Objectives

Find the slope of a line.

Use slopes to identify parallel and perpendicular lines.

Vocabulary

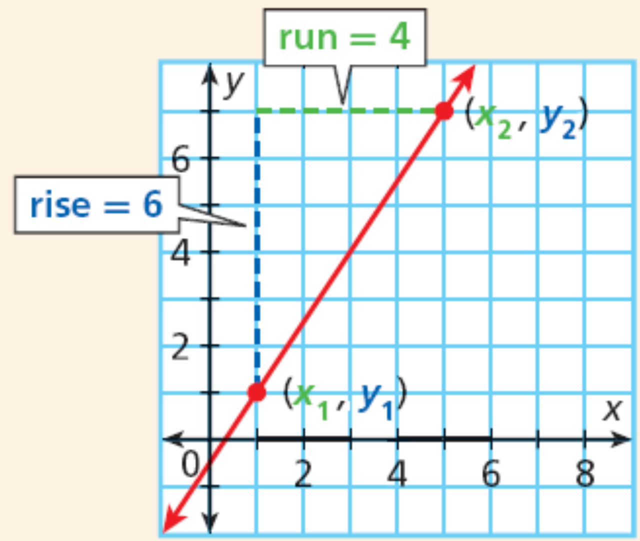
rise

run

slope

The **slope** of a line in a coordinate plane is a number that describes the steepness of the line. Any two points on a line can be used to determine the slope.

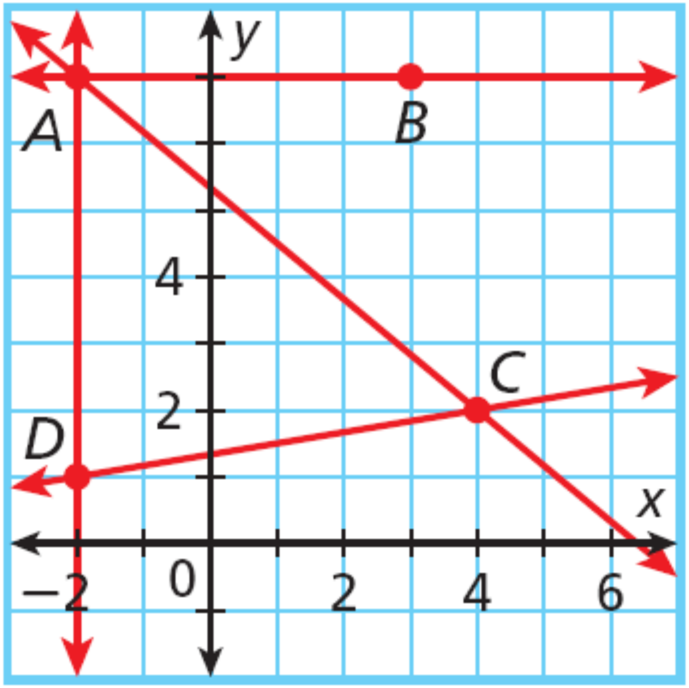
Slope of a Line

DEFINITION	EXAMPLE
The **rise** is the difference in the *y*-values of two points on a line.	
The **run** is the difference in the *x*-values of two points on a line.	
The **slope** of a line is the ratio of the rise to run. If (x_1, y_1) and (x_2, y_2) are any two points on a line, the slope of the line is $m = \frac{y_2 - y_1}{x_2 - x_1}$.	slope $= \frac{6}{4} = \frac{3}{2}$

Example 1A: Finding the Slope of a Line

Use the slope formula to determine the slope of each line.

$\overleftrightarrow{AB}$



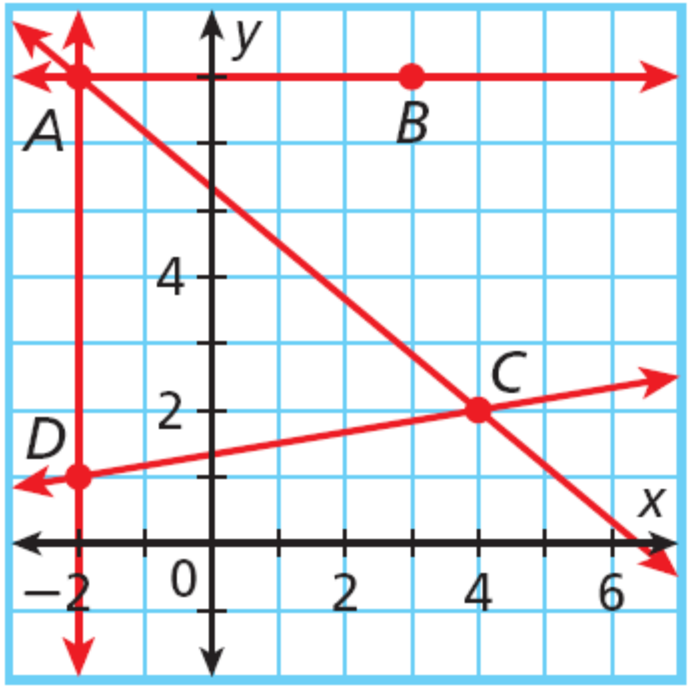
Substitute (–2, 7) for (x_1, y_1) and (3, 7) for (x_2, y_2) in the slope formula and then simplify.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7-7}{3-(-2)} = \frac{0}{5} = 0$$

