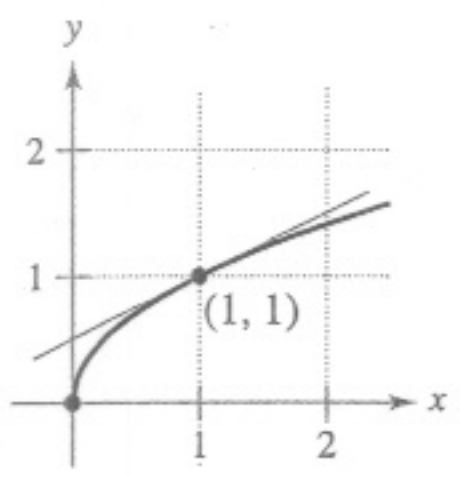
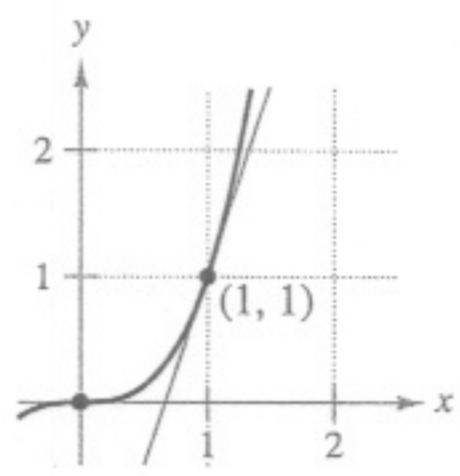
Exercises for Section 2.2

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

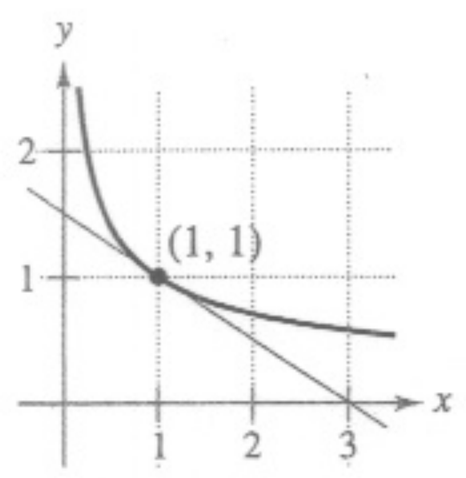
In Exercises 1 and 2, use the graph to estimate the slope of the tangent line to $y = x^n$ at the point (1, 1). Verify your answer analytically. To print an enlarged copy of the graph, go to the website *www.mathgraphs.com*.

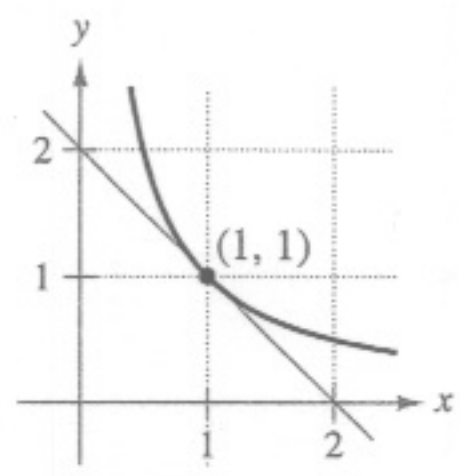
1. (a) $y = x^{1/2}$ (b) $y = x^3$





2. (a) $y = x^{-1/2}$ (b) $y = x^{-1}$





In Exercises 3–24, find the derivative of the function.

3. $y = 8$
4. $f(x) = -2$
5. $y = x^6$
6. $y = x^8$
7. $y = \dfrac{1}{x^7}$
8. $y = \dfrac{1}{x^8}$
9. $f(x) = \sqrt[5]{x}$
10. $g(x) = \sqrt[4]{x}$
11. $f(x) = x + 1$
12. $g(x) = 3x - 1$
13. $f(t) = -2t^2 + 3t - 6$
14. $y = t^2 + 2t - 3$
15. $g(x) = x^2 + 4x^3$
16. $y = 8 - x^3$
17. $s(t) = t^3 - 2t + 4$
18. $f(x) = 2x^3 - x^2 + 3x$
19. $y = \dfrac{\pi}{2}\sin\theta - \cos\theta$
20. $g(t) = \pi\cos t$
21. $y = x^2 - \frac{1}{2}\cos x$
22. $y = 5 + \sin x$
23. $y = \dfrac{1}{x} - 3\sin x$
24. $y = \dfrac{5}{(2x)^3} + 2\cos x$

In Exercises 25–30, complete the table.

