Algebra 2/Trig – Answers to Colored Rev. Wksts

6-7 MODELING REAL-WORLD DATA

34. $A = 4\pi r^2$

$\frac{A}{4\pi} = r^2$

$\sqrt{\frac{A}{4\pi}} = r$

$r(A) = \sqrt{\frac{A}{4\pi}}$; r is the radius for a sphere with a given surface area.

35a. $f(x) \approx 23.96(1.02)^x$

b. $f(85) \approx 23.96(1.02)^{85} \approx 129.2$
About 129.0 million gal will be used.

c. $23.96(1.02)^x = 50$

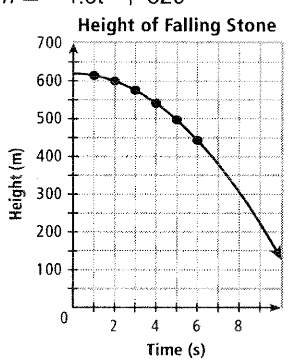
$1.02^x \approx 2.087$

$x \approx 37$

The temperature is about 37°F.

CHAPTER TEST CHAPTER 6

1a. $h = -4.9t^2 + 620$



b. The cliff is $h(0) = 620$ m high.

c. $h = -4.9(10)^2 + 620 = 130$
The stone is 130 m high after 10 seconds.

d. $-4.9t^2 + 620 = 0$

$-4.9t^2 = -620$

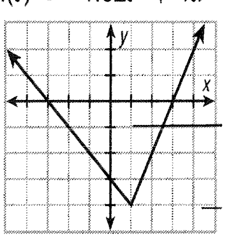
$t^2 \approx 126.53$

$t \approx 11.25$

The stone will hit the ground in about 11.25 s.

2. Using a graphing calculator and quadratic regression, $h(t) \approx -1.62t^2 + 4t + 1$ or $h(t) \approx -1.62t^2 + 4t$.

3.



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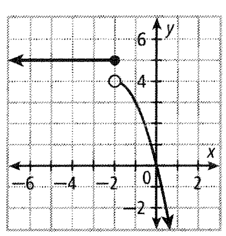

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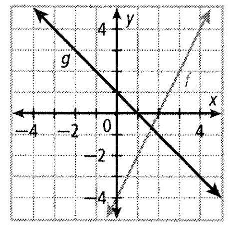
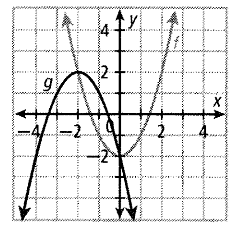


6.



7. $(f - g)(4) = f(4) - g(4)$

$= \left(4(4)^2 - 9\right) - \left(2(4) + 3\right)$

$= 55 - 11$

$= 44$

8. $f(3) = 4(3)^2 - 9 = 27$

$g(27) = 2(27) + 3 = 57$

$g(f(3)) = 57$

9. $(fg)(5) = f(5) \cdot g(5)$

$= \left(4(5)^2 - 9\right) \cdot \left(2(5) + 3\right)$

$= 91 \cdot 13$

$= 1183$

10. $\left(\frac{g}{f}\right)(x) = \frac{g(x)}{f(x)}$

$= \frac{2x + 3}{4x^2 - 9}$

$= \frac{2x + 3}{(2x + 3)(2x - 3)}$

$= \frac{1}{2x - 3}$, where $x \neq \pm\frac{3}{2}$

11. $P(c) = (1 + 10\%) \cdot 150\% \cdot c = 1.65c$

12. $y = 12 - 5x$

$x = 12 - 5y$

$5y = 12 - x$

$y = \frac{12 - x}{5}$

$f^{-1}(x) = \frac{-x + 12}{5}$

D: $\mathbb{R}$;
R: $\mathbb{R}$;
it is a function

13. $y = \frac{10}{x + 4}$

$x = \frac{10}{y + 4}$

$x(y + 4) = 10$

$y + 4 = \frac{10}{x}$

$y = \frac{10}{x} - 4$

$g^{-1}(x) = \frac{10}{x} - 4$

D: $\{x \mid x \neq 0\}$;
R: $\{y \mid y \neq -4\}$;
it is a function

14. $y = \frac{(x + 5)^2}{2}$

$x = \frac{(y + 5)^2}{2}$

$2x = (y + 5)^2$

$\pm\sqrt{2x} = y + 5$

$\pm\sqrt{2x} - 5 = y$

$y = \pm\sqrt{2x} - 5$

D: $\{x \mid x \geq 0\}$;
R: $\mathbb{R}$;
it is not a function

15a. $P(x) = 128{,}800(0.959)^x$

b. $P(20) = 128{,}800(0.959)^{20} \approx 55{,}949$
The average sales price is \$55,949.

