11.3 Geometric Series

$a_n = a_1 \cdot r^{n-1}$

std. 22.0

sum of the 1st *n* terms of a geometric series:

$$S_n = a_1\left(\frac{1-r^n}{1-r}\right) \quad (r \neq 1)$$

proof:

$$S_n = a_1 \cdot r^0 + \cancel{a_1 \cdot r^1} + \cancel{a_1 \cdot r^2} + \cdots + \cancel{a_1 r^{n-1}}$$

$$-\, r(S_n) \qquad \cancel{a_1 r^1} + \cancel{a_1 r^2} + \cdots + \cancel{a_1 r^{n-1}} + a_1 r^n$$

$$S_n - rS_n = a_1 - a_1 r^n$$

$$\frac{S_n(\cancel{1-r})}{\cancel{1-r}} = \frac{a_1(1-r^n)}{1-r} \quad ☆$$

ex. 1 **Using the "birthday money" sequence from yesterday, find the total amount of money Ryker could receive, up through his 18th birthday.**

$1+2+4+\cdots$

$$S_{18} = 1\left(\frac{1-2^{18}}{1-2}\right) = \$262{,}143$$

(-1) $\quad 2^{18}-1$

ex. 2 **Find the sum** $\sum_{n=1}^{8} -2\left(\frac{1}{5}\right)^{n-1}$

$$= \overset{a_1}{-2} + \frac{-2}{5} + \frac{-2}{25} + \cdots$$

$$S_8 = -2\left(\frac{1-.2^8}{1-.2}\right) \approx -2.50$$

ex. 3

For the series 1 + 4 + 16 + . . . find *n* $= 9$
so that $S_n = 87381$.

$$n \cancel{\log 4} = \frac{\log 262144}{\log 4}$$

$$S = a_1\left(\frac{1-r^n}{1-r}\right)$$

$$87381 = 1\left(\frac{1-4^n}{-3}\right)$$

$$-262143 = 1-4^n$$

$$4^n = 262144$$