Original Function	Rewrite	Differentiate	Simplify
25. $y = \dfrac{5}{2x^2}$			
26. $y = \dfrac{2}{3x^2}$			
27. $y = \dfrac{3}{(2x)^3}$			
28. $y = \dfrac{\pi}{(3x)^2}$			
29. $y = \dfrac{\sqrt{x}}{x}$			
30. $y = \dfrac{4}{x^{-3}}$			

In Exercises 31–38, find the slope of the graph of the function at the given point. Use the *derivative* feature of a graphing utility to confirm your results.

Function	Point
31. $f(x) = \dfrac{3}{x^2}$	$(1, 3)$
32. $f(t) = 3 - \dfrac{3}{5t}$	$\left(\frac{3}{5}, 2\right)$
33. $f(x) = -\frac{1}{2} + \frac{7}{5}x^3$	$\left(0, -\frac{1}{2}\right)$
34. $y = 3x^3 - 6$	$(2, 18)$
35. $y = (2x + 1)^2$	$(0, 1)$
36. $f(x) = 3(5 - x)^2$	$(5, 0)$
37. $f(\theta) = 4\sin\theta - \theta$	$(0, 0)$
38. $g(t) = 2 + 3\cos t$	$(\pi, -1)$

In Exercises 39–52, find the derivative of the function.

39. $f(x) = x^2 + 5 - 3x^{-2}$
40. $f(x) = x^2 - 3x - 3x^{-2}$
41. $g(t) = t^2 - \dfrac{4}{t^3}$
42. $f(x) = x + \dfrac{1}{x^2}$
43. $f(x) = \dfrac{x^3 - 3x^2 + 4}{x^2}$
44. $h(x) = \dfrac{2x^2 - 3x + 1}{x}$
45. $y = x(x^2 + 1)$
46. $y = 3x(6x - 5x^2)$
47. $f(x) = \sqrt{x} - 6\sqrt[3]{x}$
48. $f(x) = \sqrt[3]{x} + \sqrt[5]{x}$
49. $h(s) = s^{4/5} - s^{2/3}$
50. $f(t) = t^{2/3} - t^{1/3} + 4$
51. $f(x) = 6\sqrt{x} + 5\cos x$
52. $f(x) = \dfrac{2}{\sqrt[3]{x}} + 3\cos x$

In Exercises 53–56, (a) find an equation of the tangent line to the graph of f at the given point, (b) use a graphing utility to graph the function and its tangent line at the point, and (c) use the *derivative* feature of a graphing utility to confirm your results.

Function	Point
53. $y = x^4 - 3x^2 + 2$	$(1, 0)$
54. $y = x^3 + x$	$(-1, -2)$
55. $f(x) = \dfrac{2}{\sqrt[4]{x^3}}$	$(1, 2)$
56. $y = (x^2 + 2x)(x + 1)$	$(1, 6)$

In Exercises 57–62, determine the point(s) (if any) at which the graph of the function has a horizontal tangent line.

57. $y = x^4 - 8x^2 + 2$

58. $y = x^3 + x$

59. $y = \dfrac{1}{x^2}$

60. $y = x^2 + 1$

61. $y = x + \sin x, \quad 0 \le x < 2\pi$

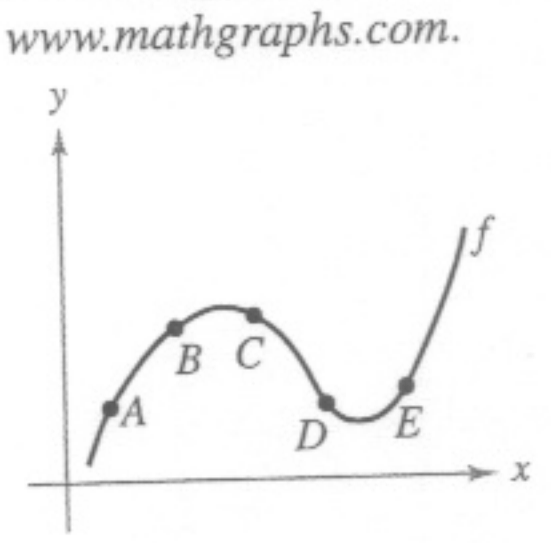
62. $y = \sqrt{3}x + 2\cos x, \quad 0 \le x < 2\pi$

In Exercises 63–66, find k such that the line is tangent to the graph of the function.

	Function	*Line*
63.	$f(x) = x^2 - kx$	$y = 4x - 9$
64.	$f(x) = k - x^2$	$y = -4x + 7$
65.	$f(x) = \dfrac{k}{x}$	$y = -\dfrac{3}{4}x + 3$
66.	$f(x) = k\sqrt{x}$	$y = x + 4$

Writing About Concepts

67. Use the graph of f to answer each question. To print an enlarged copy of the graph, go to the website *www.mathgraphs.com*.



(a) Between which two consecutive points is the average rate of change of the function greatest?

(b) Is the average rate of change of the function between A and B greater than or less than the instantaneous rate of change at B?