7-5 COMPOUND EVENTS

34. Each coupon offers only 1 discount.

35. $P(10\% \cup 15\%) = P(10\%) + P(15\%)$
$$= \frac{1}{3} + \frac{1}{2} = \frac{5}{6}$$

36. $P(\text{red} \cup 5) = P(\text{red}) + P(5) - P(\text{red} \cap 5)$
$$= \frac{26}{52} + \frac{4}{52} - \frac{2}{52} = \frac{7}{13}$$

37. $P(\text{club} \cup \text{heart}) = P(\text{club}) + P(\text{heart})$
$$= \frac{13}{52} + \frac{13}{52} = \frac{1}{2}$$

38. $P(\text{passed} \cup \text{male})$
$$= P(\text{passed}) + P(\text{male}) - P(\text{passed} \cap \text{male})$$
$$= \frac{170}{300} - \frac{120}{300} - \frac{80}{300} = \frac{7}{10}$$

CHAPTER TEST CHAPTER 7

1. $6 \cdot 4 \cdot 8 = 192$
The mannequin can be dressed in 192 ways.

2. ${}_8P_3 = \frac{8!}{(8-3)!}$
$$= 8 \cdot 7 \cdot 6 = 336$$
There are 336 ways to award first, second, and third places.

3. ${}_{30}C_3 = \frac{30!}{3!(30-3)!}$
$$= \frac{30 \cdot 29 \cdot 28}{3 \cdot 2 \cdot 1} = 4060$$

4. $P(4 \text{ jacks, queens, or kings})$
$$= 3 \cdot \frac{4!}{{}_{52}P_4}$$
$$= \frac{3}{270{,}725} \approx 0.000011$$

5. $P(\text{T, T}) = \frac{6}{20} = \frac{3}{10}$

6. Replacing the first letter means that the occurrence of the first selection does not affect the probability of the second selection. The events are independent.
$P(D, \text{ then } J) = \frac{1}{26} \cdot \frac{1}{26} = \frac{1}{676}$

7. Not replacing the vowel means that there will be fewer vowels to chose from, affecting the probability of the second and third selections. The events are dependent.
$P(\text{vowel, then vowel, then vowel})$
$$= \frac{5}{26} \cdot \frac{4}{25} \cdot \frac{3}{24} = \frac{1}{260}$$

8. $P(C \cup \text{even}) = \frac{1}{9} + \frac{2}{9} = \frac{1}{3}$

9. $P(\text{odd} \cup \text{multiple of } 3)$
$$= P(\text{odd}) + P(\text{multiple of } 3) - P(\text{odd} \cap \text{multiple of } 3)$$
$$= \frac{3}{9} + \frac{3}{9} - \frac{2}{9} = \frac{4}{9}$$

10. expected value
$$= 0\left(\frac{7}{20}\right) + 1\left(\frac{5}{20}\right) + 2\left(\frac{4}{20}\right) + 3\left(\frac{3}{20}\right) + 4\left(\frac{1}{20}\right)$$
$$= \frac{13}{10} = 1.3$$

11. The total number of students voting is 349. Calculate joint relative frequencies by dividing each entry by the total number. Then add each row and column to calculate the marginal relative frequencies.

		Plays a Sport		
		Yes	**No**	**Total**
Mascot	**Aardvark**	$\frac{9}{349} \approx 0.026$	$\frac{75}{349} \approx 0.215$	0.241
	Fruit bat	$\frac{35}{349} \approx 0.100$	$\frac{56}{349} \approx 0.160$	0.260
	Plankton	$\frac{51}{349} \approx 0.146$	$\frac{123}{349} \approx 0.352$	0.498
	Total	0.272	0.727	1

12. The marginal relative frequency for the row with the condition "Fruit bat" is about 0.260, or 26.0%. Out of these, about 0.100, or 10.0% also play on a sports team. The conditional relative frequency is $\frac{0.100}{0.260} \approx$ 0.385, or about 38.5%.