Example 1B: Finding the Slope of a Line

Use the slope formula to determine the slope of each line.

$\overleftrightarrow{AC}$



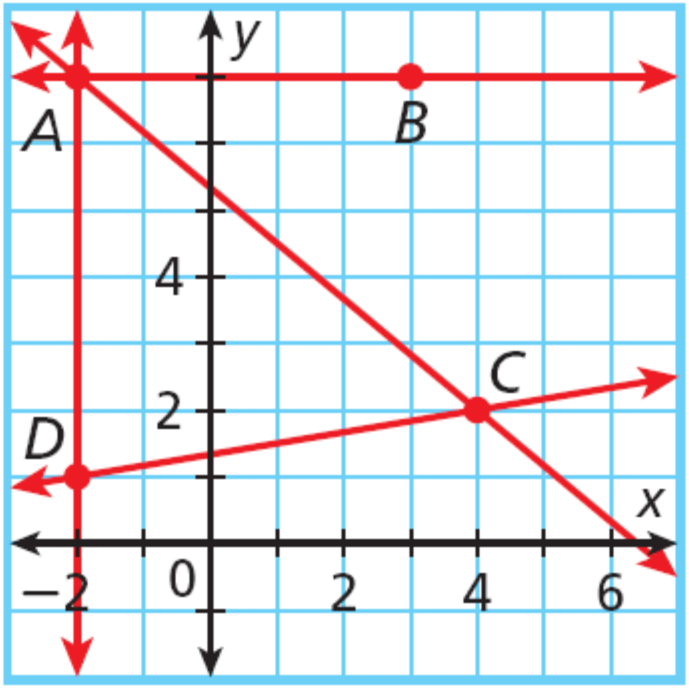
Substitute (–2, 7) for (x_1, y_1) and (4, 2) for (x_2, y_2) in the slope formula and then simplify.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 7}{4 - (-2)} = \frac{-5}{6} = -\frac{5}{6}$$

Example 1C: Finding the Slope of a Line

Use the slope formula to determine the slope of each line.

$\overleftrightarrow{AD}$



Substitute (–2, 7) for (x_1, y_1) and (–2, 1) for (x_2, y_2) in the slope formula and then simplify.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-7}{2-(-2)} = \frac{-6}{0}$$

The slope is undefined.

Remember!

A fraction with zero in the denominator is undefined because it is impossible to divide by zero.

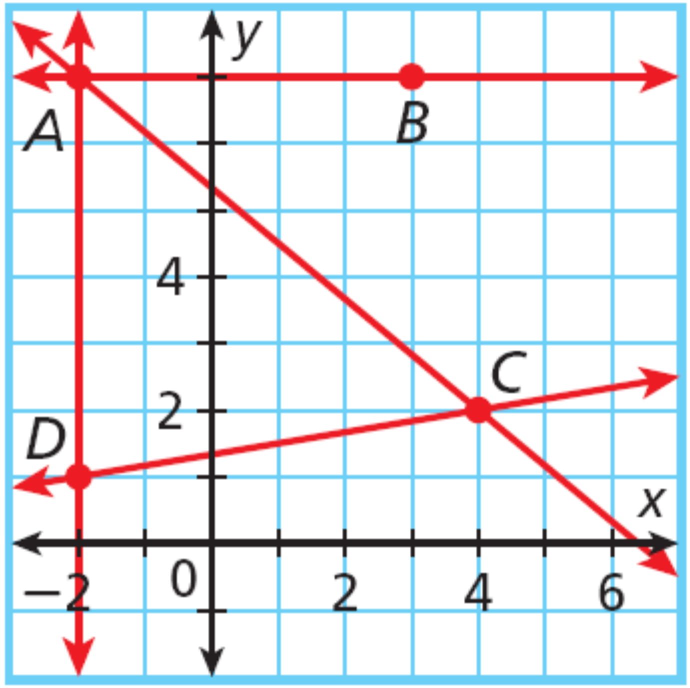
Example 1D: Finding the Slope of a Line

Use the slope formula to determine the slope of each line.

