10. (a) $\dfrac{x^2+x-6}{x-2} = x+3$

The left hand side is NOT defined for $x=2$ but the right hand side is

(b) $\lim\limits_{x \to 2} \dfrac{x^2+x-6}{x-2} = \lim\limits_{x \to 2} (x+3)$

We are approaching $x=2$ not considering $x=2$

16. $\lim\limits_{x \to -1} \dfrac{x^2-4x}{x^2-3x-4} = \lim\limits_{x \to -1} \dfrac{x(x-4)}{(x-4)(x+1)}$

$= \lim\limits_{x \to -1} \dfrac{x}{x+1}$

$\lim\limits_{x \to -1^-} \dfrac{x}{x+1} = \infty$ $\left[\begin{array}{l} \text{as } x \to -1^-,\ x \to -1 \\ \text{as } x \to -1^-,\ (x+1) \to 0^- \end{array}\right]$

$\lim\limits_{x \to -1^+} \dfrac{x}{x+1} = -\infty$ $\left[\begin{array}{l} \text{as } x \to -1^+,\ x \to -1 \\ \text{as } x \to -1^+,\ (x+1) \to 0^+ \end{array}\right]$

$\lim\limits_{x \to -1} \dfrac{x^2-4x}{x^2-3x-4}$ DNE since

$\lim\limits_{x \to -1^-} \dfrac{x^2-4x}{x^2-3x-4} \neq \lim\limits_{x \to -1^+} \dfrac{x^2-4x}{x^2-3x-4}$

18. $\lim\limits_{x \to 1} \dfrac{x^3-1}{x^2-1} = \lim\limits_{x \to 1} \dfrac{(x-1)(x^2+x+1)}{(x+1)(x-1)}$

$= \lim\limits_{x \to 1} \dfrac{x^2+x+1}{x+1}$

$= \dfrac{(1)^2+1+1}{1+1}$

$= \dfrac{3}{2}$

20. $\lim_{h\to 0} \frac{(2+h)^3-8}{h} = \lim_{h\to 0} \frac{(8+12h+6h^2+h^3)-8}{h}$

$= \lim_{h\to 0} \frac{h(12+6h+h^2)}{h}$

$= \lim_{h\to 0} (12+6h+h^2)$

$= 12+6(0)+0^2$

$= 12$

22. $\lim_{h\to 0} \frac{\sqrt{1+h}-1}{h} = \lim_{h\to 0} \frac{\sqrt{1+h}-1}{h} \cdot \frac{\sqrt{1+h}+1}{\sqrt{1+h}+1}$

$= \lim_{h\to 0} \frac{(1+h)-1}{h(\sqrt{1+h}+1)}$

$= \lim_{h\to 0} \frac{h}{h(\sqrt{1+h}+1)}$

$= \lim_{h\to 0} \frac{1}{\sqrt{1+h}+1}$

$= \frac{1}{\sqrt{1+0}+1}$

$= \frac{1}{2}$

24. $\lim_{x\to -1} \frac{x^2+2x+1}{x^4-1} = \lim_{x\to -1} \frac{(x+1)^2}{(x^2-1)(x^2+1)}$

$= \lim_{x\to -1} \frac{(x+1)^2}{(x+1)(x-1)(x^2+1)}$

$= \lim_{x\to -1} \frac{(x+1)}{(x-1)(x^2+1)}$

$= \frac{-1+1}{(-1-1)[(-1)^2+1]}$

$= 0$

26. $\lim_{t\to 0}\left(\frac{1}{t}-\frac{1}{t^2+t}\right)=\lim_{t\to 0}\left(\frac{1}{t}-\frac{1}{t(t+1)}\right)$

$=\lim_{t\to 0}\frac{(t+1)-1}{t(t+1)}$

$=\lim_{t\to 0}\frac{t}{t(t+1)}$

$=\lim_{t\to 0}\frac{1}{t+1}$

$=\frac{1}{0+1}$

$=1$

28. $\lim_{h\to 0}\frac{(3+h)^{-1}-3^{-1}}{h}=\lim_{h\to 0}\frac{\frac{1}{3+h}-\frac{1}{3}}{h}$

