2. If $f(x)=(2x+1)^4$, then the 4th derivative of $f(x)$ at $x=0$ is

(A) 0 (B) 24 (C) 48 (D) 240 (E) 384

3. If $y=\dfrac{3}{4+x^2}$, then $\dfrac{dy}{dx}=$

(A) $\dfrac{-6x}{\left(4+x^2\right)^2}$ (B) $\dfrac{3x}{\left(4+x^2\right)^2}$ (C) $\dfrac{6x}{\left(4+x^2\right)^2}$ (D) $\dfrac{-3}{\left(4+x^2\right)^2}$ (E) $\dfrac{3}{2x}$

6. If $f(x)=x$, then $f'(5)=$

(A) 0 (B) $\dfrac{1}{5}$ (C) 1

(D) 5 (E) $\dfrac{25}{2}$

2. E

$f'(x)=4(2x+1)^3\cdot 2$, $f''(1)=4\cdot 3(2x+1)^2\cdot 2^2$, $f'''(1)=4\cdot 3\cdot 2(2x+1)^1\cdot 2^3$,
$f^{(4)}(1)=4!\cdot 2^4=384$

3. A

$y=3(4+x^2)^{-1}$ so $y'=-3(4+x^2)^{-2}(2x)=\dfrac{-6x}{(4+x^2)^2}$

Or using the quotient rule directly gives $y'=\dfrac{\left(4+x^2\right)(0)-3(2x)}{\left(4+x^2\right)^2}=\dfrac{-6x}{\left(4+x^2\right)^2}$

6. C $f'(x)=1\Rightarrow f'(5)=1$

10. If $y = 10^{(x^2-1)}$, then $\frac{dy}{dx} =$

(A) $(\ln 10)10^{(x^2-1)}$ (B) $(2x)10^{(x^2-1)}$ (C) $(x^2-1)10^{(x^2-2)}$

(D) $2x(\ln 10)10^{(x^2-1)}$ (E) $x^2(\ln 10)10^{(x^2-1)}$

18. If $y = \cos^2 x - \sin^2 x$, then $y' =$

(A) -1 (B) 0 (C) $-2\sin(2x)$

(D) $-2(\cos x + \sin x)$ (E) $2(\cos x - \sin x)$

20. If $y = \arctan(\cos x)$, then $\frac{dy}{dx} =$

(A) $\frac{-\sin x}{1+\cos^2 x}$ (B) $-(\text{arcsec}(\cos x))^2 \sin x$ (C) $(\text{arcsec}(\cos x))^2$

(D) $\frac{1}{(\arccos x)^2 + 1}$ (E) $\frac{1}{1+\cos^2 x}$

10. D $y' = 10^{(x^2-1)} \cdot \ln(10) \cdot \frac{d}{dx}\left((x^2-1)\right) = 2x \cdot 10^{(x^2-1)} \cdot \ln(10)$

18. C $y = \cos^2 x - \sin^2 x = \cos 2x, \;\; y' = -2\sin 2x$

20. A $\frac{dy}{dx} = \frac{1}{1+\cos^2 x} \cdot \frac{d}{dx}(\cos x) = \frac{-\sin x}{1+\cos^2 x}$

23. $\frac{d}{dx}\left(\frac{1}{x^3}-\frac{1}{x}+x^2\right)$ at $x=-1$ is

(A) -6 (B) -4 (C) 0

(D) 2 (E) 6

42. $\frac{d}{dx}\int_2^x \sqrt{1+t^2}\,dt =$

(A) $\frac{x}{\sqrt{1+x^2}}$ (B) $\sqrt{1+x^2}-5$ (C) $\sqrt{1+x^2}$

(D) $\frac{x}{\sqrt{1+x^2}}-\frac{1}{\sqrt{5}}$ (E) $\frac{1}{2\sqrt{1+x^2}}-\frac{1}{2\sqrt{5}}$