13. The marginal relative frequency for the column with the condition "Plays a Sport" is about 0.272, or 27.2%. Out of these, about 0.100, or 10.0%, also voted for fruit bat. The conditional relative frequency is $\frac{0.100}{0.272} \approx 0.368$, or about 36.8%.

14. The marginal relative frequency for the column with the condition "Plays a Sport" is about 0.272, or 27.2%. Out of these, those who did not vote for fruit bat are $0.026 + 0.146 = 0.172$. The conditional relative frequency is $\frac{0.172}{0.272} \approx 0.632$, or about 63.2%.

8-7 FITTING TO A NORMAL DISTRIBUTION

23. $z = \frac{73-77}{4} = -1$; "At least" indicates $\geq$ so the area to the right of z is needed. The probability is $1 - 0.16 = 0.84$.

24. $z = \frac{83-77}{4} = 1.5$; "At most" indicates $\leq$ so the area to the left of z is needed. The probability is 0.93.

25. $z = \frac{69-77}{4} = -2$ and $z = \frac{75-77}{4} = -0.5$. Use the area to the left of both z-values, then subtract the overlapping part. The area to the left of -2 is 0.02 and the area to the left of -0.5 is 0.31. Subtracting yields a probability of $0.31 - 0.02 = 0.29$.

26. $z = \frac{425-400}{25} = 1$; "Greater than" means using the area to the right of z. The probability is $1 - 0.84 = 0.16$.

27. $z = \frac{350-400}{25} = -2$ and $z = \frac{412.5-400}{25} = 0.5$. Use the area to the left of both z-values, then subtract the overlapping part. The area to the left of -2 is 0.02 and the area to the left of 0.5 is 0.69. Subtracting yields a probability of $0.69 - 0.02 = 0.67$.

28. $z = \frac{375-400}{25} = -1$; "Less than" means using the area to the left of z, so the probability is 0.16. $z = \frac{450-400}{25} = 2$; "Greater than" means using the area to the right of z, so the probability is $1 - 0.98 = 0.02$. Because $P(A \text{ or } B) = P(A) + P(B)$, then the probability that $x < 375$ or $x > 450$ is $0.16 + 0.02 = 0.18$.

8-8 ANALYZING DECISIONS

29. $EV = \frac{1}{6}(1) + \frac{2}{6}(2) + \frac{1}{6}(3) + \frac{2}{6}(5) = 3$

30. $EV = \frac{1}{6}(1) + \frac{3}{6}(2) + \frac{1}{6}(3) + \frac{1}{6}(5) = 2.5$

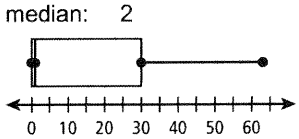
31. $EV = 26\frac{2}{3}$ cents;

There are 3 possibilities for the two coins: a quarter and a dime, a dime and a nickel, or a quarter and a nickel. Each is equally likely, with a probability of $\frac{1}{3}$.

Using the total value for each pair of coins: $EV = \frac{1}{3}(0.35) + \frac{1}{3}(0.15) + \frac{1}{3}(0.30) = 26\frac{2}{3}$.

CHAPTER TEST CHAPTER 8

1. minimum: 0
maximum: 63
median: 2
first quartile: 0.5
third quartile: 30
IQR: 29.5



2. No; mean $\approx$ 15.6; standard deviation $\approx$ 20.5; 3 standard deviations above the mean $\approx$ 77.1, and $77.1 > 63$.

3. 12; the mean decreases from 104.4 to 96.7, and the standard deviation increases from $\approx$ 10.6 to $\approx$ 27.5.