$=\lim_{h\to 0}\frac{3-(3+h)}{3(3+h)}\cdot\frac{1}{h}$

$=\lim_{h\to 0}\frac{-h}{3h(3+h)}$

$=\lim_{h\to 0}\frac{-1}{3(3+h)}$

$=\frac{-1}{3(3+0)}$

$=\frac{-1}{9}$

30. $$\lim_{x\to -4} \frac{\sqrt{x^2+9}-5}{x+4} = \lim_{x\to -4} \frac{\sqrt{x^2+9}-5}{x+4} \cdot \frac{\sqrt{x^2+9}+5}{\sqrt{x^2+9}+5}$$

$$= \lim_{x\to -4} \frac{(x^2+9)-25}{(x+4)[\sqrt{x^2+9}+5]}$$

$$= \lim_{x\to -4} \frac{x^2-16}{(x+4)(\sqrt{x^2+9}+5)}$$

$$= \lim_{x\to -4} \frac{(x+4)(x-4)}{(x+4)(\sqrt{x^2+9}+5)}$$

$$= \lim_{x\to -4} \frac{x-4}{\sqrt{x^2+9}+5}$$

$$= \frac{-4-4}{\sqrt{(-4)^2+9}+5}$$

$$= \frac{-4}{5}$$

39. $$\lim_{x\to 3} (2x+|x-3|)$$

$$|x-3| = \begin{cases} x-3 & \text{if } x-3 \leq 0 \\ -(x-3) & \text{if } x-3 < 0 \end{cases}$$

$$|x-3| = \begin{cases} x-3 & \text{if } x \leq 3 \\ -(x-3) & \text{if } x < 3 \end{cases}$$

$$\lim_{x\to 3^-} (2x+|x-3|) = \lim_{x\to 3^-} (2x-(x-3)) = \lim_{x\to 3^-} (x+3) = 6$$

$$\lim_{x\to 3^+} (2x+|x-3|) = \lim_{x\to 3^+} (2x+(x-3)) = \lim_{x\to 3^+} (3x-3) = 6$$

$$\therefore \lim_{x\to 3} (2x+|x-3|) = 6$$

43. $\lim_{x \to 0^-} \left(\frac{1}{x} - \frac{1}{|x|} \right)$ $\qquad |x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$

$= \lim_{x \to 0^-} \left(\frac{1}{x} - \frac{1}{-x} \right)$

$= \lim_{x \to 0^-} \left(\frac{1}{x} + \frac{1}{x} \right)$

$= \lim_{x \to 0^-} \frac{2}{x}$

$= -\infty$

44. $\lim_{x \to 0^+} \left(\frac{1}{x} - \frac{1}{|x|} \right) = \lim_{x \to 0^+} \left(\frac{1}{x} - \frac{1}{x} \right)$

$= \lim_{x \to 0^+} 0$

$= 0$

45.

$\operatorname{sgn} x = \begin{cases} -1 & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ 1 & \text{if } x > 0 \end{cases}$

b) i) $\lim_{x \to 0^+} \operatorname{sgn} x = \lim_{x \to 0^+} 1 = 1$

ii) $\lim_{x \to 0^-} \operatorname{sgn} x = \lim_{x \to 0^-} (-1) = -1$

iii) $\lim_{x \to 0^+} \operatorname{sgn} x = 1$ and $\lim_{x \to 0^-} \operatorname{sgn} x = -1$

so $\lim_{x \to 0} \operatorname{sgn} x$ DNE since $\lim_{x \to 0^-} \operatorname{sgn} x \neq \lim_{x \to 0^+} \operatorname{sgn} x$

46. $f(x) = \begin{cases} 4-x^2 & \text{if } x \le 2 \\ x-1 & \text{if } x > 2 \end{cases}$

(a) $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} (4-x^2) = 4-(2)^2 = 0$

$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} (x-1) = 2-1 = 1$

(b) $\lim_{x\to 2^-} f(x) = 0$ and $\lim_{x\to 2^+} f(x) = 1$

So $\lim_{x\to 2} f(x)$ DNE since $\lim_{x\to 2^-} f(x) \neq \lim_{x\to 2^+} f(x)$

48. $g(x) = \begin{cases} x & \text{if } x < 1 \\ 3 & \text{if } x = 1 \\ 2-x^2 & \text{if } 1 < x \le 2 \\ x-3 & \text{if } x > 2 \end{cases}$

(a) (i) $\lim_{x\to 1^-} g(x) = \lim_{x\to 1^-} x = 1$

(ii) $\lim_{x\to 1^+} g(x) = \lim_{x\to 1^+} (2-x^2) = 2-(1)^2 = 1$

then $\lim_{x\to 1} g(x) = 1$ [since $\lim_{x\to 1^-} g(x) = 1 = \lim_{x\to 1^+} g(x)$]

(iii) $g(1) = 3$

(iv) $\lim_{x\to 2^-} g(x) = \lim_{x\to 2^-} (2-x^2) = 2-2^2 = -2$

(v) $\lim_{x\to 2^+} g(x) = \lim_{x\to 2^+} (x-3) = 2-3 = -1$

(vi) $\lim_{x\to 2^-} g(x) = -2$ and $\lim_{x\to 2^+} g(x) = -1$

So $\lim_{x\to 2} g(x)$ DNE since $\lim_{x\to 2^-} g(x) \neq \lim_{x\to 2^+} g(x)$