Parametric equations express x and y in terms of a parameter, t.

Example 1. Graph the parametric equations $x = 2t - 8$ and $y = -t + 4$, for $0 \le t \le 5$.
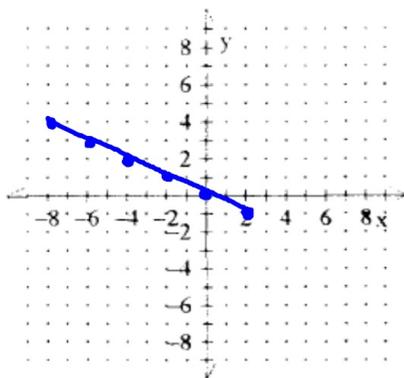
t	x	y
0	-8	4
1	-6	3
2	-4	2
3	-2	1
4	0	0
5	2	-1

Example 2. Rewrite the pair of parametric equations as an *xy*-equation. Find the domain.

$$x = -2t + 1, \quad y = -4t + 6, \quad \text{for } 0 \le t \le 5.$$

$t =$

$x - 1 = -2t$

$\dfrac{x-1}{-2} = t$

$y = -4\left(\dfrac{x-1}{-2}\right) + 6$

$y = 2x + 4$

$$\begin{array}{cc} t & x \end{array}$$

$x = -2 \cdot 0 + 1 = 1$

$x = -2 \cdot 5 + 1 = -9$
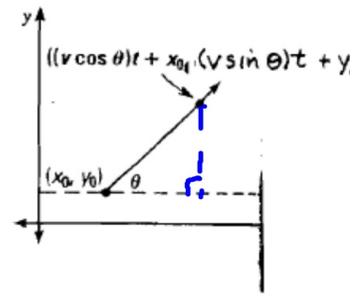
Domain: $-9 \le x \le 1$

Linear Motion

Consider an object that is moving with constant speed v along a straight line that makes an angle θ measured counterclockwise from a line parallel to the x-axis. The position of the object at any time t can be represented by the parametric equations

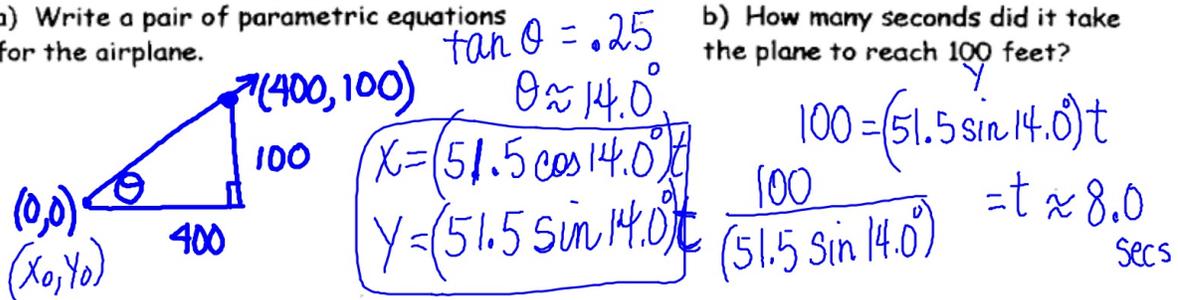$$x = (v \cos \theta)t + x_0$$
$$y = (v \sin \theta)t + y_0$$

where (x_0, y_0) is the object's location when $t = 0$.

$((v \cos \theta)t + x_0, (v \sin \theta)t + y_0)$

(x_0, y_0) θ t

Example 3. A model airplane takes off and is 100 feet high and 400 feet away horizontally. Assume the plane is flying in a straight line at 51.5 ft/sec. (v)

a) Write a pair of parametric equations for the airplane.

(400, 100)
100
(0,0) θ
(x_0, y_0)
400

$\tan \theta = .25$
$\theta \approx 14.0°$

$$X = (51.5 \cos 14.0°)t$$
$$Y = (51.5 \sin 14.0°)t$$

b) How many seconds did it take the plane to reach 100 feet?

$$100 = (51.5 \sin 14.0°)t$$
$$\frac{100}{(51.5 \sin 14.0°)} = t \approx 8.0 \text{ secs}$$

Projectile Motion

Parametric equations can also be used to model nonlinear motion in a plane. For instance, consider an object that is projected into the air at an angle θ with an initial speed v. The object's parabolic path can be modeled with the parametric equations

$$x = (v \cos \theta)t + x_0$$
$$y = -\frac{1}{2}gt^2 + (v \sin \theta)t + y_0$$

where (x_0, y_0) is the object's location when $t = 0$. The constant g is the acceleration due to gravity. Its value is 32 ft/sec² or 9.8 m/sec². (Note that this model neglects air resistance.)

$((v \cos \theta)t + x_0, -\frac{1}{2}gt^2 + (v \sin \theta)t + y_0)$

(x_0, y_0) θ
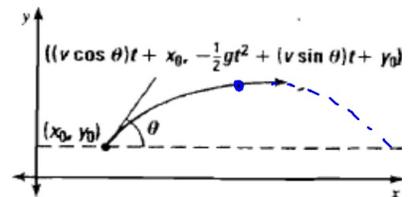
Example 4. At a watermelon seed spitting contest, a seed is launched with an initial velocity of 35 ft/sec, at an angle of 35°, and from an initial height of 5 feet.

a) Write a set of parametric equations this motion.

$\theta \cdot (0,5)$
5
$(x, 0)$

$$X = (35 \cos 35°)t$$
$$Y = -16t^2 + (35 \sin 35°)t + 5$$

b) How far does the seed travel? $X \approx 43.0$ ft

$$0 = -16t^2 + 20.1t + 5$$
$$t = \frac{-20.1 \pm \sqrt{20.1^2 - 4(-80)}}{-32}$$
$$t \approx 1.5 \text{ sec}$$

$$X \approx (35 \cos 35°)(1.5)$$