(c) Sketch a tangent line to the graph between C and D such that the slope of the tangent line is the same as the average rate of change of the function between C and D.

68. Sketch the graph of a function f such that $f' > 0$ for all x and the rate of change of the function is decreasing.

In Exercises 69 and 70, the relationship between f and g is given. Explain the relationship between f' and g'.

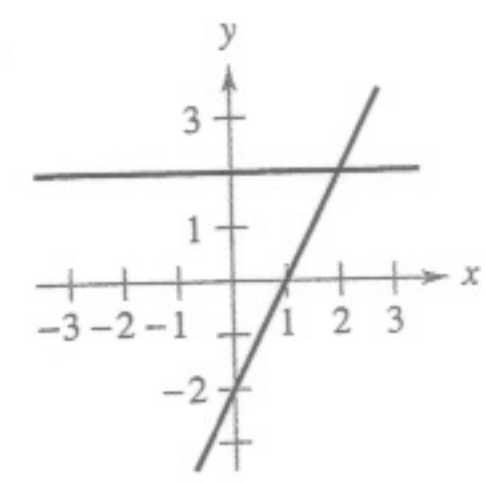
69. $g(x) = f(x) + 6$

70. $g(x) = -5f(x)$

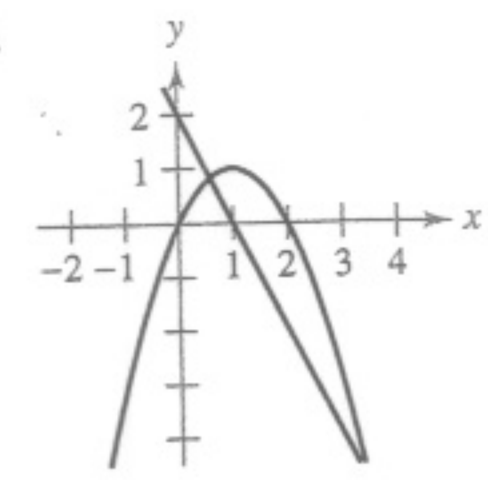
Writing About Concepts (continued)

In Exercises 71 and 72, the graphs of a function f and its derivative f' are shown on the same set of coordinate axes. Label the graphs as f or f' and write a short paragraph stating the criteria used in making the selection. To print an enlarged copy of the graph, go to the website *www.mathgraphs.com*.

71.



72.



73. Sketch the graphs of $y = x^2$ and $y = -x^2 + 6x - 5$, and sketch the two lines that are tangent to both graphs. Find equations of these lines.

74. Show that the graphs of the two equations $y = x$ and $y = 1/x$ have tangent lines that are perpendicular to each other at their point of intersection.

75. Show that the graph of the function

$$f(x) = 3x + \sin x + 2$$

does not have a horizontal tangent line.

76. Show that the graph of the function

$$f(x) = x^5 + 3x^3 + 5x$$

does not have a tangent line with a slope of 3.

In Exercises 77 and 78, find an equation of the tangent line to the graph of the function f through the point (x_0, y_0) not on the graph. To find the point of tangency (x, y) on the graph of f, solve the equation

$$f'(x) = \frac{y_0 - y}{x_0 - x}.$$

77. $f(x) = \sqrt{x}$
$(x_0, y_0) = (-4, 0)$

78. $f(x) = \dfrac{2}{x}$
$(x_0, y_0) = (5, 0)$

79. *Linear Approximation* Use a graphing utility, with a square window setting, to zoom in on the graph of

$$f(x) = 4 - \tfrac{1}{2}x^2$$

to approximate $f'(1)$. Use the derivative to find $f'(1)$.

80. *Linear Approximation* Use a graphing utility, with a square window setting, to zoom in on the graph of

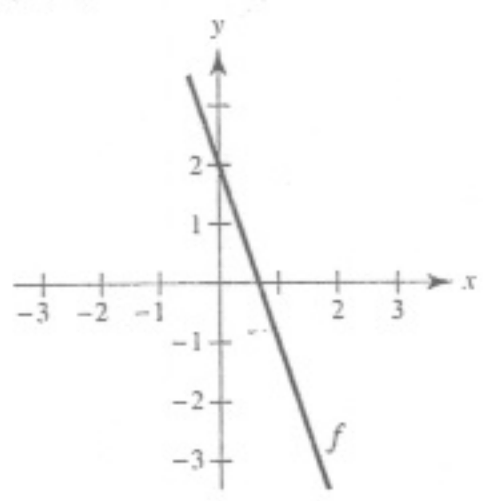
$$f(x) = 4\sqrt{x} + 1$$

to approximate $f'(4)$. Use the derivative to find $f'(4)$.