6. If $f(x)=\frac{x}{\tan x}$, then $f'\left(\frac{\pi}{4}\right)=$

(A) 2 (B) $\frac{1}{2}$ (C) $1+\frac{\pi}{2}$

(D) $\frac{\pi}{2}-1$ (E) $1-\frac{\pi}{2}$

23. B $\frac{d}{dx}\left(x^{-3}-x^{-1}+x^{2}\right)\Big|_{x=-1}=\left(-3x^{-4}+x^{-2}+2x\right)\Big|_{x=-1}=-3+1-2=-4$

42. C This is a direct application of the Fundamental Theorem of Calculus: $f'(x)=\sqrt{1+x^{2}}$

6. E $f(x)=\frac{x}{\tan x},\ f'(x)=\frac{\tan x-x\sec^{2}x}{\tan^{2}x},\ f'\left(\frac{\pi}{4}\right)=\frac{1-\frac{\pi}{4}\cdot\left(\sqrt{2}\right)^{2}}{1}=1-\frac{\pi}{2}$

11. $\frac{d}{dx}\ln\left(\frac{1}{1-x}\right)=$

(A) $\frac{1}{1-x}$ (B) $\frac{1}{x-1}$ (C) $1-x$

(D) $x-1$ (E) $(1-x)^2$

17. If $f(x)=x\ln\left(x^2\right)$, then $f'(x)=$

(A) $\ln\left(x^2\right)+1$ (B) $\ln\left(x^2\right)+2$ (C) $\ln\left(x^2\right)+\frac{1}{x}$

(D) $\frac{1}{x^2}$ (E) $\frac{1}{x}$

19. If f and g are twice differentiable functions such that $g(x)=e^{f(x)}$ and $g''(x)=h(x)e^{f(x)}$, then $h(x)=$

(A) $f'(x)+f''(x)$ (B) $f'(x)+\left(f''(x)\right)^2$ (C) $\left(f'(x)+f''(x)\right)^2$

(D) $\left(f'(x)\right)^2+f''(x)$ (E) $2f'(x)+f''(x)$

11. A $$\frac{d}{dx}\left(\ln\left(\frac{1}{1-x}\right)\right)=\frac{d}{dx}(-\ln(1-x))=-\left(\frac{-1}{1-x}\right)=\frac{1}{1-x}$$

17. B $$f'\left(x\right)=(1)\cdot\ln(x^2)+x\cdot\frac{\frac{d}{dx}(x^2)}{x^2}=\ln(x^2)+\frac{2x^2}{x^2}=\ln(x^2)+2$$

19. D $$g(x)=e^{f(x)},\ g'(x)=e^{f(x)}\cdot f'(x),\ g''(x)=e^{f(x)}\cdot f''(x)+f'(x)\cdot e^{f(x)}\cdot f'(x)$$
$$g''(x)=e^{f(x)}\left(f''(x)+\left(f'(x)^2\right)\right)=h(x)e^{f(x)}\Rightarrow h(x)=f''(x)+\left(f'(x)^2\right)$$

26. For $0 < x < \frac{\pi}{2}$, if $y = (\sin x)^x$, then $\frac{dy}{dx}$ is

(A) $x\ln(\sin x)$ (B) $(\sin x)^x \cot x$ (C) $x(\sin x)^{x-1}(\cos x)$

(D) $(\sin x)^x(x\cos x + \sin x)$ (E) $(\sin x)^x(x\cot x + \ln(\sin x))$

1. If $y = x^2e^x$, then $\frac{dy}{dx} =$

(A) $2xe^x$ (B) $x(x+2e^x)$ (C) $xe^x(x+2)$

(D) $2x+e^x$ (E) $2x+e$

6. If $y = \frac{\ln x}{x}$, then $\frac{dy}{dx} =$

(A) $\frac{1}{x}$ (B) $\frac{1}{x^2}$ (C) $\frac{\ln x - 1}{x^2}$

(D) $\frac{1-\ln x}{x^2}$ (E) $\frac{1+\ln x}{x^2}$

26. E

Apply the log function, simplify, and differentiate. $\ln y = \ln(\sin x)^x = x\ln(\sin x)$

$$\frac{y'}{y} = \ln(\sin x) + x\cdot\frac{\cos x}{\sin x} \Rightarrow y' = y\left(\ln(\sin x) + x\cdot\cot x\right) = (\sin x)^x\left(\ln(\sin x) + x\cdot\cot x\right)$$

1. C $\quad \dfrac{dy}{dx} = x^2\cdot\dfrac{d}{dx}(e^x) + e^x\cdot\dfrac{d}{dx}(x^2) = x^2e^x + 2xe^x = xe^x(x+2)$

6. D $\quad \dfrac{dy}{dx} = \dfrac{x\cdot\dfrac{d}{dx}(\ln x) - \ln x\cdot\dfrac{d}{dx}(x)}{x^2} = \dfrac{x\cdot\left(\dfrac{1}{x}\right) - \ln x\cdot(1)}{x^2} = \dfrac{1-\ln x}{x^2}$

