} = ae^v + bv^{-1} + cv^{-2} \Rightarrow y' = ae^v - bv^{-2} - 2cv^{-3} = ae^v - \dfrac{b}{v^2} - \dfrac{2c}{v^3}$

29. $u = \sqrt[5]{t} + 4\sqrt{t^5} = t^{1/5} + 4t^{5/2} \Rightarrow u' = \tfrac{1}{5}t^{-4/5} + 4\left(\tfrac{5}{2}t^{3/2}\right) = \tfrac{1}{5}t^{-4/5} + 10t^{3/2}$ or $1/\left(5\sqrt[5]{t^4}\right) + 10\sqrt{t}$

Key Stewart # 34 - 37

$y = \sqrt[4]{x} = x^{1/4} \Rightarrow y' = \frac{1}{4}x^{-3/4} = \frac{1}{4\sqrt[4]{x^3}}$. At $(1,1)$, $y' = \frac{1}{4}$ and an equation of the tangent line is

$y - 1 = \frac{1}{4}(x-1)$ or $y = \frac{1}{4}x + \frac{3}{4}$.

34→ $y = x^4 + 2x^2 - x \Rightarrow y' = 4x^3 + 4x - 1$. At $(1,2)$, $y' = 7$ and an equation of the tangent line is

$y - 2 = 7(x-1)$ or $y = 7x - 5$.

$y = x^4 + 2e^x \Rightarrow y' = 4x^3 + 2e^x$. At $(0,2)$, $y' = 2$ and an equation of the tangent line is $y - 2 = 2(x-0)$

or $y = 2x + 2$. The slope of the normal line is $-\frac{1}{2}$ (the negative reciprocal of 2) and an equation of the normal line is

$y - 2 = -\frac{1}{2}(x-0)$ or $y = -\frac{1}{2}x + 2$.

$y = (1+2x)^2 = 1 + 4x + 4x^2 \Rightarrow y' = 4 + 8x$. At $(1,9)$, $y' = 12$ and an equation of the tangent line is

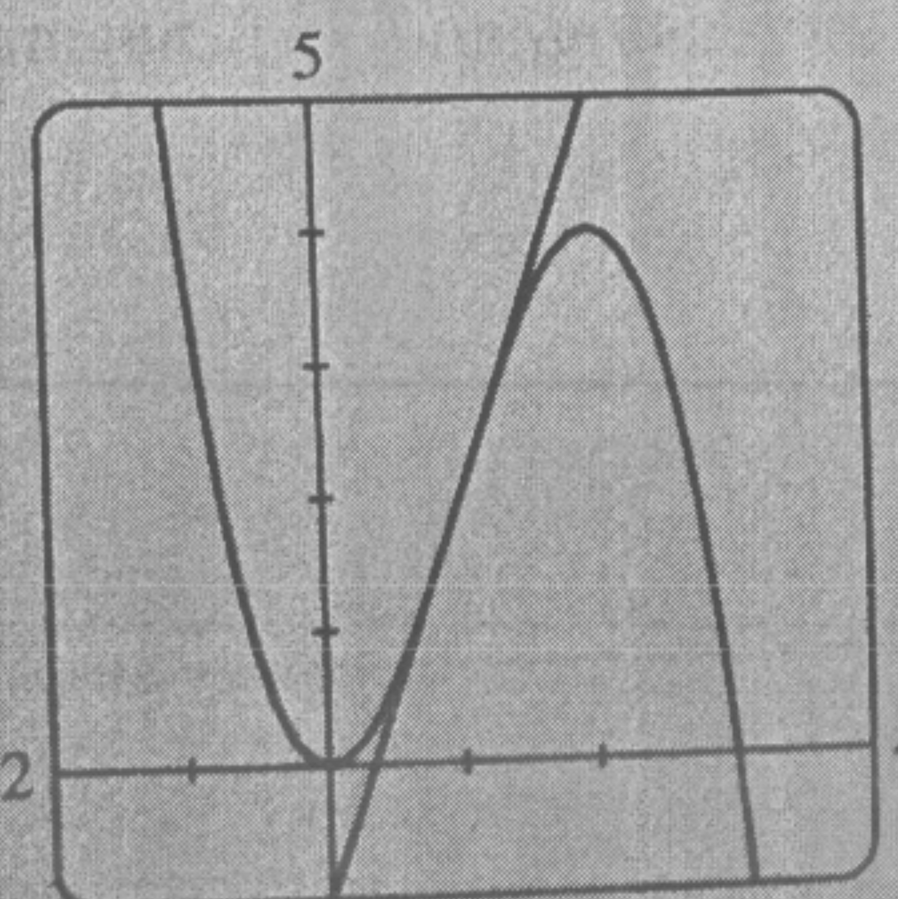
$y - 9 = 12(x-1)$ or $y = 12x - 3$. The slope of the normal line is $-\frac{1}{12}$ (the negative reciprocal of 12) and an equation of the

normal line is $y - 9 = -\frac{1}{12}(x-1)$ or $y = -\frac{1}{12}x + \frac{109}{12}$.

