Chapter 11 **Mathematical Induction** Date________________________

Mathematical Induction is a method of proof used to show that an algebraic statement is true for all positive integers.

To prove a statement is true for all positive integers *n* :

1. Show that the statement is true for $n = 1$.

2. Assume that the statement is true for $n = k - 1$, where $k - 1$ is any positive integer

3. Show that the statement is true for the next positive integer, $n = k$.

Example 1. **Prove by induction:** $1 + 4 + 7 + \ldots + (3n - 2) = \dfrac{n(3n-1)}{2}$

① For $n=1$, $(3 \cdot 1 - 2) = \dfrac{1(3 \cdot 1 - 1)}{2}$

$$1 = 1\left(\frac{2}{2}\right)$$

$$1 = 1 \checkmark$$

② Assume that $1+4+7+\cdots+\left(3(K-1)-2\right) = \frac{(K-1)\left(3(K-1)-1\right)}{2}$

③ Show that $1+4+7+\cdots+(3K-2) = \frac{K(3K-1)}{2}$

Proof: $1+4+7+\cdots+\left(3(K-1)-2\right) = \frac{(K-1)(3K-4)}{2}$ $+(3K-2)$

$$1+4+7+\cdots+\left(3(K-1)-2\right)+(3K-2) = \frac{(K-1)(3K-4)}{2}$$

$$= \frac{3K^2-7K+4+6K-4}{2}$$

$$= \frac{3K^2-K}{2}$$

$$= \frac{K(3K-1)}{2}$$