12. If $f(x)=\sin x$, then $f'\left(\frac{\pi}{3}\right)=$

(A) $-\frac{1}{2}$ (B) $\frac{1}{2}$ (C) $\frac{\sqrt{2}}{2}$

(D) $\frac{\sqrt{3}}{2}$ (E) $\sqrt{3}$

15. If $f(x)=\sqrt{2x}$, then $f'(2)=$

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) $\frac{\sqrt{2}}{2}$

(D) 1 (E) $\sqrt{2}$

18. If $y=2\cos\left(\frac{x}{2}\right)$, then $\frac{d^2y}{dx^2}=$

(A) $-8\cos\left(\frac{x}{2}\right)$ (B) $-2\cos\left(\frac{x}{2}\right)$ (C) $-\sin\left(\frac{x}{2}\right)$

(D) $-\cos\left(\frac{x}{2}\right)$ (E) $-\frac{1}{2}\cos\left(\frac{x}{2}\right)$

12. B $$f'\left(\frac{\pi}{3}\right)=\cos\left(\frac{\pi}{3}\right)=\frac{1}{2}$$

15. B $$f(x)=\sqrt{2x}=\sqrt{2}\cdot\sqrt{x};\ f'(x)=\sqrt{2}\cdot\frac{1}{2\sqrt{x}};\ f'(2)=\sqrt{2}\cdot\frac{1}{2\sqrt{2}}=\frac{1}{2}$$

18. E

$$y'=2\cdot\left(-\sin\left(\frac{x}{2}\right)\cdot\frac{1}{2}\right)=-\sin\left(\frac{x}{2}\right);\ y''=-\left(\cos\left(\frac{x}{2}\right)\cdot\left(\frac{1}{2}\right)\right)=-\frac{1}{2}\cos\left(\frac{x}{2}\right)$$

24. $\frac{d}{dx}\left(x^{\ln x}\right) =$

(A) $x^{\ln x}$ (B) $(\ln x)^x$ (C) $\frac{2}{x}(\ln x)\left(x^{\ln x}\right)$

(D) $(\ln x)\left(x^{\ln x - 1}\right)$ (E) $2(\ln x)\left(x^{\ln x}\right)$

3. If $f(x) = \ln\left(\sqrt{x}\right)$, then $f''(x) =$

(A) $-\frac{2}{x^2}$ (B) $-\frac{1}{2x^2}$ (C) $-\frac{1}{2x}$

(D) $-\frac{1}{2x^{\frac{3}{2}}}$ (E) $\frac{2}{x^2}$

8. If $f(x) = e^x$, then $\ln\left(f'(2)\right) =$

(A) 2 (B) 0 (C) $\frac{1}{e^2}$

(D) $2e$ (E) e^2

24. C

Let $y = x^{\ln x}$ and take the ln of each side. $\ln y = \ln x^{\ln x} = \ln x \cdot \ln x$. Take the derivative of each side with respect to x. $\frac{y'}{y} = 2\ln x \cdot \frac{1}{x} \Rightarrow y' = 2\ln x \cdot \frac{1}{x} \cdot x^{\ln x}$