$\overleftrightarrow{CD}$

Substitute (4, 2) for (x_1, y_1) and (–2, 1) for (x_2, y_2) in the slope formula and then simplify.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-2}{-2-4} = \frac{-1}{-6} = \frac{1}{6}$$



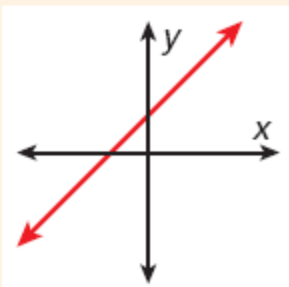
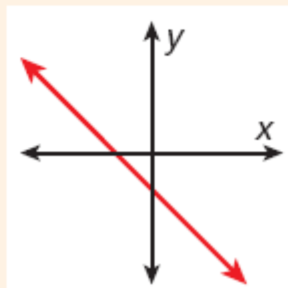
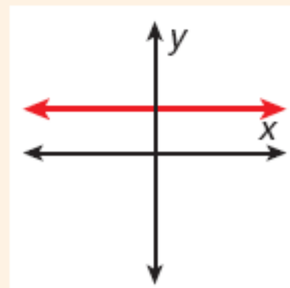
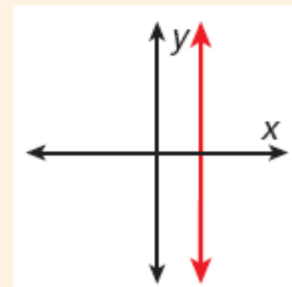
Check It Out! Example 1

Use the slope formula to determine the slope of $\overleftrightarrow{JK}$ through $J(3, 1)$ and $K(2, -1)$.

Substitute (3, 1) for (x_1, y_1) and (2, −1) for (x_2, y_2) in the slope formula and then simplify.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1-1}{2-3} = \frac{-2}{-1} = 2$$

Summary: Slope of a Line

Positive Slope	Negative Slope	Zero Slope	Undefined Slope
			

One interpretation of slope is a *rate of change*. If *y* represents miles traveled and *x* represents time in hours, the slope gives the rate of change in miles per hour.

Slopes of Parallel and Perpendicular Lines

3-5-1 Parallel Lines Theorem

In a coordinate plane, two nonvertical lines are parallel if and only if they have the same slope. Any two vertical lines are parallel.

3-5-2 Perpendicular Lines Theorem

In a coordinate plane, two nonvertical lines are perpendicular if and only if the product of their slopes is −1. Vertical and horizontal lines are perpendicular.

If a line has a slope of $\frac{a}{b}$, then the slope of a perpendicular line is $-\frac{b}{a}$.

The ratios $\frac{a}{b}$ and $-\frac{b}{a}$ are called *opposite reciprocals*.

Caution!

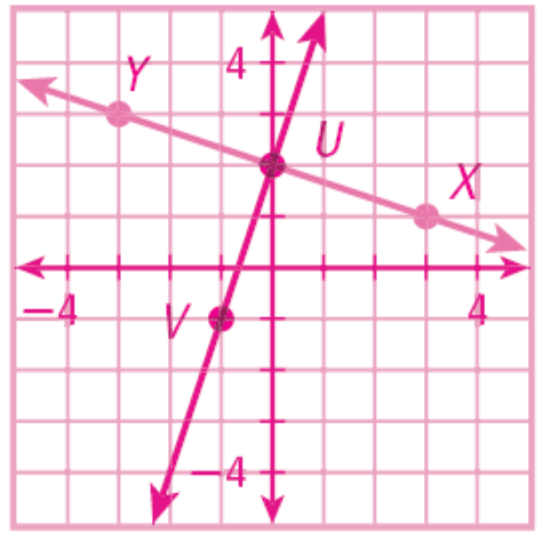
Four given points do not always determine two lines.

Graph the lines to make sure the points are not collinear.

Example 3A: Determining Whether Lines Are Parallel, Perpendicular, or Neither

Graph each pair of lines. Use their slopes to determine whether they are parallel, perpendicular, or neither.

