

| y | f(y) | y | f(y) |
|--------|------|--------|------|
| 2.002 | | 1.998 | |
| 2.001 | | 1.999 | |
| 2.0001 | | 1.9999 | |

4. $\lim_{x \rightarrow 0^+} f(x)$, where $f(x) = x \ln x$.

| x | 1 | 0.5 | 0.1 | 0.05 | 0.01 | 0.005 | 0.001 |
|------|---|-----|-----|------|------|-------|-------|
| f(x) | | | | | | | |

5. Determine $\lim_{x \rightarrow 0.5} f(x)$ for $f(x)$ as in Figure 9.

6. Determine $\lim_{x \rightarrow 0.5} g(x)$ for $g(x)$ as in Figure 10.

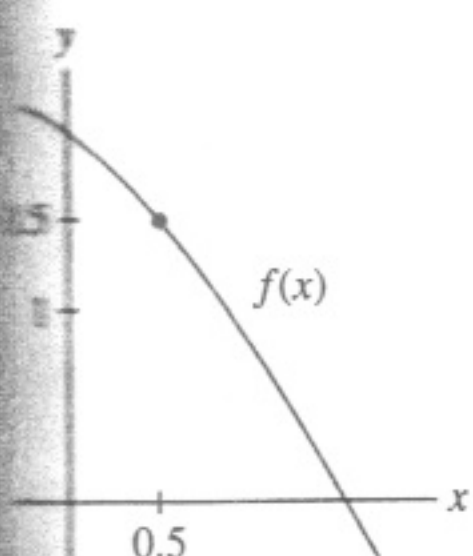


FIGURE 9

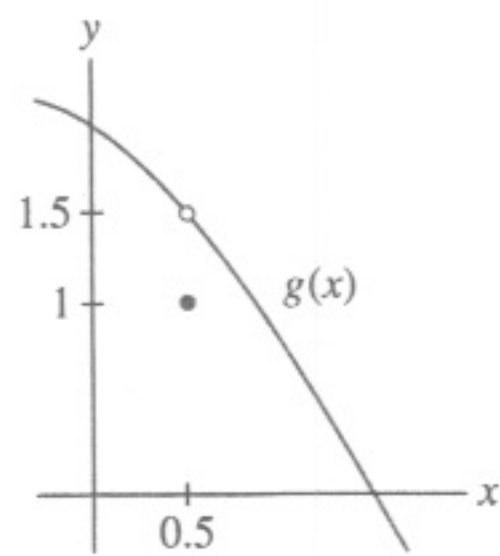


FIGURE 10

In Exercises 7–8, evaluate the limit.

7. $\lim_{x \rightarrow 21} x$

8. $\lim_{x \rightarrow 4.2} \sqrt{3}$

In Exercises 9–16, verify each limit using the limit definition. For example, in Exercise 9, show that $|3x - 12|$ can be made as small as desired by taking x close to 4.

9. $\lim_{x \rightarrow 4} 3x = 12$

10. $\lim_{x \rightarrow 5} 3 = 3$

11. $\lim_{x \rightarrow 3} (5x + 2) = 17$

12. $\lim_{x \rightarrow 2} (7x - 4) = 10$

13. $\lim_{x \rightarrow 0} x^2 = 0$

14. $\lim_{x \rightarrow 0} (3x^2 - 9) = -9$

15. $\lim_{x \rightarrow 0} (4x^2 + 2x + 5) = 5$

16. $\lim_{x \rightarrow 0} (x^3 + 12) = 12$

In Exercises 17–36, estimate the limit numerically or state that the limit does not exist. If infinite, state whether the one-sided limits are ∞ or $-\infty$.