3. B $f(x) = \ln\sqrt{x} = \frac{1}{2}\ln x;\ f'(x) = \frac{1}{2}\cdot\frac{1}{x} \Rightarrow f''(x) = -\frac{1}{2x^2}$

8. A $f'(x) = e^x,\ f'(2) = e^2,\ \ln e^2 = 2$

22. If $f(x) = \left(x^2+1\right)^x$, then $f'(x) =$

(A) $x\left(x^2+1\right)^{x-1}$

(B) $2x^2\left(x^2+1\right)^{x-1}$

(C) $x\ln\left(x^2+1\right)$

(D) $\ln\left(x^2+1\right)+\dfrac{2x^2}{x^2+1}$

(E) $\left(x^2+1\right)^x\left[\ln\left(x^2+1\right)+\dfrac{2x^2}{x^2+1}\right]$

28. $\dfrac{d}{dx}\ln\left|\cos\left(\dfrac{\pi}{x}\right)\right|$ is

(A) $\dfrac{-\pi}{x^2\cos\left(\dfrac{\pi}{x}\right)}$

(B) $-\tan\left(\dfrac{\pi}{x}\right)$

(C) $\dfrac{1}{\cos\left(\dfrac{\pi}{x}\right)}$

(D) $\dfrac{\pi}{x}\tan\left(\dfrac{\pi}{x}\right)$

(E) $\dfrac{\pi}{x^2}\tan\left(\dfrac{\pi}{x}\right)$

1. If $f(x) = x^{\frac{3}{2}}$, then $f'(4) =$

(A) -6

(B) -3

(C) 3

(D) 6

(E) 8

22. E

Quick Solution: f' must have a factor of f which makes E the only option. Or,

$$\ln f(x) = x\ln(x^2+1) \Rightarrow \frac{f'(x)}{f(x)} = x\cdot\frac{2x}{x^2+1} + \ln(x^2+1) \Rightarrow f'(x) = f(x)\cdot\left(\frac{2x^2}{x^2+1} + \ln(x^2+1)\right)$$

28. E

$$\frac{\frac{d}{dx}\left(\cos\left(\frac{\pi}{x}\right)\right)}{\cos\left(\frac{\pi}{x}\right)} = \frac{-\sin\left(\frac{\pi}{x}\right)\cdot\frac{d}{dx}\left(\frac{\pi}{x}\right)}{\cos\left(\frac{\pi}{x}\right)} = \frac{-\sin\left(\frac{\pi}{x}\right)\cdot\left(-\frac{\pi}{x^2}\right)}{\cos\left(\frac{\pi}{x}\right)} = \frac{\pi}{x^2}\tan\left(\frac{\pi}{x}\right)$$

1. C $f'(x) = \frac{3}{2}x^{\frac{1}{2}}; f'(4) = \frac{3}{2}\cdot 4^{\frac{1}{2}} = \frac{3}{2}\cdot 2 = 3$

8. If $y = \tan x - \cot x$, then $\dfrac{dy}{dx} =$

(A) $\sec x \csc x$ (B) $\sec x - \csc x$ (C) $\sec x + \csc x$

(D) $\sec^2 x - \csc^2 x$ (E) $\sec^2 x + \csc^2 x$

9. If h is the function given by $h(x) = f(g(x))$, where $f(x) = 3x^2 - 1$ and $g(x) = |x|$, then $h(x) =$

(A) $3x^3 - |x|$ (B) $|3x^2 - 1|$ (C) $3x^2|x| - 1$ (D) $3|x| - 1$ (E) $3x^2 - 1$

10. If $f(x) = (x-1)^2 \sin x$, then $f'(0) =$

(A) -2 (B) -1 (C) 0

(D) 1 (E) 2

8. E $y' = \sec^2 x + \csc^2 x$

9. E $h(x) = f\left(|x|\right) = 3|x|^2 - 1 = 3x^2 - 1$

10. D

$f'(x) = 2(x-1)\cdot \sin x + (x-1)^2 \cos x;\ \ f'(0) = (-2)\cdot 0 + 1\cdot 1 = 1$

