Lesson 7 - 5

# Compound Events

## Going Deeper

**Essential question:** *How do you find the probability of mutually exclusive events and overlapping events?*

Two events are **mutually exclusive events** if the events cannot both occur in the same trial of an experiment. For example, when you toss a coin, the coin landing heads up and the coin landing tails up are mutually exclusive events.

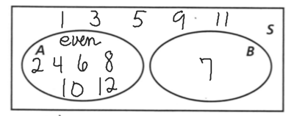
PREP FOR CC.9–12.S.CP.7

### 1 EXAMPLE Finding the Probability of Mutually Exclusive Events

**A dodecahedral number cube has 12 sides numbered 1 through 12. What is the probability that you roll the cube and the result is an even number or a 7?**

**A** Let event $A$ be the event that you roll an even number. Let event $B$ be the event that you roll a 7. Let $S$ be the sample space.

Complete the Venn diagram by writing all outcomes in the sample space in the appropriate region.




**B** You must find the probability of $A$ or $B$.

#of items

$n(S) =$ 12

$n(A \text{ or } B) = n(A) + n(B)$ — $A$ and $B$ are mutually exclusive events.

$= 6 + 1$ — Use the Venn diagram to find $n(A)$ and $n(B)$.

$= 7$ — Add.

So, $P(A \text{ or } B) = \frac{n(A \text{ or } B)}{n(S)} = \frac{7}{12}$.

REFLECT

**1a.** Does the probability you calculated seem reasonable? Why?

**1b.** Is it always true that $n(A \text{ or } B) = n(A) + n(B)$? Explain.

**1c.** How is $P(A \text{ or } B)$ related to $P(A)$ and $P(B)$? Do you think this is always true?

$P(A \text{ or } B) \quad P(A) \quad P(B)$

$\frac{7}{12} = \frac{6}{12} + \frac{1}{12}$

No, true only if mutually exclusive events

The process you used in the example can be generalized to give a formula for the probability of mutually exclusive events.

**Mutually Exclusive Events**

If $A$ and $B$ are mutually exclusive events, then $\boxed{P(A \text{ or } B) = P(A) + P(B)}$.

Two events are **overlapping events** (or *inclusive events*) if they have one or more outcomes in common.

PREP FOR CC.9–12.S.CP.7

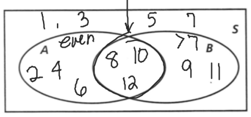
### 2 EXAMPLE Finding the Probability of Overlapping Events

**What is the probability that you roll a dodecahedral number cube and the result is an even number or a number greater than 7?**

**A** Let event $A$ be the event that you roll an even number. Let event $B$ be the event that you roll a number greater than 7. Let $S$ be the sample space.

Complete the Venn diagram by writing all outcomes in the sample space in the appropriate region.

← intersection
$A \cap B$
overlap




**B** You must find the probability of $A$ or $B$.

$n(S) =$ 12

overlap

$n(A \text{ or } B) = n(A) + n(B) - n(A \text{ and } B)$ — $A$ and $B$ are overlapping events.

$= 6 + 5 - 3$ — Use the Venn diagram.

$= 8$ — Simplify.

So, $P(A \text{ or } B) = \frac{n(A \text{ or } B)}{n(S)} = \frac{8}{12} = \frac{2}{3}$.

**REFLECT**

**2a.** Why is $n(A \text{ or } B)$ equal to $n(A) + n(B) - n(A \text{ and } B)$?

**2b.** Is $P(A \text{ or } B)$ equal to $P(A) + P(B)$ in this case? Explain.

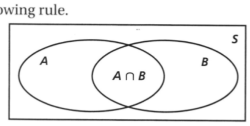
$\frac{2}{3} \neq \frac{6}{12} + \frac{5}{12}$ NO, because the events are overlapping

In the previous example you saw that for overlapping events $A$ and $B$, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$. You can convert these counts to probabilities by dividing each term by $n(S)$ as shown below.

$\cup$ union   OR   $P(A) + P(B) - P(\text{overlap})$

$$\frac{n(A \cup B)}{n(S)} = \frac{n(A)}{n(S)} + \frac{n(B)}{n(S)} - \frac{n(A \cap B)}{n(S)}$$

Rewriting each term as a probability results in the following rule.




**Addition Rule**

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

Notice that when $A$ and $B$ are mutually exclusive events, $P(A \text{ and } B) = 0$, and the ru becomes the simpler rule for mutually exclusive events on the previous page.

CC.9–12.S.CP.7

## 3 EXAMPLE Using the Addition Rule

**You shuffle a standard deck of playing cards and choose a card at random. What is the probability that you choose a king or a heart?**

**A** Let event $A$ be the event that you choose a king. Let event $B$ be the event that you choose a heart. Let $S$ be the sample space.

There are 52 cards in the deck, so $n(S) =$ 52.

There are 4 kings in the deck, ~~so $n(A) =$~~ ______ and $P(A) = \frac{4}{52}$.

P(King)

There are 13 hearts in the deck, ~~so $n(B) =$~~ ______ and $P(B) = \frac{13}{52}$.

P(♡)

There is one king of hearts in the deck, so $P(A \text{ and } B) = \frac{1}{52}$.

P(K and ♡)

**B** Use the Addition Rule.

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$= 4/52 + 13/52 - 1/52 \qquad \text{Substitute.}$$

$$= \frac{16}{52} \text{ or } \frac{4}{13} \qquad \text{Simplify.}$$

So, the probability of choosing a king or a heart is $\frac{4}{13}$.