# SUMMER ASSIGNMENT

1. (a) $\dfrac{x^3-9x}{x^2-7x+12} = \dfrac{x(x+3)(x-3)}{(x-4)(x-3)} = \dfrac{x(x+3)}{x-4}$

(b) $\dfrac{x^2-2x-8}{x^3+x^2-2x} = \dfrac{(x-4)(x+2)}{x(x+2)(x-1)} = \dfrac{x-4}{x(x-1)}$

(c) $\dfrac{\frac{1}{x}-\frac{1}{5}}{\frac{1}{x^2}-\frac{1}{25}} = \dfrac{\frac{5-x}{5x}}{\frac{25-x^2}{25x^2}} = \dfrac{5-x}{5x} \cdot \dfrac{25x^2}{(5-x)(5+x)}$

$= \dfrac{5x}{5+x}$

(d) $\dfrac{9-x^{-2}}{3+x^{-1}} = \dfrac{9-\frac{1}{x^2}}{3+\frac{1}{x}} = \dfrac{\frac{9x^2-1}{x^2}}{\frac{3x+1}{x}}$

$= \dfrac{(3x+1)(3x-1)}{x^2} \cdot \dfrac{x}{3x+1}$

$= \dfrac{3x-1}{x}$

(e) $\dfrac{2}{\sqrt{3}+\sqrt{2}} \cdot \dfrac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}} = \dfrac{2(\sqrt{3}-\sqrt{2})}{3-2} = 2(\sqrt{3}-\sqrt{2})$

(f) $\dfrac{\sqrt{x-2}+\frac{5}{\sqrt{x-2}}}{x-2} = \dfrac{x-2+5}{\sqrt{x-2}} \cdot \dfrac{1}{x-2}$

$= \dfrac{x+3}{(\sqrt{x-2})(x-2)} \cdot \dfrac{\sqrt{x-2}}{\sqrt{x-2}} = \dfrac{(x+3)(\sqrt{x-2})}{(x-2)^2}$