4. $(3x + y)^4$
$= {}_4C_0(3x)^4y^0 + {}_4C_1(3x)^3y^1 + {}_4C_2(3x)^2y^2 + {}_4C_3(3x)^1y^3 + {}_4C_4(3x)^0y^4$
$= 1 \cdot 81x^4 + 4 \cdot 27x^3y + 6 \cdot 9x^2y^2 + 4 \cdot 3xy^3 + 1 \cdot y^4$
$= 81x^4 + 108x^3y + 54x^2y^2 + 12xy^3 + y^4$

5. $P(2) = {}_{10}C_2(0.15)^2(0.85)^8$
≈ 0.28

6. $P(\text{at least } 2)$
$= 1 - P(0 \text{ or } 1)$
$= 1 - \left[{}_{10}C_0(0.15)^0(0.85)^{10} + {}_{10}C_1(0.15)^1(0.85)^9\right]$
≈ 0.46

7. $\frac{5}{30} = \frac{x}{372}$; $x = 62$

8. Since it would be unethical to risk the respiratory systems of participants in an experiment, an observational study would be the best way to study this topic.

9. Since the machine is unlikely to cause any damage, an experiment is acceptable.

10. $z = \frac{0.161-0.17}{\frac{0.03}{\sqrt{50}}} \approx -2.12$; Because $-2.12 > 1.96$, there is enough evidence to reject the claim.

11. simple random

12. Because $\mu + \sigma$ is the mean plus one standard deviation, this can be represented by a z-value of 1. "Less than" indicates finding the area to the left of the z-value, so the probability is 0.84.

13. $EV = \frac{5}{30}(1) + \frac{5}{30}(-1) + \frac{5}{30}(4) + \frac{5}{30}(-2) + \frac{5}{30}(-6) + \frac{5}{30}(13) + \frac{5}{30}(-4.5) = 0.6875$

60. $r = \frac{1}{3} < 1$. $a_1 = \frac{9}{3^1} = 3$.

$S = \frac{3}{1 - \frac{1}{3}} = \frac{3}{\frac{2}{3}} = \frac{9}{2} = 4.5$

61. $r = \frac{3}{5} < 1$. $a_1 = -7\left(\frac{3}{5}\right) = -\frac{21}{5}$.

$S = \frac{-\frac{21}{5}}{1 - \frac{3}{5}} = -\frac{\frac{21}{5}}{\frac{2}{5}} = -\frac{21}{2} = -10.5.$

62. $a_1 = (-1)^2\left(\frac{1}{8^1}\right) = \frac{1}{8}$. $r = -\frac{1}{8}$. $\left|-\frac{1}{8}\right| < 1$

$S = \frac{\frac{1}{8}}{1 - \left(-\frac{1}{8}\right)} = \frac{\frac{1}{8}}{\frac{9}{8}} = \frac{1}{9}.$

63. $r = \frac{4}{3} > 1$. So, no sum exists.

64. Base Case: Prove it is true for $n = 1$.

$2^1 = 2^{1+1} - 2 = 2.$

Thus, it is true for the base case.

Assume it is true for a natural number k.

Thus, $2 + 4 + ... + 2^k = 2^{k+1} - 2$.

Look at $n = k + 1$.

$2 + 4 + ... + 2^k + 2^{k+1}$

$= 2^{k+1} - 2 + 2^{k+1}$ (By Step 2)

$= 2(2^{k+1}) - 2$

$= 2^{k+2} - 2$

$= 2^{(k+1)+1} - 2.$

Thus, by induction, $2 + ... + 2^n = 2^{n+1} - 2$

65. Base Case: Prove true for $n = 1$

$5^{1-1} = \frac{5^1 - 1}{4} = 1.$

Thus, it is true for the base case.

Assume it is true for a natural number k.

Thus, $1 + 5 + ... + 5^{k-1} = \frac{5^k - 1}{4}$

Look at $n = k + 1$.

$1 + ... + 5^{k-1} + 5^{k+1-1}$

$= \frac{5^k - 1}{4} + 5^k$

$= \frac{5^k - 1}{4} + \frac{4(5)^k}{4}$

$= \frac{5^k + 4(5)^k - 1}{4}$

$= \frac{(4+1)5^k - 1}{4}$

$= \frac{5(5)^k - 1}{4}$

$= \frac{5^{k+1} - 1}{4}$

Thus, by induction, $1 + 5 + ... + 5^{n-1} = \frac{5^n - 1}{4}$

66. Base Case: Prove it is true for $n = 1$.

$\frac{1}{4(1^2) - 1} = \frac{1}{2(1) + 1} = \frac{1}{3}$

Thus, it is true for the base case.