$\overleftrightarrow{UV}$ and $\overleftrightarrow{XY}$ for $U(0, 2)$, $V(–1, –1)$, $X(3, 1)$, and $Y(–3, 3)$



$$\text{slope of } \overleftrightarrow{UV} = \frac{-1-2}{-1-0} = \frac{-3}{-1} = 3$$

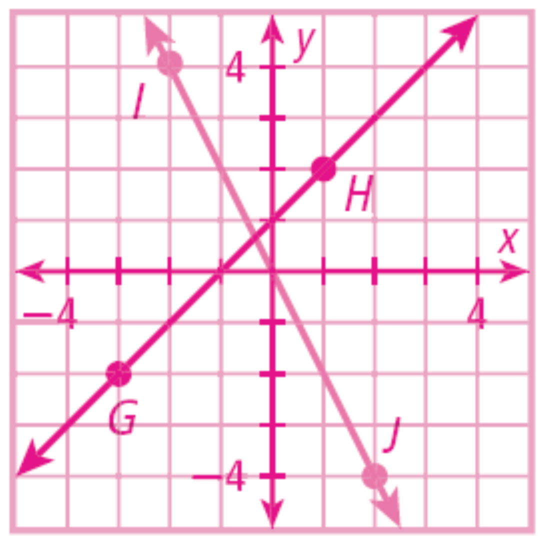
$$\text{slope of } \overleftrightarrow{XY} = \frac{3-1}{-3-3} = \frac{2}{-6} = -\frac{1}{3}$$

The products of the slopes is –1, so the lines are perpendicular.

Example 3B: Determining Whether Lines Are Parallel, Perpendicular, or Neither

Graph each pair of lines. Use their slopes to determine whether they are parallel, perpendicular, or neither.

$\overleftrightarrow{GH}$ and $\overleftrightarrow{IJ}$ for $G(-3, -2)$, $H(1, 2)$, $I(-2, 4)$, and $J(2, -4)$



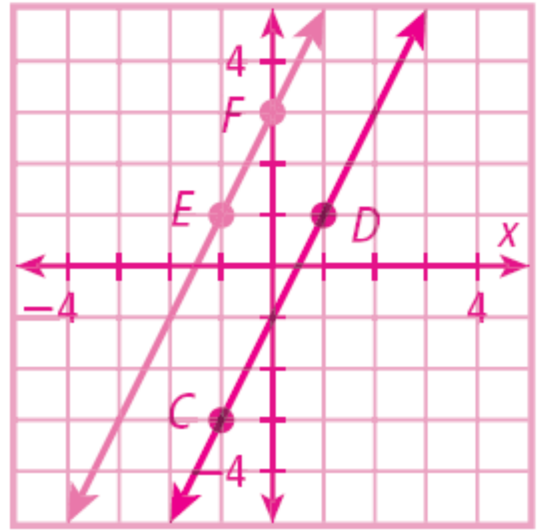
$$\text{slope of } \overleftrightarrow{GH} = \frac{2-(-2)}{1-(-3)} = \frac{4}{4} = 1$$

$$\text{slope of } \overleftrightarrow{IJ} = \frac{-4-4}{2-(-2)} = \frac{-8}{4} = -2$$

The slopes are not the same, so the lines are not parallel. The product of the slopes is not −1, so the lines are not perpendicular.

Example 3C: Determining Whether Lines Are Parallel, Perpendicular, or Neither

Graph each pair of lines. Use their slopes to determine whether they are parallel, perpendicular, or neither.



$\overleftrightarrow{CD}$ and $\overleftrightarrow{EF}$ for $C(-1, -3)$, $D(1, 1)$, $E(-1, 1)$, and $F(0, 3)$

$$\text{slope of } \overleftrightarrow{CD} = \frac{1-(-3)}{1-(-1)} = \frac{4}{2} = 2$$

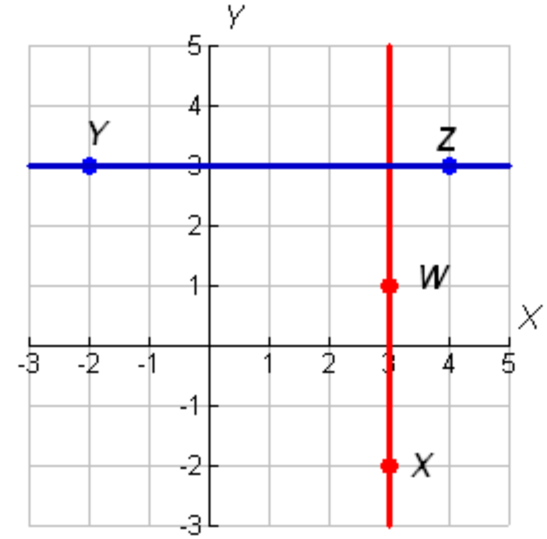
$$\text{slope of } \overleftrightarrow{EF} = \frac{3-1}{0-(-1)} = \frac{2}{1} = 2$$

The lines have the same slope, so they are parallel.