2. (a) $5^{x+1} = 25$

$5^{x+1} = 5^2$

$x+1 = 2$

$x = 1$

(b) $\frac{1}{3} = 3^{2x+2}$

$3^{-1} = 3^{2x+2}$

$-1 = 2x+2$

$x = \frac{-3}{2}$

(c) $\log_2(x) = 3$

$2^3 = x$

$x = 8$

(d) $\log_3(x^2) = 2\log_3(4) - 4\log_3(5)$

$\log_3(x^2) = \log_3(4^2) - \log_3(5^4)$

$\log_3(x^2) = \log_3\left(\frac{16}{625}\right)$

$x^2 = \frac{16}{625}$

$x = \pm\frac{4}{25}$

3. $xy' + y = 1 + y'$

$xy' - y' = 1 - y$

$y'(x-1) = 1 - y$

$y' = \frac{1-y}{x-1}$

4. (a) $\log_2(5) + \log_2(x^2-1) - \log_2(x-1)$

$= \log_2\left(\frac{5(x+1)(x-1)}{x-1}\right)$

$= \log_2(5(x+1))$

→ (b) $2\log_4(9) - \log_2(3) = 2\log_4(3^2) - \log_2(3)$

$= 4\log_4(3) - \log_2(3)$

$= \frac{4\log(3)}{\log(4)} - \frac{\log(3)}{\log(2)}$

$= \frac{4\log(3)}{\log(2^2)} - \frac{\log(3)}{\log(2)}$

$= \frac{4\log(3)}{2\log(2)} - \frac{\log(3)}{\log(2)}$

$= \frac{2\log(3)}{\log(2)} - \frac{\log(3)}{\log(2)}$

$= \frac{\log(3)}{\log(2)}$

(c) $3^{2\log_3(5)} = 3^{\log_3(25)} = 25$

(d) $\log_{10}(10^{\frac{1}{2}}) = \frac{1}{2}$

(e) $2\log_{10}(\sqrt{x}) + 3\log_{10}(x^{\frac{1}{3}})$

$= 2\log_{10}(x^{\frac{1}{2}}) + 3\log_{10}(x^{\frac{1}{3}})$

$= \log_{10}(x) + \log_{10}(x)$

$= \log_{10}(x^2) = \log(x^2)$

5. (a) $4x^2 - 21x - 18 = 0$

$(x-6)(4x+3) = 0$

$x = 6 \quad x = -\frac{3}{4}$

(b) $2x^2 - 5x + 3 = 0$

$(x-1)(2x-3) = 0$

$x = 1 \quad x = \frac{3}{2}$

(c) $x^4 - 9x^2 + 8 = 0$

$(x^2-8)(x^2-1) = 0$

$(x^2-8)(x+1)(x-1) = 0$

$x = \pm 2\sqrt{2} \quad x = -1 \quad x = 1$

6. $(2,4)$ $\quad 2x+3y-8=0$

$3y = -2x+8$

$y = -\frac{2}{3}x + \frac{8}{3} \quad m = -\frac{2}{3}$

$y-y_1 = m(x-x_1)$

$y-4 = -\frac{2}{3}(x-2)$

$y-4 = -\frac{2}{3}x + \frac{4}{3}$

$y = -\frac{2}{3}x + \frac{16}{3}$

7. $(1,2)$ $\quad 2x+3y-8=0$

$m = -\frac{2}{3}$

$\perp m = \frac{3}{2}$

$y-2 = \frac{3}{2}(x-1)$

$y-2 = \frac{3}{2}x - \frac{3}{2}$

$y = \frac{3}{2}x + \frac{1}{2}$

8. $m=5 \quad (-1,3)$

$y-3 = 5(x+1)$

$y-3 = 5x+5$

$y = 5x+8$ passes x-axis when $y=0$

$0 = 5x+8$

$x = -\frac{8}{5}$

line crosses x-axis at $(-\frac{8}{5}, 0)$

9. $(1,-3) \quad (-2,4)$

$m = \frac{4+3}{-2-1} = \frac{7}{-3}$

$y+3 = -\frac{7}{3}(x-1)$

$y+3 = -\frac{7}{3}x + \frac{7}{3}$

$y = -\frac{7}{3}x - \frac{2}{3}$ passes y-axis when $x=0$

$y = -\frac{7}{3}(0) - \frac{2}{3}$

$y = -\frac{2}{3}$

line crosses y-axis at $(0, -\frac{2}{3})$

10. (a) $\cos(0) = 1$

(b) $\sin(0) = 0$

(c) $\tan(\frac{\pi}{2}) =$ und.

(d) $\cos(\frac{\pi}{4}) = \frac{\sqrt{2}}{2}$

(e) $\sin(\frac{\pi}{2}) = 1$

(f) $\sin(\pi) = 0$

(g) $\arccos(\frac{\sqrt{3}}{2}) = \frac{\pi}{6}$

(h) $\arctan(1) = \frac{\pi}{4}$

(i) $\arcsin(-\frac{1}{2}) = -\frac{\pi}{6}$

11. $0 < \theta \leq 2\pi$

(a) $2\sin^2(\theta) = 1 - \sin(\theta)$

$2\sin^2(\theta) + \sin(\theta) - 1 = 0$

$[2\sin(\theta) - 1][\sin(\theta) + 1] = 0$

$2\sin(\theta) - 1 = 0$ $\quad$ $\sin(\theta) + 1 = 0$

$\sin(\theta) = \frac{1}{2}$ $\quad$ $\sin(\theta) = -1$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$ $\quad$ $\theta = \frac{3\pi}{2}$

