Algebra 2--H Chapter 6

UPPER AND LOWER BOUNDS
GRAPHING POLYNOMIAL FUNCTIONS
END BEHAVIOR OF FUNCTIONS

UPPER AND LOWER BOUNDS

A *positive* integer, a, is the ***upper bound*** of the real zeros of a polynomial function $f(x)$ if $f(x) \div (x-a)$ results in a polynomial function with *nonnegative* coefficients and remainder.

A *negative* integer, b, is the ***lower bound*** of the real zeros of a polynomial function $f(x)$ if $f(x) \div (x-b)$ results in a polynomial function that has coefficients and remainder with alternating signs.

***Note: A coefficient of 0 may be positive or negative as needed to fit the pattern for upper or lower bound.*

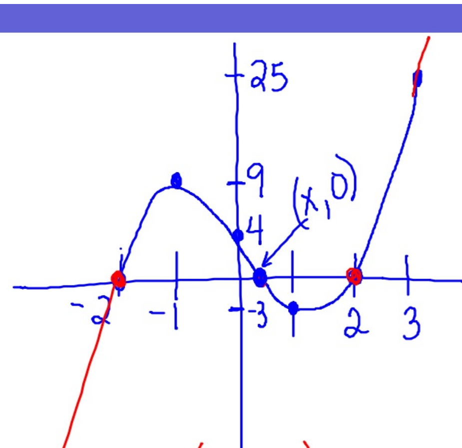
LOWER BOUND $\le$ REAL ZEROS $\le$ UPPER BOUND

Ex. 1 f(x) = 2x^3 - x^2 - 8x + 4

(0,4)

x	2	-1	-8	4	
1	2	1	-7	-3	← y
2	2	3	-2	0	
UB 3	2	5	7	25	
-1	2	-3	-5	9	
-2	2	-5	2	0	

$-2 \le$ Real zeros ≤ 3
LB UB



$\frac{p}{q} = \frac{\pm(1,2,4)}{\pm(1,2)}$

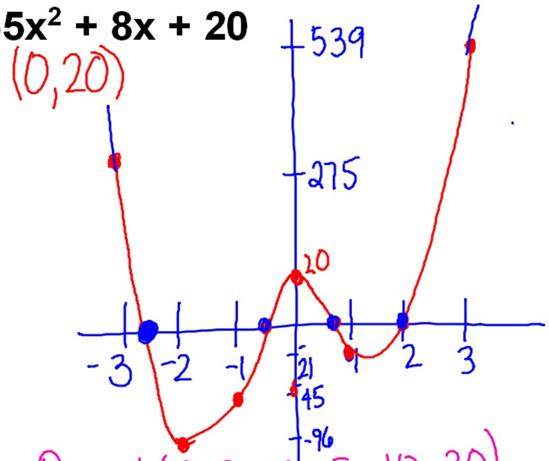
possible rational zeros

$\frac{1}{2}$

Ex. 2 $f(x) = 12x^4 + 4x^3 - 65x^2 + 8x + 20$

x	12	4	-65	8	20	
1	12	16	-49	-41	-21	← y
2	12	28	-9	-10	0	
3 UB	12	40	55	173	539	
-1	12	-8	-57	65	-45	
-2	12	-20	-25	58	-96	
-3 LB	12	-32	31	-85	275	

$-3 \le \text{Real zeros} \le 3$



$\frac{p}{q} = \frac{\pm(1, 2, 4, 5, 10, 20)}{\pm(1, 2, 3, 4, 6, 12)}$

possible rational zeros

$\pm\left(\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{6}, \frac{1}{12}, \frac{2}{3}, \frac{5}{6}, \frac{5}{12}\right)$

$-\frac{5}{2}$