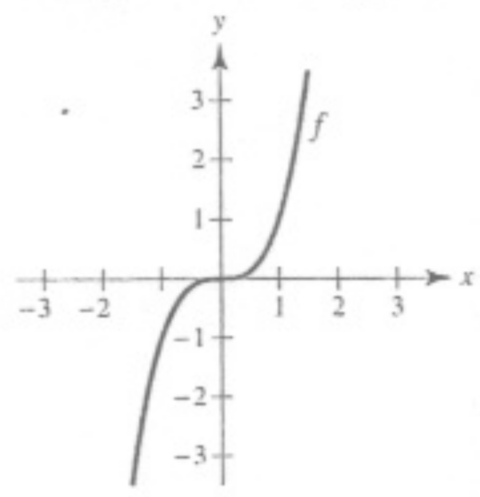
49. $f(x) = 5 - 3x$
$c = 1$

51. $f(x) = -x^2$
$c = 6$

53. $f(x) = -3x + 2$



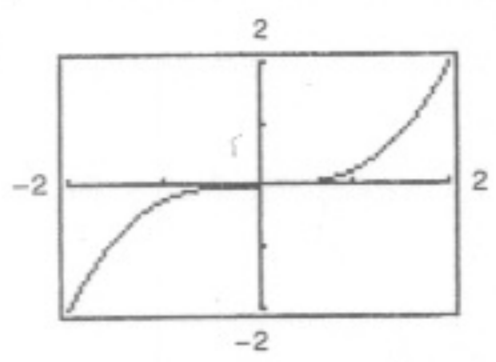
55. Answers will vary.
Sample answer: $f(x) = x^3$



57. $y = 2x + 1;\ y = -2x + 9$

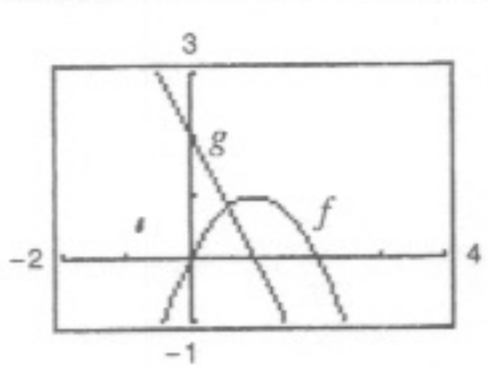
59. (a) -3 (b) 0
(c) The graph is moving downward to the right when $x = 1$.
(d) The graph is moving upward to the right when $x = -4$.
(e) Positive. Because $g'(x) > 0$ on $[3, 6]$, the graph of g is moving upward to the right.
(f) No. Knowing only $g'(2)$ is not sufficient information. $g'(2)$ remains the same for any vertical translation of g.

61.



x	-2	-1.5	-1	-0.5	0	0.5	1	1.5	2
$f(x)$	-2	$-\frac{27}{32}$	$-\frac{1}{4}$	$-\frac{1}{32}$	0	$\frac{1}{32}$	$\frac{1}{4}$	$\frac{27}{32}$	2
$f'(x)$	3	$\frac{27}{16}$	$\frac{3}{4}$	$\frac{3}{16}$	0	$\frac{3}{16}$	$\frac{3}{4}$	$\frac{27}{16}$	3

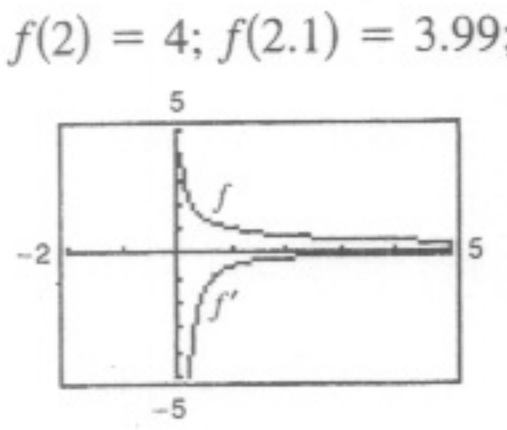
63.



$g(x) \approx f'(x)$

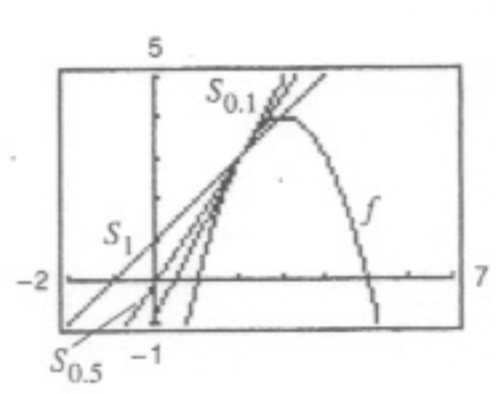
65. $f(2) = 4;\ f(2.1) = 3.99;\ f'(2) \approx -0.1$

67.



As x approaches infinity, the graph of f approaches a line of slope 0. Thus $f'(x)$ approaches 0.

