

## 6-5: Factoring Polynomials and Finding Zeros

**ex. 1** Use synthetic division:  $(x^3 + 2x^2 - 6x - 15) \div (x + 3)$

$$\begin{array}{r|rrrr} -3 & 1 & 2 & -6 & -15 \\ & & -3 & 3 & 9 \\ \hline & 1 & -1 & -3 & -6 = R \end{array}$$

$x^2 - x - 3 - \frac{6}{x+3}$

If  $f(x) = x^3 + 2x^2 - 6x - 15$ , find  $f(-3)$ . ← Remainder

$$f(-3) = -27 + 2(9) + 18 - 15 = -10$$

**Remainder Theorem:** If a polynomial  $f(x)$  is divided by  $x - k$ , then the remainder  $R = f(k)$ .

**Factor Theorem:** A polynomial  $f(x)$  has a factor  $x - k$  if and only if  $f(k) = 0$ .  $k$  is a zero of  $f(x)$

**ex. 2** Factor  $f(x) = 3x^3 + 13x^2 + 2x - 8$  given that  $f(-4) = 0$

$$\begin{array}{r|rrrr} -4 & 3 & 13 & 2 & -8 \\ & & -12 & -4 & 8 \\ \hline & 3 & 1 & -2 & 0 \end{array}$$

$x + 4$  is a factor

$$f(x) = (x + 4)(3x^2 + x - 2) = (x + 4)(3x - 2)(x + 1)$$

**ex. 3** Is -2 a zero of  $f(x) = x^2 + 5x - 6$ ? (Is  $f(-2) = 0$ ?)

$$f(-2) = 16 + 5(4) - 6 = 30 \quad \text{NO}$$

ex. 4

Given one zero of  $f(x)$ , find the other zeros. (solutions)

$$f(x) = x^3 - 3x^2 + x + 1; 1 \leftarrow \text{zero}$$

$$\begin{array}{r|rrrr} .1 & 1 & -3 & 1 & 1 \\ & & 1 & -2 & -1 \\ \hline & 1 & -2 & -1 & 0 \end{array} \quad f(x)$$
$$(x-1)(x^2 - 2x - 1) = 0$$

$$x=1$$
$$x=1 \pm \sqrt{2}$$

$$x = \frac{2 \pm \sqrt{4 - 4(-1)}}{2} = \frac{2 \pm \sqrt{8}}{2}$$
$$= \frac{2 \pm 2\sqrt{2}}{2}$$