Assume it is true for a natural number k.

Thus, $\frac{1}{3} + ... + \frac{1}{4k^2 - 1} = \frac{k}{2k+1}$

Look at $n = k + 1$.

$\frac{1}{3} + ... + \frac{1}{4k^2 - 1} + \frac{1}{4(k+1)^2 - 1}$

$= \frac{k}{2k+1} + \frac{1}{4(k+1)^2 - 1}$

$= \frac{k}{2k+1} + \frac{1}{4k^2 + 8k + 3}$

$= \frac{k}{2k+1} + \frac{1}{(2k+1)(2k+3)}$

$= \frac{k(2k+3)}{(2k+1)(2k+3)} + \frac{1}{(2k+1)(2k+3)}$

$= \frac{2k^2 + 3k + 1}{(2k+1)(2k+3)}$

$= \frac{(2k+1)(k+1)}{(2k+1)(2k+3)}$

$= \frac{k+1}{2k+3}$

$= \frac{k+1}{2(k+1)+1}.$

Thus, by induction $\frac{1}{3} + ... + \frac{1}{4n^2 - 1} = \frac{n}{2n+1}$

67a. $a_1 = 9$. $r = 0.85$. Thus, $S = \sum_{k=1}^{\infty} 9(0.85)^{k-1}$

b. $0.85 < 1$. Thus, this is an infinite converging geometric series. So:

$S = \frac{a_1}{1 - r} = \frac{9}{1 - 0.85} = \frac{9}{0.15} = 60$ ft.

CHAPTER TEST

CHAPTER 9

1.

n	$n^2 - 4$	a_n
1	$(1)^2 - 4$	−3
2	$(2)^2 - 4$	0
3	$(3)^2 - 4$	5
4	$(4)^2 - 4$	12
5	$(5)^2 - 4$	21

2.

n	$\frac{1}{2}a_{n-1} - 8$	a_n
1	Given	48
2	24 − 8	16
3	8 − 8	0
4	0 − 8	−8

5	(−4) − 8	−12

3. This is an arithemetic sequence, with
$d = -2 - (-4) = 2$
Thus, $a_n = -4 + 2(n - 1) = -4 + 2n - 2 = 2n - 6$.

4. This a geometric sequence with $r = \frac{18}{54} = \frac{1}{3}$.
$$a_n = 54\left(\frac{1}{3}\right)^{n-1}$$

5. $5(1)^3 + 5(2)^3 + 5(3)^3 + 5(4)^3$
$= 5 + 40 + 135 + 320 = 500$

6. $(-1)^{1+1}(1) + (-1)^{2+1}(2) + (-1)^{3+1}(3) + (-1)^{4+1}(4) + (-1)^{5+1}(5) + (-1)^{6+1}(6) + (-1)^{7+1}(7)$
$= 1 - 2 + 3 - 4 + 5 - 6 + 7 = 4$

7. $d = -13 - (-19) = 6$. $a_1 = -19$.
Thus, $a_9 = -19 + 6(9 - 1) = 48 - 19 = 29$.

8. $(5 - 2)d = 5 - 11.6 = -6.6$. So, $d = -2.2$.
$a_1 = a_2 - d = 11.6 - (-2.2) = 13.8$
$a_9 = 13.8 - 2.2(9 - 1) = 13.8 - 17.6 = -3.8$

9. $3d = 65 - 125 = -60$. Thus, $d = -20$.
So, the next terms are $125 - 20 = 105$ and $125 - 2(20) = 85$.

10. $d = 7 - 4 = 3$. $a_1 = 4$. So, $a_{20} = 4 + 3(20 - 1)\ 61$.
$$S_{20} = 20\left(\frac{4+61}{2}\right) = 20\left(\frac{65}{2}\right) = 650.$$

11. $a_1 = -9(1) + 8 = -1$. $a_{12} = -9(12) + 8 = -100$.
$$S_{12} = 12\left(\frac{-1-100}{2}\right) = 12\left(\frac{-101}{2}\right) = -606$$

12. $a_1 = 16$. $d = 2$. $a_{12} = 16 + 2(12 - 1) = 38$.
$$S_{12} = 12\left(\frac{16+38}{2}\right) = 12\left(\frac{54}{2}\right) = 324.$$