69. (a)



(b) The graphs of S for decreasing values of Δx are secant lines approaching the tangent line to the graph of f at the point $(2, f(2))$.

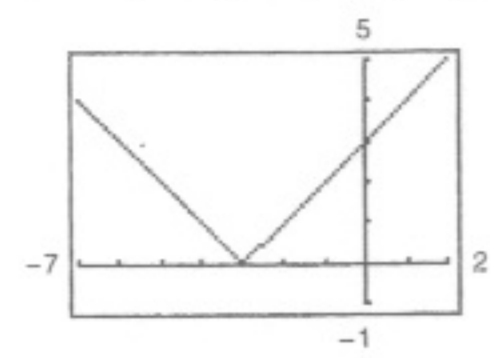
71. 4 **73.** 4 **75.** $g(x)$ is not differentiable at $x = 0$.

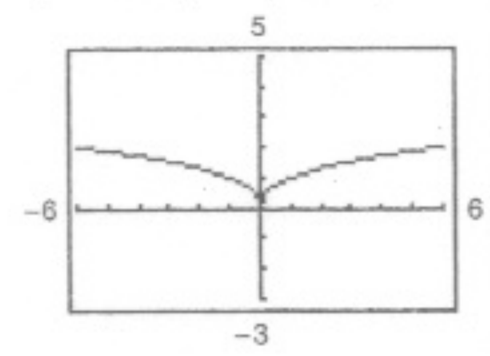
77. $f(x)$ is not differentiable at $x = 6$.

79. $h(x)$ is not differentiable at $x = -5$.

81. $(-\infty, -1) \cup (-1, \infty)$ **83.** $(-\infty, 3) \cup (3, \infty)$ **85.** $(1, \infty)$

87. $(-\infty, -3) \cup (-3, \infty)$ **89.** $(-\infty, 0) \cup (0, \infty)$



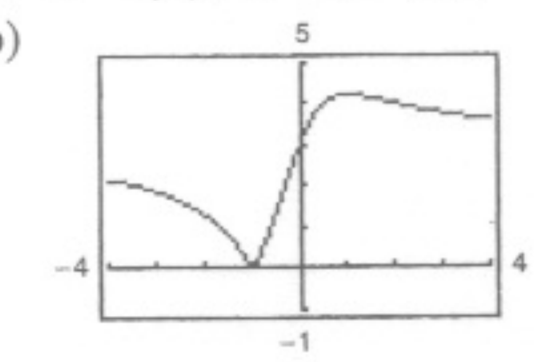


91. The derivative from the left is -1 and the derivative from the right is 1, so f is not differentiable at $x = 1$.

93. The derivatives from both the right and the left are 0, so $f'(1) = 0$.

95. f is differentiable at $x = 2$.

97. (a) $d = (3|m + 1|)/\sqrt{m^2 + 1}$
(b)



Not differentiable at $m = -1$

99. False. The slope is $\lim_{\Delta x \to 0} \dfrac{f(2 + \Delta x) - f(2)}{\Delta x}$.

101. False. For example: $f(x) = |x|$. The derivative from the left and the derivative from the right both exist but are not equal.

103. Proof

Section 2.2 (page 115)

1. (a) $\frac{1}{2}$ (b) 3 **3.** 0 **5.** $6x^5$ **7.** $-7/x^8$ **9.** $1/(5x^{4/5})$

11. 1 **13.** $-4t + 3$ **15.** $2x + 12x^2$ **17.** $3t^2 - 2$

19. $\dfrac{\pi}{2}\cos\theta + \sin\theta$ **21.** $2x + \dfrac{1}{2}\sin x$ **23.** $-\dfrac{1}{x^2} - 3\cos x$

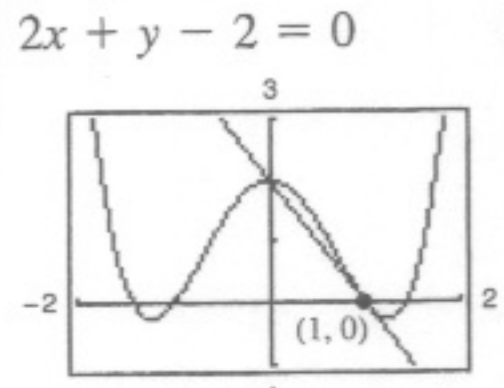
	Function	*Rewrite*	*Derivative*	*Simplify*
25.	$y = \dfrac{5}{2x^2}$	$y = \dfrac{5}{2}x^{-2}$	$y' = -5x^{-3}$	$y' = -\dfrac{5}{x^3}$
27.	$y = \dfrac{3}{(2x)^3}$	$y = \dfrac{3}{8}x^{-3}$	$y' = -\dfrac{9}{8}x^{-4}$	$y' = -\dfrac{9}{8x^4}$
29.	$y = \dfrac{\sqrt{x}}{x}$	$y = x^{-1/2}$	$y' = -\dfrac{1}{2}x^{-3/2}$	$y' = -\dfrac{1}{2x^{3/2}}$