(b) $2\tan(\theta) - \sec^2(\theta) = 0$

$2\tan(\theta) - [1 + \tan^2(\theta)] = 0$

$\tan^2(\theta) - 2\tan(\theta) + 1 = 0$

$[\tan(\theta) - 1][\tan(\theta) - 1] = 0$

$\tan(\theta) - 1 = 0$

$\tan(\theta) = 1$

$\theta = \frac{\pi}{4}, \frac{5\pi}{4}$

(c) $\sin(2\theta) + \sin(\theta) = 0$

$2\sin(\theta)\cos(\theta) + \sin(\theta) = 0$

$\sin(\theta)[2\cos(\theta) + 1] = 0$

$\sin(\theta) = 0$ $\quad$ $2\cos(\theta) + 1 = 0$

$\theta = \pi, 2\pi$ $\quad$ $\cos(\theta) = -\frac{1}{2}$

$\theta = \frac{2\pi}{3}, \frac{4\pi}{3}$

(d) $3\tan^3(\theta) - 3\tan^2(\theta) - \tan(\theta) + 1 = 0$

$3\tan^2(\theta)[\tan(\theta) - 1] - [\tan(\theta) - 1] = 0$

$[\tan(\theta) - 1][3\tan^2(\theta) - 1] = 0$

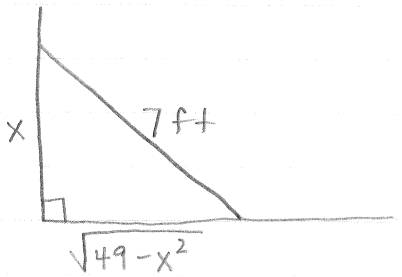
$\tan(\theta) - 1 = 0$ $\quad$ $3\tan^2(\theta) - 1 = 0$

$\tan(\theta) = 1$ $\quad$ $\tan^2(\theta) = \frac{1}{3}$

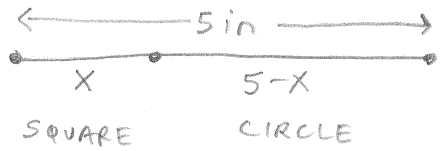
$\theta = \frac{\pi}{4}, \frac{5\pi}{4}$ $\quad$ $\tan(\theta) = \pm\frac{\sqrt{3}}{3}$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

12.




13.