17. $\lim_{x \rightarrow 1} \frac{\sqrt{x} - 1}{x - 1}$

18. $\lim_{x \rightarrow -4} \frac{2x^2 - 32}{x + 4}$

19. $\lim_{x \rightarrow 2} \frac{x^2 + x - 6}{x^2 - x - 2}$

20. $\lim_{x \rightarrow 3} \frac{x^3 - 2x^2 - 9}{x^2 - 2x - 3}$

21. $\lim_{x \rightarrow 0} \frac{\sin 2x}{x}$

22. $\lim_{x \rightarrow 0} \frac{\sin 5x}{x}$

23. $\lim_{\theta \rightarrow 0} \frac{\cos \theta - 1}{\theta}$

24. $\lim_{x \rightarrow 0} \frac{\sin x}{x^2}$

25. $\lim_{x \rightarrow 4} \frac{1}{(x - 4)^3}$

26. $\lim_{x \rightarrow 1^-} \frac{3 - x}{x - 1}$

27. $\lim_{x \rightarrow 3^+} \frac{x - 4}{x^2 - 9}$

28. $\lim_{h \rightarrow 0} \frac{3^h - 1}{h}$

29. $\lim_{h \rightarrow 0} \sin h \cos \frac{1}{h}$

30. $\lim_{h \rightarrow 0} \cos \frac{1}{h}$

31. $\lim_{x \rightarrow 0} |x|^x$

32. $\lim_{x \rightarrow 1^+} \frac{\sec^{-1} x}{\sqrt{x - 1}}$

33. $\lim_{t \rightarrow e} \frac{t - e}{\ln t - 1}$

34. $\lim_{r \rightarrow 0} (1 + r)^{1/r}$

35. $\lim_{x \rightarrow 1^-} \frac{\tan^{-1} x}{\cos^{-1} x}$

36. $\lim_{x \rightarrow 0} \frac{\tan^{-1} x - x}{\sin^{-1} x - x}$

37. The **greatest integer function** is defined by $[x] = n$, where n is the unique integer such that $n \leq x < n + 1$. Sketch the graph of $y = [x]$. Calculate, for c an integer:

(a) $\lim_{x \rightarrow c^-} [x]$

(b) $\lim_{x \rightarrow c^+} [x]$

38. Determine the one-sided limits at $c = 1, 2,$ and 4 of the function $g(x)$ shown in Figure 11, and state whether the limit exists at these points.

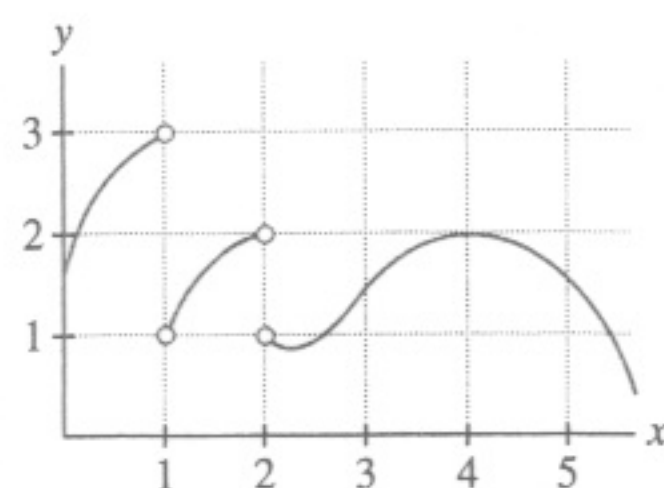


FIGURE 11

In Exercises 39–46, determine the one-sided limits numerically or graphically. If infinite, state whether the one-sided limits are ∞ or $-\infty$, and describe the corresponding vertical asymptote. In Exercise 46, $[x]$ is the greatest integer function defined in Exercise 37.

39. $\lim_{x \rightarrow 0^\pm} \frac{\sin x}{|x|}$

40. $\lim_{x \rightarrow 0^\pm} |x|^{1/x}$

41. $\lim_{x \rightarrow 0^\pm} \frac{x - \sin |x|}{x^3}$

42. $\lim_{x \rightarrow 4^\pm} \frac{x + 1}{x - 4}$

43. $\lim_{x \rightarrow -2^\pm} \frac{4x^2 + 7}{x^3 + 8}$

44. $\lim_{x \rightarrow -3^\pm} \frac{x^2}{x^2 - 9}$

45. $\lim_{x \rightarrow 1^\pm} \frac{x^5 + x - 2}{x^2 + x - 2}$

46. $\lim_{x \rightarrow 2^\pm} \cos\left(\frac{\pi}{2}(x - [x])\right)$

47. Determine the one-sided limits at $c = 2, 4$ of the function $f(x)$ in Figure 12. What are the vertical asymptotes of $f(x)$?

48. Determine the infinite one- and two-sided limits in Figure 13.

3.

| | | | |
|------|---------|---------|----------|
| y | 1.998 | 1.999 | 1.9999 |
| f(y) | 0.59984 | 0.59992 | 0.599992 |

| | | | |
|------|----------|---------|----------|
| y | 2.0001 | 2.001 | 2.02 |
| f(y) | 0.600008 | 0.60008 | 0.601594 |

The limit as $y \rightarrow 2$ is $\frac{3}{5}$.