31. -6 **33.** 0 **35.** 4 **37.** 3 **39.** $2x + 6/x^3$

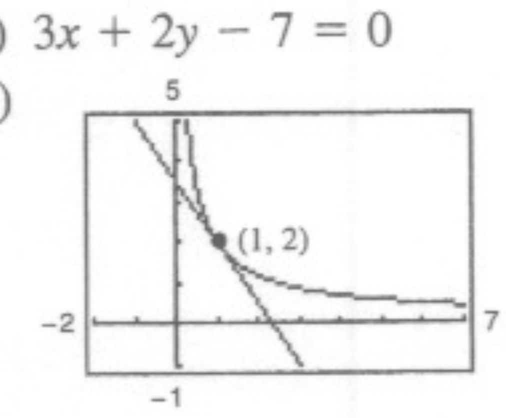
41. $2t + 12/t^4$ **43.** $(x^3 - 8)/x^3$ **45.** $3x^2 + 1$

47. $\dfrac{1}{2\sqrt{x}} - \dfrac{2}{x^{2/3}}$ **49.** $\dfrac{4}{5s^{1/5}} - \dfrac{2}{3s^{1/3}}$ **51.** $\dfrac{3}{\sqrt{x}} - 5\sin x$

53. (a) $2x + y - 2 = 0$
(b)



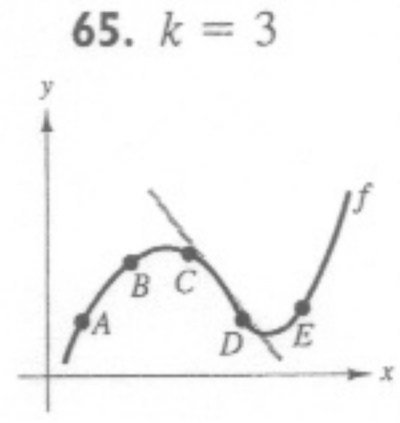
55. (a) $3x + 2y - 7 = 0$
(b)



57. $(0, 2), (-2, -14), (2, -14)$ **59.** No horizontal tangents

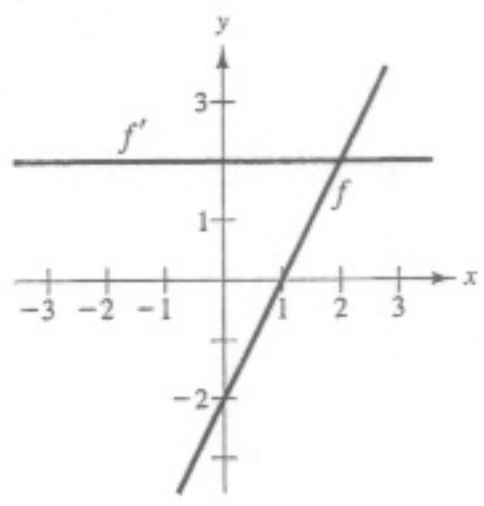
61. (π, π) **63.** $k = 2,\ k = -10$ **65.** $k = 3$

67. (a) A and B (b) Greater (c)



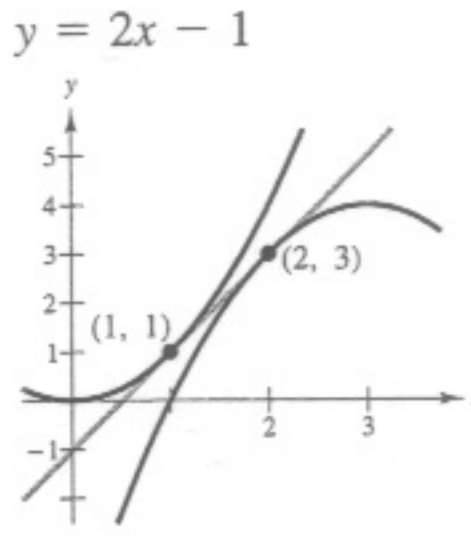
69. $g'(x) = f'(x)$

71.

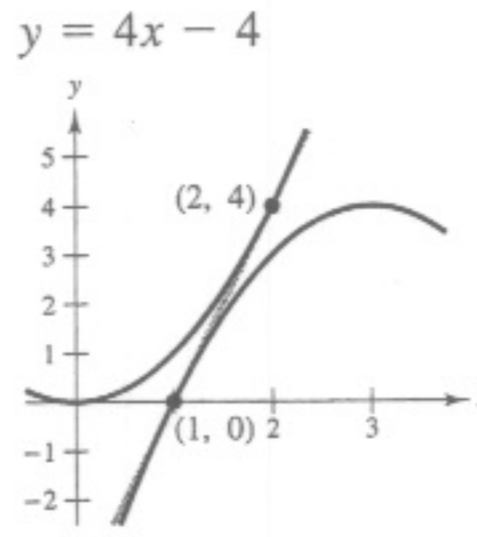


The rate of change of f is constant and therefore f' is a constant function.

