6-4 Factoring / Solving Polynomial Equations

Std. 4.0

Examples: factor completely

ex. 1 $30x^4 + 5x^2 - 60 = 5(6x^4 + x^2 - 12)$

product $-72x^4$, sum $9x^2, -8x^2$

$= 5(3x^2 - 4)(2x^2 + 3)$

ex. 2 $25x^8 - 30x^4 + 9$

$(5x^4)^2 \quad 2(15x^4)\ (3)^2 \quad = (5x^4 - 3)^2$

ex. 3 $3x^{20n} - 48 = 3(x^{20n} - 16)$

$= 3(x^{10n} + 4)(x^{10n} - 4)$

$= 3(x^{10n} + 4)(x^{5n} - 2)(x^{5n} + 2)$

Factoring by Grouping

ex. 4 $x^2y^2 - 3x^2 - 4y^2 + 12$

$(x^2y^2 - 4y^2) + (-3x^2 + 12)$

$y^2(x^2 - 4) - 3(x^2 - 4)$ $\quad a(c) - b(c)$

$(y^2 - 3)(x^2 - 4) = (y^2 - 3)(x + 2)(x - 2)$ $\quad (a - b)(c)$

ex. 5 $4x^4 + 4x^3 - 8x^2 - 8x$

$4x(x^3 + x^2 - 2x - 2) = 4x[(x^3 + x^2) + (-2x - 2)]$

$4x[x^2(x + 1) - 2(x + 1)]$

$4x(x^2 - 2)(x + 1)$

Factoring Cubes

cube roots — prime

sum of cubes: $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

difference of cubes: $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

ex. 6 $x^3 + 64$

$(x)^3 + (4)^3 = (x+4)(x^2 - 4x + 16)$

4^2

ex. 7 $512x^6 - y^3$

$(8x^2)^3 - (y)^3 = (8x^2 - y)(64x^4 + 8x^2y + y^2)$

$(8x^2)^2$

$\sqrt[3]{x} \leftarrow 512$

$3 \rightarrow x\sqrt{y} \leftarrow 512$

$y^{1/x}$

$y \boxed{\wedge} x$

$512 \boxed{\wedge} (1/3)$

Find all real solutions:

ex. 8 $2x^5 - 10x^3 = -8x$

$x^5 - 5x^3 + 4x = 0$

$x(x^4 - 5x^2 + 4) = 0$

$x = 0, \pm 2, \pm 1$

$x(x^2 - 4)(x^2 - 1) = 0$

$x^2 = 4 \quad x^2 = 1$

ex. 9 $x^3 = -27$

$x^3 + 27 = 0$

$(x+3)(x^2 - 3x + 9) = 0$

$x = -3$

"i"

$b^2 - 4ac = 9 - 4(9) = -27$