13. $r = \frac{\frac{3}{64}}{\frac{3}{256}} = 4$. $a_1 = \frac{3}{256}$. So:
$$a_{10} = \frac{3}{256}(4)^{10-1} = \frac{3}{256}(4^9) = 3072$$

14. $r = \frac{a_5}{a_4} = \frac{8}{2} = 4$.
$a_4 = 2 = a_1 4^{4-1}$. So, $a_1 = \frac{1}{32}$.
$$a_{10} = \frac{1}{32}(4)^{10-1} = \frac{4^9}{32} = 8192$$

15. $\sqrt{ab} = \sqrt{4 \cdot 25} = \sqrt{100} = 10$.

16. $r = \frac{1}{2}$. $n = 6$. $a_1 = 2$.
$$S = 2\left(\frac{1 - \left(\frac{1}{2}\right)^6}{1 - \frac{1}{2}}\right) = \frac{63}{16}$$

17. $r = -\frac{1}{5}$. $a_1 = 250$. $n = 6$.
$$S = 250\left(\frac{1 - \left(\frac{-1}{5}\right)^6}{1 - \frac{-1}{5}}\right) = 208.32$$

18. $a_1 = 1000$. $r = 1.05$. $a_{10} = 1000(1.05)^{10-1}$
$= \$1628.89$.
$$S_{10} = 1000\left(\frac{1 - 1.05^{10}}{1 - 1.05}\right) = \$13{,}206.79$$

19. $r = -\frac{100}{200} = -\frac{1}{2}$. $\left|-\frac{1}{2}\right| < 1$, so the sum exists.
$$S = \frac{200}{1 - \left(-\frac{1}{2}\right)} = \frac{200}{\frac{3}{2}} = \frac{400}{3} = 133.\overline{3}.$$

20. $r = \frac{7}{8}$. $a_1 = \frac{14}{8}$.
$$S = \frac{\frac{14}{8}}{1 - \frac{7}{8}} = \frac{\frac{14}{8}}{\frac{1}{8}} = 14.$$

Answers to Final Review: Chapter 10 (ID: 1)

1) 10;(3, 4)

2) 5.39; $(-1, \frac{5}{2})$

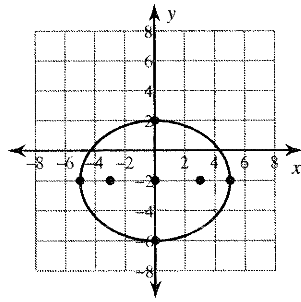
3) 5.83; $(-\frac{3}{2}, -\frac{7}{2})$

4) 3; $(3, \frac{9}{2})$

5) y = 8x + 65

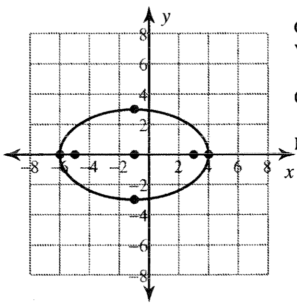
6) $y = \frac{1}{3}x + \frac{20}{3}$

7)



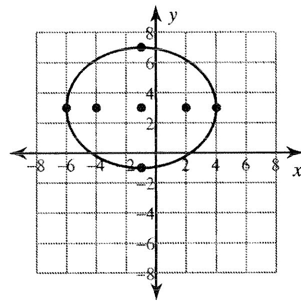
Center: (0, −2)
Vertices: (5, −2)
(−5, −2)
Co-vertices: (0, 2)
(0, −6)
Foci: (3, −2)
(−3, −2)

8)



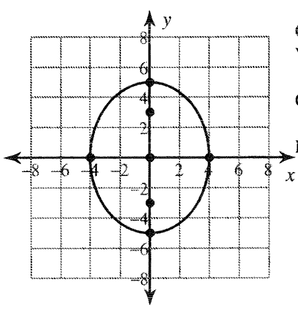
Center: (−1, 0)
Vertices: (4, 0)
(−6, 0)
Co-vertices: (−1, 3)
(−1, −3)
Foci: (3, 0)
(−5, 0)

9)



Center: (−1, 3)
Vertices: (4, 3)
(−6, 3)
Co-vertices: (−1, 7)
(−1, −1)
Foci: (2, 3)
(−4, 3)