T = total area

area of square $= (\frac{1}{4}x)^2 = \frac{x^2}{16}$

area of circle $= \pi(\frac{5-x}{2})^2$

$T(x) = \frac{1}{16}x^2 + \pi(\frac{5-x}{2})^2$

14. (a) $h(x) = \dfrac{1}{4x^2 - 21x - 18}$

$h(x) = \dfrac{1}{(4x+3)(x-6)}$

Domain: $\mathbb{R}$ except $x = \frac{-3}{4}$ and $x = 6$

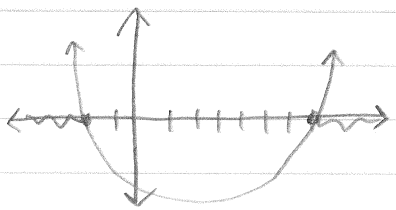
(b) $k(x) = \sqrt{x^2 - 5x - 14}$

$x^2 - 5x - 14 \geq 0$

$(x-7)(x+2) = 0$

$x = 7 \quad x = -2$

$x \leq -2$ or $x \geq 7$



Domain: $(-\infty, -2] \cup [7, \infty)$

(c) $y = \ln(2x - 12)$

$2x - 12 > 0$

$x > 6$

Domain: $(6, \infty)$

SKIP
15, 16, 17

18 (a) $2x+1 = \frac{5}{x+2}$

$(2x+1)(x+2) = 5$

$2x^2+5x+2 = 5$

$2x^2+5x-3 = 0$

$(2x-1)(x+3) = 0$

$x = \frac{1}{2} \quad x = -3$

(b) $\frac{x+1}{x} - \frac{x}{x+1} = 0$

$\frac{x+1}{x} = \frac{x}{x+1}$

$(x+1)^2 = x^2$

$x^2+2x+1 = x^2$

$2x+1 = 0$

$x = -\frac{1}{2}$

19. (a) $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$

$\frac{x}{a} = 1 - \frac{y}{b} - \frac{z}{c}$

$\frac{x}{a} = \frac{bc - cy - bz}{bc}$

$a(bc - cy - bz) = bcx$

$a = \frac{bcx}{bc - cy - bz}$

(b) $V = 2(ab+bc+ca)$

$\frac{1}{2}V = ab+bc+ca$

$\frac{1}{2}V - bc = ab+ca$

$\frac{1}{2}V - bc = a(b+c)$

$\frac{\frac{1}{2}V - bc}{b+c} = a$

$a = \frac{V - 2bc}{2(b+c)}$

(c) $A = 2\pi r^2 + 2\pi rh$

$2\pi r^2 + 2\pi rh - A = 0$ use Quadratic Formula

$$r = \frac{-2\pi h \pm \sqrt{(2\pi h)^2 - 4(2\pi)(-A)}}{2(2\pi)}$$

$$r = \frac{-2\pi h \pm \sqrt{4\pi^2 h^2 + 8\pi A}}{4\pi}$$

(d) $\frac{2x}{4\pi} + \frac{1-x}{2} = 0$

$\frac{1-x}{2} = -\frac{2x}{4\pi}$

$4\pi(1-x) = 2(-2x)$

$4\pi - 4\pi x = -4x$

$4\pi = -4x + 4\pi x$

$4\pi = x(-4 + 4\pi)$

$x = \frac{4\pi}{-4+4\pi} = \frac{\pi}{-1+\pi}$

20. (a) $x^5 - 4x^4 + x^3 + 0x^2 - 7x + 1$ divided by $x+2$

| | | | | | | |
|---|---|---|---|---|---|---|
| $-2$ | 1 | $-4$ | 1 | 0 | $-7$ | 1 |
| | ↓ | $-2$ | 12 | $-26$ | 52 | $-90$ |
| | 1 | $-6$ | 13 | $-26$ | 45 | $-89$ |

remainder is $-89$

(b)

$$x^3+1 \;\overline{)\; x^5 - x^4 + x^3 + 2x^2 - x + 4}$$

$$x^2 - x + 1 + \frac{x^2+3}{x^3+1}$$

$-(x^5 \qquad + x^2)$ ↓

$-x^4 + x^3 + x^2 - x$

$-(x^4 \qquad - x)$ ↓

$x^3 + x^2 + 0x + 4$

$-(x^3 \qquad + 1)$

$x^2 \qquad + 3$

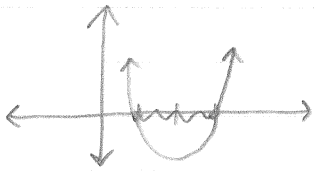
remainder: $x^2+3$

21. (a) $x^2 + 2x - 3 \leq 0$

$(x+3)(x-1) = 0$

$x = -3 \quad x = 1$

$-3 \leq x \leq 1$



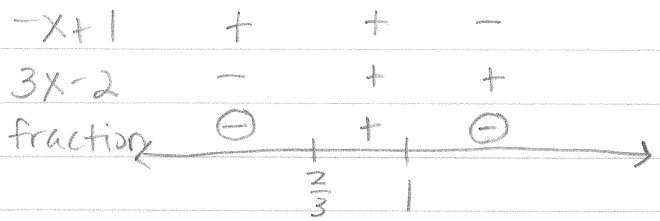
(b) $\frac{2x-1}{3x-2} \leq 1$ note: $x \neq \frac{2}{3}$

$\frac{2x-1}{3x-2} - 1 \leq 0$

$\frac{2x-1-(3x-2)}{3x-2} \leq 0$

$\frac{-x+1}{3x-2} \leq 0$

test points: $x=1$, $x=\frac{2}{3}$




$x < \frac{2}{3}$ or $x \geq 1$

(c) $x^2+2x+1>0$

$(x+1)(x+1)>0$

$(x+1)^2>0$ $\Rightarrow$ ALWAYS TRUE

$\mathbb{R}$

22 $x^2+y^2+6x-4y+3=0$

$(x^2+6x)+(y^2-4y)=-3$

$(x^2+6x+\frac{(6)^2}{2})+(y^2-4y+\frac{(-4)^2}{2})=-3+\underline{9}+\underline{4}$

$(x+3)^2+(y-2)^2=10$

center: $(-3,2)$

$r=\sqrt{10}$

23 slope between center and $(-2,5)$: $m=\frac{2-5}{-3+2}=\frac{-3}{-1}=3$

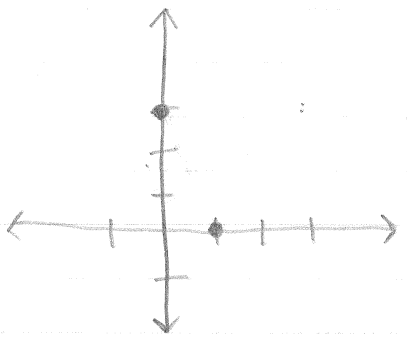
$m=3$

tangent slope $\Rightarrow \perp m=-\frac{1}{3}$

$y-5=-\frac{1}{3}(x+2)$

$y=-\frac{1}{3}x+\frac{13}{3}$

24.