24. If $f(x)=(x^2-2x-1)^{\frac{2}{3}}$, then $f'(0)$ is

(A) $\frac{4}{3}$ (B) 0 (C) $-\frac{2}{3}$ (D) $-\frac{4}{3}$ (E) -2

25. $\frac{d}{dx}\left(2^x\right)=$

(A) 2^{x-1} (B) $(2^{x-1})x$ (C) $(2^x)\ln 2$

(D) $(2^{x-1})\ln 2$ (E) $\frac{2x}{\ln 2}$

31. If $f(x)=e^{3\ln(x^2)}$, then $f'(x)=$

(A) $e^{3\ln(x^2)}$ (B) $\frac{3}{x^2}e^{3\ln(x^2)}$ (C) $6(\ln x)e^{3\ln(x^2)}$

(D) $5x^4$ (E) $6x^5$

24. A $f'(x)=\frac{2}{3}\left(x^2-2x-1\right)^{-\frac{1}{3}}(2x-2),\ f'(0)=\frac{2}{3}\cdot(-1)\cdot(-2)=\frac{4}{3}$

25. C $\frac{d}{dx}(2^x)=2^x\cdot\ln 2$

31. E $f(x)=e^{3\ln(x^2)}=e^{\ln(x^6)}=x^6;\ f'(x)=6x^5$

41. $\frac{d}{dx}\int_0^x \cos(2\pi u)\,du$ is

(A) 0 (B) $\frac{1}{2\pi}\sin x$ (C) $\frac{1}{2\pi}\cos(2\pi x)$

(D) $\cos(2\pi x)$ (E) $2\pi\cos(2\pi x)$

8. If $f(x) = \ln(e^{2x})$, then $f'(x) =$

(A) 1 (B) 2 (C) $2x$

(D) e^{-2x} (E) $2e^{-2x}$

15. If $f(x) = e^{\tan^2 x}$, then $f'(x) =$

(A) $e^{\tan^2 x}$ (D) $2\tan x\sec^2 x\, e^{\tan^2 x}$

(B) $\sec^2 x\, e^{\tan^2 x}$ (E) $2\tan x\, e^{\tan^2 x}$

(C) $\tan^2 x\, e^{\tan^2 x - 1}$

41. D Answer follows from the Fundamental Theorem of Calculus.

8. B $f(x) = \ln e^{2x} = 2x,\ f'(x) = 2$

15. D $f'(x) = e^{\tan^2 x} \cdot \dfrac{d\left(\tan^2 x\right)}{dx} = 2\tan x \cdot \sec^2 x \cdot e^{\tan^2 x}$

18. If $e^{f(x)} = 1 + x^2$, then $f'(x) =$

(A) $\dfrac{1}{1+x^2}$ (B) $\dfrac{2x}{1+x^2}$ (C) $2x(1+x^2)$

(D) $2x\left(e^{1+x^2}\right)$ (E) $2x\ln(1+x^2)$

21. The value of the derivative of $y = \dfrac{\sqrt[3]{x^2+8}}{\sqrt[4]{2x+1}}$ at $x = 0$ is

(A) -1 (B) $-\dfrac{1}{2}$ (C) 0

(D) $\dfrac{1}{2}$ (E) 1

26. If $y = \arctan(e^{2x})$, then $\dfrac{dy}{dx} =$

(A) $\dfrac{2e^{2x}}{\sqrt{1-e^{4x}}}$ (B) $\dfrac{2e^{2x}}{1+e^{4x}}$ (C) $\dfrac{e^{2x}}{1+e^{4x}}$

(D) $\dfrac{1}{\sqrt{1-e^{4x}}}$ (E) $\dfrac{1}{1+e^{4x}}$

18. B $$f'(x)\cdot e^{f(x)} = 2x \Rightarrow f'(x) = \frac{2x}{e^{f(x)}} = \frac{2x}{1+x^2}$$

21. A

Use logarithms.

$\ln y = \frac{1}{3}\ln\left(x^2+8\right) - \frac{1}{4}\ln(2x+1);\ \frac{y'}{y} = \frac{2x}{3\left(x^2+8\right)} - \frac{2}{4(2x+1)};$ at $(0,2)$, $y' = -1$.

26. B $$\frac{2e^{2x}}{1+(e^{2x})^2} = \frac{2e^{2x}}{1+e^{4x}}$$

2. If $f(x) = x\sqrt{2x-3}$, then $f'(x) =$

(A) $\dfrac{3x-3}{\sqrt{2x-3}}$

(B) $\dfrac{x}{\sqrt{2x-3}}$

(C) $\dfrac{1}{\sqrt{2x-3}}$

(D) $\dfrac{-x+3}{\sqrt{2x-3}}$

(E) $\dfrac{5x-6}{2\sqrt{2x-3}}$

4. If $f(x) = -x^3 + x + \dfrac{1}{x}$, then $f'(-1) =$

(A) 3 (B) 1 (C) –1

(D) –3 (E) –5

7. $\dfrac{d}{dx}\cos^2(x^3) =$

(A) $6x^2 \sin(x^3)\cos(x^3)$

(B) $6x^2 \cos(x^3)$

(C) $\sin^2(x^3)$

(D) $-6x^2 \sin(x^3)\cos(x^3)$

(E) $-2\sin(x^3)\cos(x^3)$

2. A

$$f(x) = x(2x-3)^{\frac{1}{2}}; \quad f'(x) = (2x-3)^{\frac{1}{2}} + x(2x-3)^{-\frac{1}{2}} = (2x-3)^{-\frac{1}{2}}(3x-3) = \frac{(3x-3)}{\sqrt{2x-3}}$$