$y = 3x^2 - x^3 \Rightarrow y' = 6x - 3x^2$.

At $(1,2)$, $y' = 6 - 3 = 3$, so an equation of the tangent line is

$y - 2 = 3(x-1)$ or $y = 3x - 1$.



Key Stewart #30-34

30. $v = \left(\sqrt{x} + \frac{1}{\sqrt[3]{x}}\right)^2 = \left(\sqrt{x}\right)^2 + 2\sqrt{x} \cdot \frac{1}{\sqrt[3]{x}} + \left(\frac{1}{\sqrt[3]{x}}\right)^2 = x + 2x^{1/2-1/3} + 1/x^{2/3} = x + 2x^{1/6} + x^{-2/3}$

$v' = 1 + 2\left(\frac{1}{6}x^{-5/6}\right) - \frac{2}{3}x^{-5/3} = 1 + \frac{1}{3}x^{-5/6} - \frac{2}{3}x^{-5/3}$ or $1 + \frac{1}{3\sqrt[6]{x^5}} - \frac{2}{3\sqrt[3]{x^5}}$

31. $z = \frac{A}{y^{10}} + Be^y = Ay^{-10} + Be^y \Rightarrow z' = -10Ay^{-11} + Be^y = -\frac{10A}{y^{11}} + Be^y$

32. $y = e^{x+1} + 1 = e^x e^1 + 1 = e \cdot e^x + 1 \Rightarrow y' = e \cdot e^x = e^{x+1}$

33. $y = \sqrt[4]{x} = x^{1/4} \Rightarrow y' = \frac{1}{4}x^{-3/4} = \frac{1}{4\sqrt[4]{x^3}}$. At $(1, 1)$, $y' = \frac{1}{4}$ and an equation of the tangent line is

$y - 1 = \frac{1}{4}(x - 1)$ or $y = \frac{1}{4}x + \frac{3}{4}$.

34. $y = x^4 + 2x^2 - x \Rightarrow y' = 4x^3 + 4x - 1$. At $(1, 2)$, $y' = 7$ and an equation of the tangent line is

$y - 2 = 7(x - 1)$ or $y = 7x - 5$.

35. $y = x^4 + 2e^x \Rightarrow y' = 4x^3 + 2e^x$. At $(0, 2)$, $y' = 2$ and an equation of the tangent line is $y - 2 = 2(x -$

or $y = 2x + 2$. The slope of the normal line is $-\frac{1}{2}$ (the negative reciprocal of 2) and an equation of the normal li

$y - 2 = \ldots$

39. $y' = \dfrac{\quad}{(x^4+1)^2} = \dfrac{\quad}{(x^4+1)^2}$

The tangent is horizontal when $y' = 0$ or $2x - 2x^5 = 0$. Then $2x(1-x^4) = 0$ an[d] $x = 0, \pm 1$. Given $y = \dfrac{x^2}{x^4+1}$, we see that for $x = 0$, $y = 0$, and for $x = \pm 1$, $y = 1/2$. Thus, the points on the graph are $(0,0)$, $(1,1/2)$, and $(-1,1/2)$.

Zill key # 40 − 43

40. $y' = \dfrac{(x^2-6x)\cdot 0 - 1\cdot(2x-6)}{(x^2-6x)^2} = \dfrac{-2x+6}{(x^2-6x)^2}$

The tangent is horizontal when $y' = 0$, so $-2x+6 = 0$ and $x = 3$. Given $y = \dfrac{1}{x^2-6x}$ we see that for $x = 3$, $y = -1/9$. Thus, the point on the graph is $(3, -1/9)$.

41. $y' = -4x^{-2}$. Since the slope of the tangent line is -64, $-4x^{-2} = -64$, so $x^2 = 1/16$ and $x = \pm 1/4$. Given $y = 4x^{-1}$ we see that for $x = 1/4$, $y = 16$, and for $x = -1/4$, $y = -16$. Thus, the points on the graph are $(1/4, 16)$ and $(-1/4, -16)$.

42. $y = x - x^{-2}$; $y' = 1 + 2x^{-3}$. Since the slope of the tangent line is 55, $1 + 2x^{-3} = 55$, so $x^3 = 1/27$ and $x = 1/3$. Given $y = x - 1/x^2$, we see that for $x = 1/3$, $y = -26/3$. Thus, the point on the graph is $(1/3, -26/3)$.

43. $y' = \dfrac{(x+1)\cdot 1 - (x+3)\cdot 1}{(x+1)^2} = \dfrac{-2}{(x+1)^2}$

Since the slope of the tangent line is $-1/8$, $-2/(x+1)^2 = -1/8$, so $x^2 + 2x - 15 = 0$ and $x = 3$, -5. Given $y = \dfrac{x+3}{x+1}$, we see that for $x = 3$, $y = 3/2$, and for $x = -5$, $y = 1/2$. Thus, the points are $(3, 3/2)$ & $(-5, 1/2)$