Since the circle is tangent we know y-coord of center must be 3.
Also know center is on ⊥ bisector of segment connecting 2 points.

$(0,3)$ $(1,0)$

$m=\frac{0-3}{1-0}=-3 \rightarrow \perp m=\frac{1}{3}$

Midpt $\rightarrow M(\frac{0+1}{2},\frac{3+0}{2})=M(\frac{1}{2},\frac{3}{2})$

perp bisector $\rightarrow$ $y-\frac{3}{2}=\frac{1}{3}(x-\frac{1}{2})$

$y=\frac{1}{3}x+\frac{4}{3}$

The point on ⊥ bisector with y-coord 3 is

$3=\frac{1}{3}x+\frac{4}{3}$

$x=5$ Thus center is $(5,3)$

equation of circle: $(x-h)^2+(y-k)^2=r^2$

$(x-5)^2+(y-3)^2=25$

x-int when $y=0 \rightarrow$ $(x-5)^2+(0-3)^2=25$

$(x-5)^2+9=25$

$(x-5)^2=16$

$x-5=\pm 4$

$x=5\pm 4$

$x=9 \quad x=1$

x-int: 1, 9

25. (a) $f(x)=2x+3$

$x=2y+3$

$y=\frac{1}{2}x-\frac{3}{2}$

$f^{-1}(x)=\frac{1}{2}x-\frac{3}{2}$

(b) $f(x)=\frac{x+2}{5x-1}$

$x=\frac{y+2}{5y-1}$

$x(5y-1)=y+2$

$5xy-x=y+2$

$5xy-y=x+2$

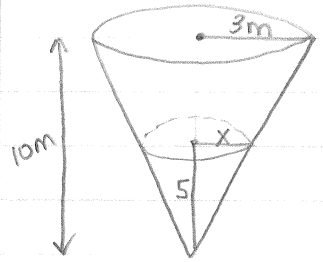
$y(5x-1)=x+2$

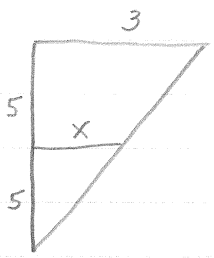
$y=\frac{x+2}{5x-1}$

$f^{-1}(x)=\frac{x+2}{5x-1}$

SKIP #26

27.




$\sim \Delta s$

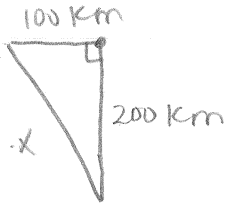
$\frac{5}{x} = \frac{10}{3}$

$10x = 15$

$x = \frac{3}{2}$

$SA = \pi(\frac{3}{2})^2 = \frac{9\pi}{4}\ m^2$

28.




$100^2 + 200^2 = x^2$

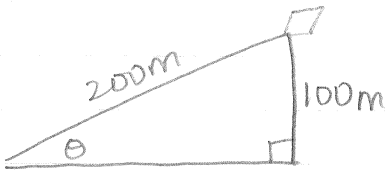
$10000 + 40000 = x^2$

$50000 = x^2$

$x = \sqrt{50000}$

$x = 100\sqrt{5}$ km

29.




$\sin(\theta) = \frac{100}{200}$

$\sin(\theta) = \frac{1}{2}$

$\theta = \frac{\pi}{6}$

30. (a) $y = x^2 + 4x + 3$

$y - 3 + \underline{4} = x^2 + 4x + \underline{(\frac{4}{2})^2}$

$y + 1 = (x+2)^2$

$y = (x+2)^2 - 1$

(b) $9y^2 - 6y - 9 - x = 0$

$9y^2 - 6y = 9 + x$

$9(y^2 - \frac{2}{3}y + \underline{(\frac{1}{3})^2}) = 9 + x + \underline{1}$

$9(y - \frac{1}{3})^2 = 10 + x \rightarrow (y - \frac{1}{3})^2 = \frac{1}{9}(x + 10)$