4. D

$$f(x) = -x^3 + x + \frac{1}{x}; \;\; f'(x) = -3x^2 + 1 - \frac{1}{x^2}; \;\; f'(-1) = -3(-1)^2 + 1 - \frac{1}{(-1)^2} = -3 + 1 - 1 = -3$$

7. D

$$\frac{d}{dx}\cos^2(x^3) = 2\cos(x^3)\left(\frac{d}{dx}(\cos(x^3)\right) = 2\cos(x^3)(-\sin(x^3)\left(\frac{d}{dx}(x^3)\right)$$

$$= 2\cos(x^3)(-\sin(x^3)(3x^2)$$

19. If $f(x) = \ln\left|x^2 - 1\right|$, then $f'(x) =$

(A) $\left|\dfrac{2x}{x^2 - 1}\right|$

(B) $\dfrac{2x}{\left|x^2 - 1\right|}$

(C) $\dfrac{2|x|}{x^2 - 1}$

(D) $\dfrac{2x}{x^2 - 1}$

(E) $\dfrac{1}{x^2 - 1}$

76. If $f(x) = \dfrac{e^{2x}}{2x}$, then $f'(x) =$

(A) 1

(B) $\dfrac{e^{2x}(1 - 2x)}{2x^2}$

(C) e^{2x}

(D) $\dfrac{e^{2x}(2x + 1)}{x^2}$

(E) $\dfrac{e^{2x}(2x - 1)}{2x^2}$

4. $\dfrac{d}{dx}\left(xe^{\ln x^2}\right) =$

(A) $1 + 2x$ (B) $x + x^2$ (C) $3x^2$

(D) x^3 (E) $x^2 + x^3$

19. D $f(x) = \ln\left|x^2 - 1\right|$; $f'(x) = \frac{1}{x^2 - 1} \cdot \frac{d}{dx}(x^2 - 1) = \frac{2x}{x^2 - 1}$

76. E $f(x) = \frac{e^{2x}}{2x}$; $f'(x) = \frac{2e^{2x} \cdot 2x - 2e^{2x}}{4x^2} = \frac{e^{2x}(2x - 1)}{2x^2}$

4. C $e^{\ln x^2} = x^2$; so $xe^{\ln x^2} = x^3$ and $\frac{d}{dx}(x^3) = 3x^2$

5. If $f(x) = (x-1)^{\frac{3}{2}} + \frac{e^{x-2}}{2}$, then $f'(2) =$

(A) 1 (B) $\frac{3}{2}$ (C) 2

(D) $\frac{7}{2}$ (E) $\frac{3+e}{2}$

15. If $F(x) = \int_0^x \sqrt{t^3+1}\, dt$, then $F'(2) =$

(A) -3 (B) -2 (C) 2

(D) 3 (E) 18

16. If $f(x) = \sin\left(e^{-x}\right)$, then $f'(x) =$

(A) $-\cos(e^{-x})$

(B) $\cos(e^{-x}) + e^{-x}$

(C) $\cos(e^{-x}) - e^{-x}$

(D) $e^{-x}\cos(e^{-x})$

(E) $-e^{-x}\cos(e^{-x})$

5. C

$$f(x)=(x-1)^{\frac{3}{2}}+\frac{1}{2}e^{x-2};\ f'(x)=\frac{3}{2}(x-1)^{\frac{1}{2}}+\frac{1}{2}e^{x-2};\ f'(2)=\frac{3}{2}+\frac{1}{2}=2$$

15 D

By the Fundamental Theorem of Calculus, $F'(x)=\sqrt{x^3+1}$, thus $F'(2)=\sqrt{2^3+1}=\sqrt{9}=3$.