Check It Out! Example 3a

Graph each pair of lines. Use slopes to determine whether the lines are parallel, perpendicular, or neither.

$\overleftrightarrow{WX}$ and $\overleftrightarrow{YZ}$ for $W(3, 1)$, $X(3, -2)$, $Y(-2, 3)$, and $Z(4, 3)$



$$\text{slope of } \overleftrightarrow{WX} = \frac{-2-1}{3-3} = \frac{-3}{0}$$

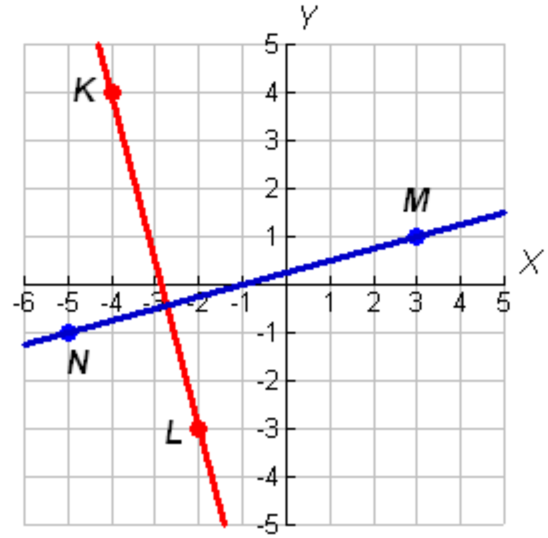
$$\text{slope of } \overleftrightarrow{YZ} = \frac{3-3}{4-(-2)} = \frac{0}{6} = 0$$

Vertical and horizontal lines are perpendicular.

Check It Out! Example 3b

Graph each pair of lines. Use slopes to determine whether the lines are parallel, perpendicular, or neither.

$\overleftrightarrow{KL}$ and $\overleftrightarrow{MN}$ for $K(-4, 4)$, $L(-2, -3)$, $M(3, 1)$, and $N(-5, -1)$



$$\text{slope of } \overleftrightarrow{KL} = \frac{-3-4}{-2-(-4)} = \frac{-7}{2}$$

$$\text{slope of } \overleftrightarrow{MN} = \frac{-1-1}{-5-3} = \frac{-2}{-8} = \frac{1}{4}$$

The slopes are not the same, so the lines are not parallel. The product of the slopes is not −1, so the lines are not perpendicular.

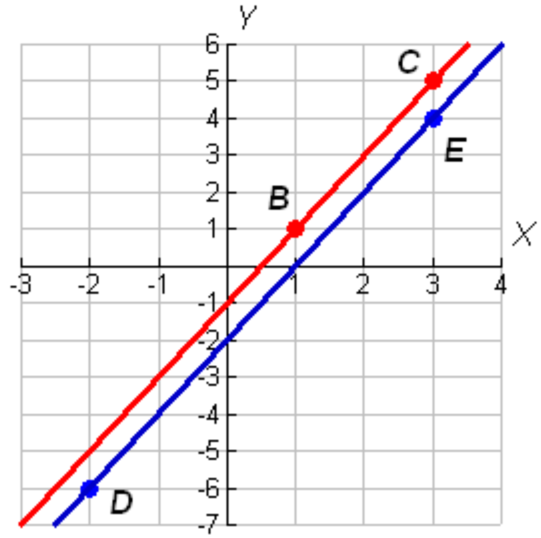
Check It Out! Example 3c

Graph each pair of lines. Use slopes to determine whether the lines are parallel, perpendicular, or neither.

$\overleftrightarrow{BC}$ and $\overleftrightarrow{DE}$ for $B(1, 1)$, $C(3, 5)$, $D(-2, -6)$, and $E(3, 4)$

$$\text{slope of } \overleftrightarrow{BC} = \frac{5-1}{3-1} = \frac{4}{2} = 2$$

$$\text{slope of } \overleftrightarrow{DE} = \frac{4-(-6)}{3-(-2)} = \frac{10}{5} = 2$$



The lines have the same slope, so they are parallel.