5. 1.5 7. 21 9. $|3x - 12| = 3|x - 4|$

11. $|(5x + 2) - 17| = |5x - 15| = 5|x - 3|$

13. Suppose $|x| < 1$, so that $|x^2 - 0| = |x + 0||x - 0| = |x||x| < |x|$

15. If $|x| < 1$, $|4x + 2|$ can be no bigger than 6, so $|4x^2 + 2x + 5 - 5| = |4x^2 + 2x| = |x||4x + 2| < 6|x|$

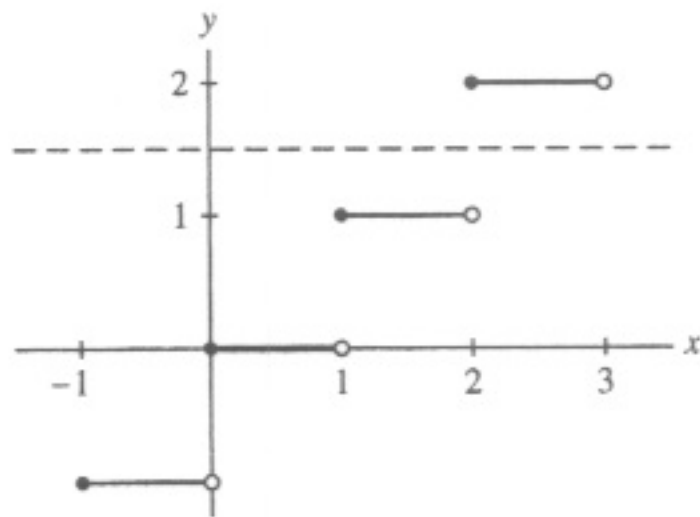
17. $\frac{1}{2}$ 19. $\frac{5}{3}$ 21. 2 23. 0

25. As $x \rightarrow 4^-$, $f(x) \rightarrow -\infty$; similarly, as $x \rightarrow 4^+$, $f(x) \rightarrow \infty$

27. $-\infty$ 29. 0 31. 1

33. 2.718 (The exact answer is e .) 35. ∞

37.



(a) $c - 1$ (b) c

39. $\lim_{x \rightarrow 0^-} f(x) = -1$, $\lim_{x \rightarrow 0^+} f(x) = 1$

41. $\lim_{x \rightarrow 0^-} f(x) = \infty$, $\lim_{x \rightarrow 0^+} f(x) = \frac{1}{6}$

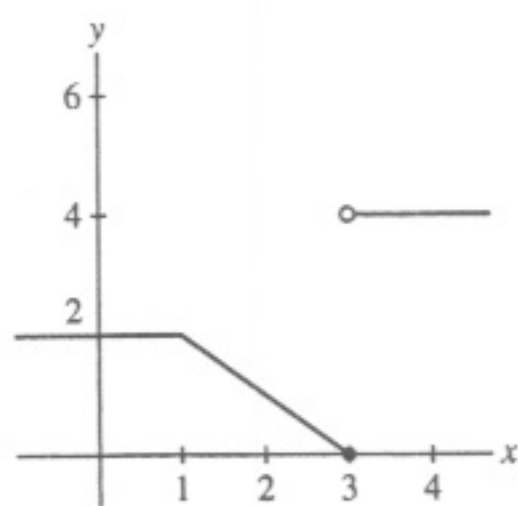
43. $\lim_{x \rightarrow -2^-} \frac{4x^2 + 7}{x^3 + 8} = -\infty$, $\lim_{x \rightarrow -2^+} \frac{4x^2 + 7}{x^3 + 8} = \infty$

45. $\lim_{x \rightarrow 1 \pm} \frac{x^5 + x - 2}{x^2 + x - 2} = 2$

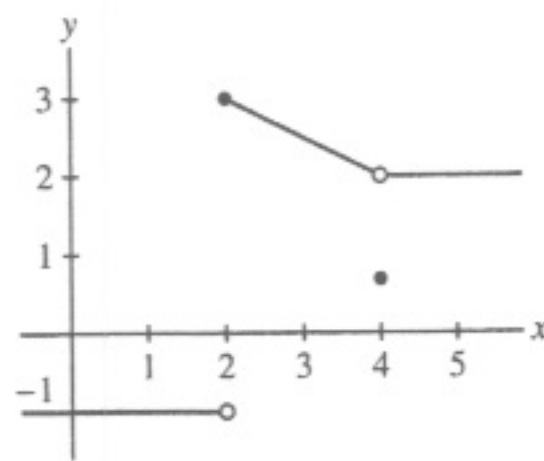
47. $\lim_{x \rightarrow 2^-} f(x) = \infty$ and $\lim_{x \rightarrow 2^+} f(x) = \infty$.
 $\lim_{x \rightarrow 4^-} f(x) = -\infty$ and $\lim_{x \rightarrow 4^+} f(x) = 10$.

The vertical asymptotes are the vertical lines $x = 2$ and $x = 4$.

49.



51.