16. E

$$f'(x)=\cos(e^{-x})\cdot\frac{d}{dx}(e^{-x})=\cos(e^{-x})\left(e^{-x}\cdot\frac{d}{dx}(-x)\right)=-e^{-x}\cos(e^{-x})$$

28. If $f(x) = \tan(2x)$, then $f'\left(\frac{\pi}{6}\right) =$

(A) $\sqrt{3}$ (B) $2\sqrt{3}$ (C) 4

(D) $4\sqrt{3}$ (E) 8

9. If $y = x^x$, then $\frac{dy}{dx} =$

(A) x^x

(B) $x^x(1 + \ln x)$

(C) $x(x^{x-1})$

(D) x^{x-1}

(E) none of the above

No Calculators

16. If $y = \cos^2 x - \sin^2 x$, then $y' =$

(A) -1

(B) 0

(C) $-2(\cos x + \sin x)$

(D) $2(\cos x + \sin x)$

(E) $-4(\cos x)(\sin x)$

No Calculator

28. E

$$f'(x) = \sec^2(2x) \cdot \frac{d}{dx}(2x) = 2\sec^2(2x);\; f'\left(\frac{\pi}{6}\right) = 2\sec^2\left(\frac{\pi}{3}\right) = 2(4) = 8$$

9. B p. 4

There are two ways to proceed with this differentiation.

I. Use the definition of a^x as $e^{x \ln a}$. From this we have:

$$y = x^x = e^{x \ln x}$$
$$\frac{dy}{dx} = e^{x \ln x}\left[x \cdot \frac{1}{x} + \ln x\right] = x^x\left[1 + \ln x\right]$$

II. Start with the function $y = x^x$ and take the natural logarithm of both sides. Then differentiate.

$$y = x^x \Rightarrow \ln y = x \ln x$$
$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = x \cdot \frac{1}{x} + 1 \cdot \ln x$$
$$\Rightarrow \frac{dy}{dx} = y \cdot \left[1 + \ln x\right] = x^x\left[1 + \ln x\right]$$

16. E p. 6

$$y = \cos^2 x - \sin^2 x$$
$$y' = -2\cos x \sin x - 2\sin x \cos x = -4 \sin x \cos x$$

22. $\frac{d}{dx}(\ln e^{3x}) =$

(A) 1

(B) 3

(C) 3x

(D) $\frac{1}{e^{3x}}$

(E) $\frac{3}{e^{3x}}$

No Calculator

No Calculator

12. Let $f(x) = \frac{\ln e^{2x}}{x - 1}$ for $x > 1$. If g is the inverse of f, then $g'(3) =$

(A) 2 (B) 1 (C) 0 (D) –1 (E) –2

22. B p. 8

$$\frac{d}{dx}(\ln e^{3x}) = \frac{d}{dx}(3x \ln e) = \frac{d}{dx}(3x) = 3$$

12. E p. 14

$$f(x) = \frac{\ln e^{2x}}{x-1} = \frac{2x}{x-1}.$$

The inverse of this function is found by solving $x = \frac{2y}{y-1}$ for y.

$$x = \frac{2y}{y-1} \Rightarrow xy - x = 2y \Rightarrow xy - 2y = x \Rightarrow y(x-2) = x$$

$$\Rightarrow g(x) = y = \frac{x}{x-2}$$

Then $g'(x) = \frac{(x-2)-x}{(x-2)^2} = \frac{-2}{(x-2)^2}$. Hence $g'(3) = -2$.

18. If $h(x) = |x - 2| - 3$, then the value of $h'(2)$ is

(A) 0

(B) 1

(C) –1

(D) 2

(E) does not exist

No Calculator

19. The derivative of $\sqrt{x} - \dfrac{1}{x\sqrt[3]{x}}$ is

(A) $\frac{1}{2}x^{-1/2} - x^{-4/3}$

(B) $\frac{1}{2}x^{-1/2} + \frac{4}{3}x^{-7/3}$

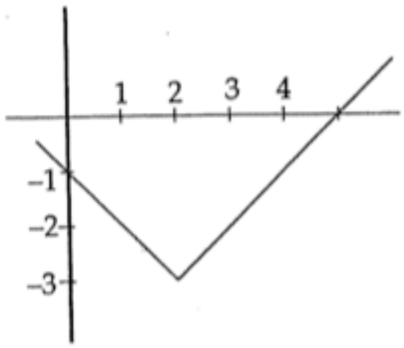
(C) $\frac{1}{2}x^{-1/2} - \frac{4}{3}x^{-1/3}$

(D) $-\frac{1}{2}x^{-1/2} + \frac{4}{3}x^{-7/3}$

(E) $-\frac{1}{2}x^{-1/2} - \frac{4}{3}x^{-1/3}$

No Calculator

18. E p. 29



The graph of $h(x) = |x-2| - 3$ is shown to the left. $h'(2)$ is the slope of the tangent line at $x = 2$, provided that slope exists. Since the left-hand slope and the right-hand slope of the graph of h are different as x approaches 2, the slope of the tangent line does not exist at $x = 2$.