73. $y = 2x - 1$

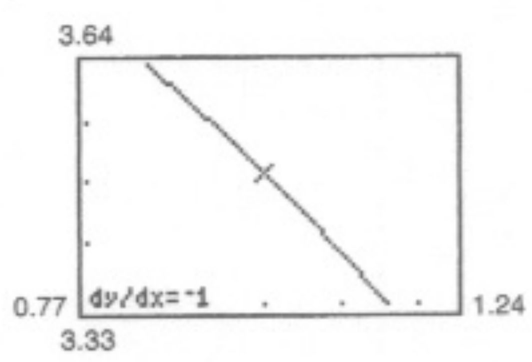


$y = 4x - 4$



75. $f'(x) = 3 + \cos x \neq 0$ for all x. **77.** $x - 4y + 4 = 0$

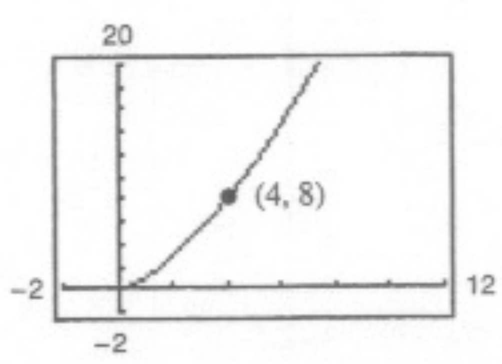
79.



$f'(1)$ appears to be close to -1.

$f'(1) = -1$

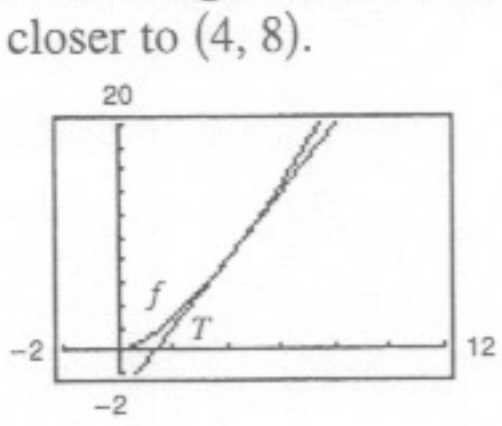
81. (a)



(3.9, 7.7019),
$S(x) = 2.981x - 3.924$

(b) $T(x) = 3(x - 4) + 8 = 3x - 4$
The slope (and equation) of the secant line approaches that of the tangent line at (4, 8) as you choose points closer and closer to (4, 8).

(c)



The approximation becomes less accurate.

(d)

Δx	-3	-2	-1	-0.5	-0.1	0
$f(4 + \Delta x)$	1	2.828	5.196	6.548	7.702	8
$T(4 + \Delta x)$	-1	2	5	6.5	7.7	8

Δx	0.1	0.5	1	2	3
$f(4 + \Delta x)$	8.302	9.546	11.180	14.697	18.520
$T(4 + \Delta x)$	8.3	9.5	11	14	17

83. False. Let $f(x) = x$ and $g(x) = x + 1$.

85. False. $dy/dx = 0$ **87.** True

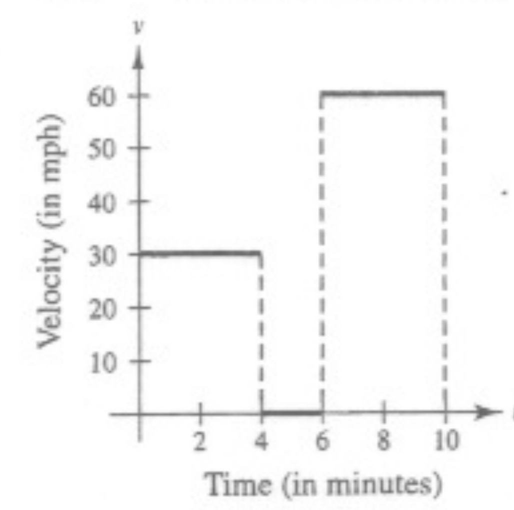
89. Average rate: 2
Instantaneous rates:
$f'(1) = 2$; $f'(2) = 2$