10)



Center: (0, 0)
Vertices: (0, 5)
(0, −5)
Co-vertices: (4, 0)
(−4, 0)
Foci: (0, 3)
(0, −3)

11) $\frac{(x-2)^2}{144} + \frac{(y+10)^2}{169} = 1$

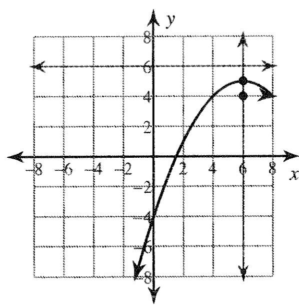
12) $\frac{(x+3)^2}{100} + \frac{(y-9)^2}{144} = 1$

13) $\frac{(x-9)^2}{16} + \frac{(y-10)^2}{25} = 1$

14) $\frac{(x+3)^2}{36} + \frac{(y-10)^2}{100} = 1$

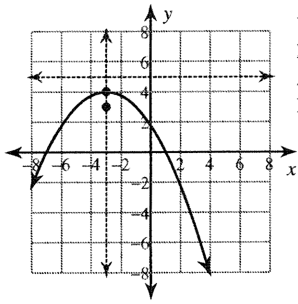
15) $\frac{(x-1)^2}{16} + \frac{(y+2)^2}{25} = 1$

16)



Vertex: (6, 5)
Focus: (6, 4)
Axis of Sym.: $x = 6$
Directrix: $y = 6$

17)



Vertex: (−3, 4)
Focus: (−3, 3)
Axis of Sym.: $x = -3$
Directrix: $y = 5$

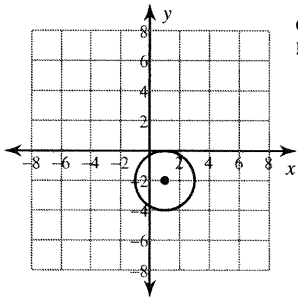
18) $-\frac{1}{5}(y-10) = (x-5)^2$

19) $\frac{1}{2}(y-9)=(x+7)^2$

20) $-(y+9)=(x-3)^2$

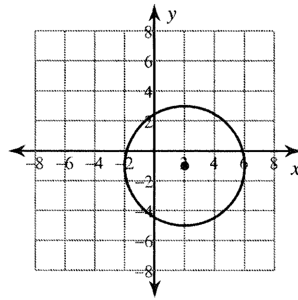
21) $-2(y-9)=(x-8)^2$

22)



Center: $(1, -2)$
Radius: 2

23)



Center: $(2, -1)$
Radius: 4

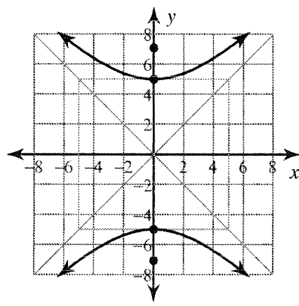
24) $(x-6)^2+(y-13)^2=4$

25) $(x-16)^2+(y+12)^2=1$

26) $(x-10)^2+(y+13)^2=9$

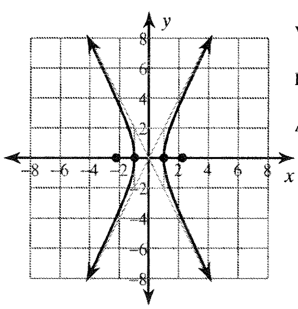
27) $(x-13)^2+(y-12)^2=25$

28)



Vertices: $(0, 5)$
$(0, -5)$
Foci: $(0, 5\sqrt{2})$
$(0, -5\sqrt{2})$
Asym.: $y = x$
$y = -x$

29)



Vertices: $(1, 0)$
$(-1, 0)$
Foci: $(\sqrt{5}, 0)$
$(-\sqrt{5}, 0)$
Asym.: $y = 2x$
$y = -2x$

30) $\frac{x^2}{9}-\frac{y^2}{16}=1$

31) $\frac{(x-6)^2}{100}-\frac{(y-1)^2}{25}=1$

32) $(2, 10)$

33) $(-9, -10)$

34) $(-1, 10), (-7, 10), (-1, -4), (-7, -4)$

35) $(2, 7), (2, -3), (-1, 9), (-1, -5)$