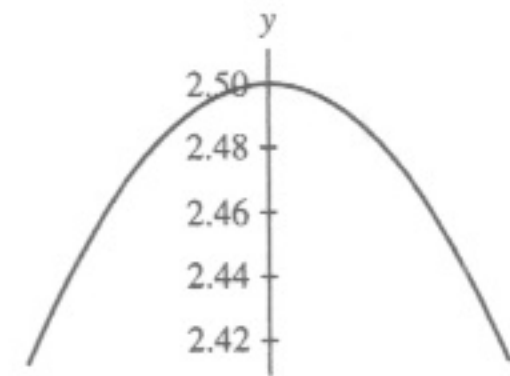


53. $\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^+} f(x) = 3$

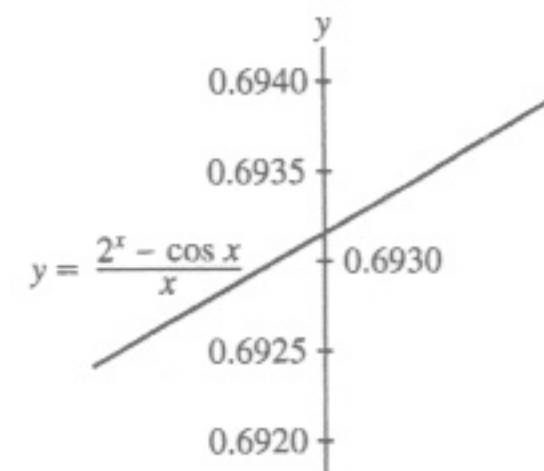
- $\lim_{x \rightarrow 3^-} f(x) = -\infty$
- $\lim_{x \rightarrow 3^+} f(x) = 4$
- $\lim_{x \rightarrow 5^-} f(x) = 2$
- $\lim_{x \rightarrow 5^+} f(x) = -3$

• $\lim_{x \rightarrow 6^-} f(x) = \lim_{x \rightarrow 6^+} f(x) = \infty$

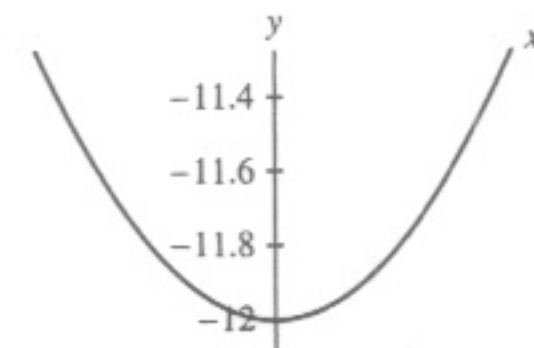
55. $\frac{5}{2}$



57. 0.693 (The exact answer is $\ln 2$.)



59. -12



61. For n even

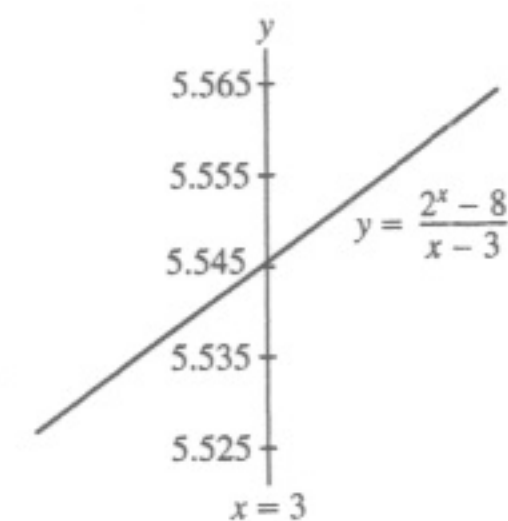
63. (a) No (b) $f(\frac{1}{2^n}) = 1$ for all integers n .

(c) At $x = 1, \frac{1}{3}, \frac{1}{5}, \dots$, the value of $f(x)$ is always -1 .

65. $\lim_{\theta \rightarrow 0} \frac{\sin n\theta}{\theta} = n$

67. $\frac{1}{2}, 2, \frac{3}{2}, \frac{2}{3}$; $\lim_{x \rightarrow 1} \frac{x^n - 1}{x^m - 1} = \frac{n}{m}$

69. (a)



(b) $L = 5.545$.

Section 2.3 Preliminary Questions

1. Suppose $\lim_{x \rightarrow c} f(x)$ and $\lim_{x \rightarrow c} g(x)$ both exist. The Sum Law states that

$$\lim_{x \rightarrow c} (f(x) + g(x)) = \lim_{x \rightarrow c} f(x) + \lim_{x \rightarrow c} g(x).$$