91. Average rate: $\frac{1}{2}$
Instantaneous rates:
$f'(1) = 1$; $f'(2) = \frac{1}{4}$

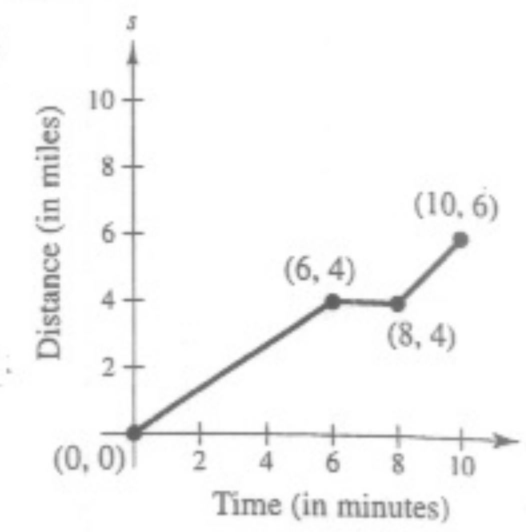
93. (a) $s(t) = -16t^2 + 1362$; $v(t) = -32t$ (b) -48 ft/sec
(c) $s'(1) = -32$ ft/sec; $s'(2) = -64$ ft/sec
(d) $t = \dfrac{\sqrt{1362}}{4} \approx 9.226$ sec (e) -295.242 ft/sec

95. $v(5) = 71$ m/sec; $v(10) = 22$ m/sec

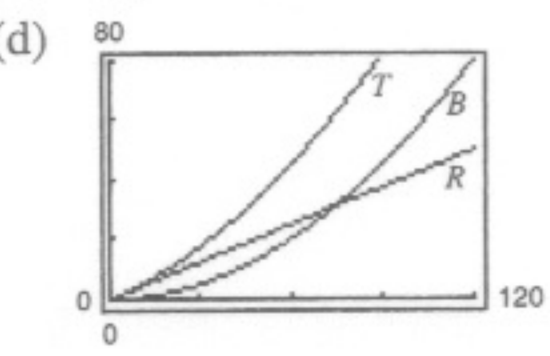
97.



99.



101. (a) $R(v) = 0.417v - 0.02$
(b) $B(v) = 0.0056v^2 + 0.001v + 0.04$
(c) $T(v) = 0.0056v^2 + 0.418v + 0.02$
(d)



(e) $T'(v) = 0.0112v + 0.418$
$T'(40) = 0.866$
$T'(80) = 1.314$
$T'(100) = 1.538$

(f) Stopping distance increases at an increasing rate.

103. $V'(4) = 48\text{ cm}^2$ **105.** Proof

107. (a) The rate of change of the number of gallons of gasoline sold when the price is $1.479
(b) In general, the rate of change when $p = 1.479$ should be negative. As prices go up, sales go down.

109. $y = 2x^2 - 3x + 1$ **111.** $y = -9x, y = -\frac{9}{4}x - \frac{27}{4}$

113. $a = \frac{1}{3}, b = -\frac{4}{3}$

115. $f_1(x) = |\sin x|$ is differentiable for all $x \neq n\pi$, n an integer.
$f_2(x) = \sin|x|$ is differentiable for all $x \neq 0$.

Section 2.3 (page 126)

1. $2(2x^3 - 3x^2 + x - 1)$ **3.** $(7t^2 + 4)/(3t^{2/3})$

5. $x^2(3\cos x - x\sin x)$ **7.** $(1 - x^2)/(x^2 + 1)^2$

9. $(1 - 8x^3)/[3x^{2/3}(x^3 + 1)^2]$ **11.** $(x\cos x - 2\sin x)/x^3$

13. $f'(x) = (x^3 - 3x)(4x + 3) + (2x^2 + 3x + 5)(3x^2 - 3)$
$= 10x^4 + 12x^3 - 3x^2 - 18x - 15$
$f'(0) = -15$

15. $f'(x) = \dfrac{x^2 - 6x + 4}{(x - 3)^2}$ **17.** $f'(x) = \cos x - x\sin x$

$f'(1) = -\dfrac{1}{4}$ $f'\left(\dfrac{\pi}{4}\right) = \dfrac{\sqrt{2}}{8}(4 - \pi)$

	Function	*Rewrite*	*Differentiate*	*Simplify*
19.	$y = \dfrac{x^2 + 2x}{3}$	$y = \dfrac{1}{3}(x^2 + 2x)$	$y' = \dfrac{1}{3}(2x + 2)$	$y' = \dfrac{2(x + 1)}{3}$
21.	$y = \dfrac{7}{3x^3}$	$y = \dfrac{7}{3}x^{-3}$	$y' = -7x^{-4}$	$y' = -\dfrac{7}{x^4}$
23.	$y = \dfrac{4x^{3/2}}{x}$	$y = 4x^{1/2}$, $x > 0$	$y' = 2x^{-1/2}$	$y' = \dfrac{2}{\sqrt{x}}$, $x > 0$

25. $\dfrac{(x^2 - 1)(-2 - 2x) - (3 - 2x - x^2)(2x)}{(x^2 - 1)^2} = \dfrac{2}{(x + 1)^2}, x \neq 1$