19. B p. 29

$$y = \sqrt{x} - \frac{1}{x\sqrt[3]{x}} = x^{1/2} - x^{-4/3}$$

$$\frac{dy}{dx} = \frac{1}{2}x^{-1/2} + \frac{4}{3}x^{-7/3}$$

Calculator Active

1. Which of the following functions have a derivative at $x = 0$?

I. $y = |x^3 - 3x^2|$

II. $y = \sqrt{x^2 + .01} - |x - 1|$

III. $y = \dfrac{e^x}{\cos x}$

(A) None (B) II only (C) III only (D) II and III only (E) I, II, III

No Calculator

9. If $f(x) = \text{Arctan}(\frac{1}{x})$, then $f'(x) =$

(A) $\dfrac{-1}{x^2 + x}$

(B) $\dfrac{x}{\sqrt{x^2 - 1}}$

(C) $\dfrac{x^2}{x^2 + 1}$

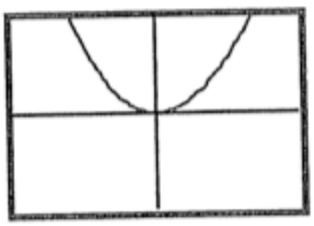
(D) $\dfrac{1}{x^2 + 1}$

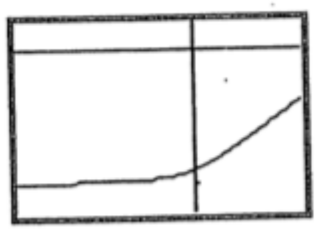
(E) $\dfrac{-1}{x^2 + 1}$

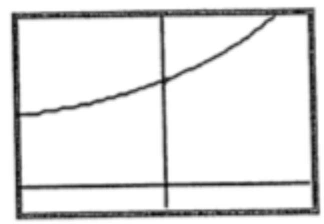
1. E P. 32

A function has a derivative at a particular x-coordinate if its graph is smooth there.

I. $y = \left|x^3 - 3x^2\right|$ II. $y = \sqrt{x^2 + .01} - \left|x - 1\right|$ III. $y = \dfrac{e^x}{\cos x}$







All three of these graphs are smooth at $x = 0$.

9. In general, if $f(x) = \text{Arctan}\, u$, then $f'(x) = \dfrac{du}{1+u^2}$.

Since $f(x) = \text{Arctan}(\frac{1}{x})$, then $f'(x) = \dfrac{-1/x^2}{1+(1/x)^2} = \dfrac{-1/x^2}{(x^2+1)/x^2} = \boxed{\dfrac{-1}{x^2+1}}$

The correct choice is (E).

No Calculator

15. If $y = 5^{(x^3-2)}$, then $\frac{dy}{dx} =$

(A) $(x^3 - 2)5^{(x^3-2)}$

(B) $3x^2(\ln 5)5^{(x^3-2)}$

(C) $(3x^2)5^{(x^3-2)}$

(D) $(\ln 5)5^{(x^3-2)}$

(E) $x^3(\ln 5)5^{(x^3-2)}$

No Calculator

24. If $f(x) = \sqrt{e^{2x} + 1}$, then $f'(0) =$

(A) $\frac{\sqrt{2}}{4}$ (B) $\sqrt{2}$ (C) $\frac{\sqrt{2}}{2}$ (D) 1 (E) $-\frac{\sqrt{2}}{2}$

15. In general, if $y = a^u$, then $y' = a^u \cdot \ln a \cdot du$

$$\text{If } y = 5^{(x^3-2)} \text{ then } \frac{dy}{dx} = 5^{(x^3-2)} \cdot \ln 5 \cdot 3x^2$$

The correct choice is (B).

24. $f(x) = \sqrt{e^{2x}+1} = \left(e^{2x}+1\right)^{\frac{1}{2}}$ and $f'(x) = \frac{1}{2}\left(e^{2x}+1\right)^{-\frac{1}{2}} \cdot 2e^{2x} = \dfrac{e^{2x}}{\sqrt{e^{2x}+1}}$

Therefore, $f'(0) = \dfrac{e^0}{\sqrt{e^0+1}} = \dfrac{1}{\sqrt{2}} = \dfrac{\sqrt{2}}{2}$

The correct choice is (C).