Lesson 2 - 5

Algebraic Proof

Going Deeper

Essential question: *What kinds of justifications can you use in writing algebraic and geometric proofs?*

PREP FOR CC.9–12.G.CO.9

1 ENGAGE **Introducing Proofs**

In mathematics, a ______ is a logical argument that uses a sequence of statements to prove a conjecture. Once the conjecture is proved, it is called a ______.

Each statement in a proof must follow logically from what has come ______ and must have a ______ to support it. The ______ may be a piece of given information, a definition, a previously proven theorem, or a mathematical property.

The table states some properties of equality that you have seen in earlier courses. You have used these properties to solve algebraic equations and you will often use these properties as reasons in a proof.

Properties of Equality	
Addition Property of Equality	
Subtraction Property of Equality	
Multiplication Property of Equality	
Division Property of Equality	
Reflexive Property of Equality	
Symmetric Property of Equality	
Transitive Property of Equality	
Substitution Property of Equality	

Remember!

The Distributive Property states that

$a(b + c) =$ ______

Example 4: Identifying Property of Equality and Congruence

Identify the property that justifies each statement.

A. $\angle QRS \cong \angle QRS$ ______

B. $m\angle 1 = m\angle 2$ so $m\angle 2 = m\angle 1$ ______

C. $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$, so $\overline{AB} \cong \overline{EF}$. ______

D. $32° = 32°$ ______

4a. $DE = GH$, so $GH = DE$. ______

4b. $94° = 94°$ ______

4c. $0 = a$, and $a = x$. So $0 = x$. ______

4d. $\angle A \cong \angle Y$, so $\angle Y \cong \angle A$ ______

REFLECT

1a. Given the equation 3 = ______ you quickly write the solution as $x =$ ______ Which property or properties of equality are you using? Explain.

1b. Give an example of an equation that you can solve using the Division Property of Equality. Explain how you would use this property to solve the equation.

Solve the equation $4m - 8 = -12$. Write a justification for each step.

$4m - 8 = -12$

$\quad +8 \qquad +8$

$4m = -4$ Simplify.

$\frac{4m}{4} = \frac{-4}{4}$

$m = -1$ Simplify.

Solve the equation $\frac{1}{2}t = -7$. Write a justification for each step.

$\frac{1}{2}t = -7$

$2\left(\frac{1}{2}\right)t = 2(-7)$

$t = -14$

Solve each equation. Write a justification for each step.

1. $\frac{z-5}{6} = -2$

$\frac{z-5}{6} = -2$

$z - 5 = -12$

$z = -7$

2. $6r - 3 = -2(r + 1)$

$6r - 3 = -2(r + 1)$

$6r - 3 = -2r - 2$

$8r - 3 = -2$

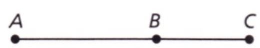
$8r = 1$

$r = \frac{1}{8}$

A ______ is a statement that is accepted as true without proof. Like undefined terms, postulates are basic building blocks of geometry. The following postulate states that the lengths of segments "add up" in a natural way.

Segment Addition Postulate

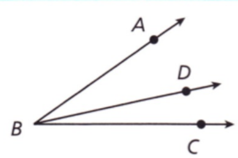
If *B* is between *A* and *C*, then ______



The Angle Addition Postulate is similar to the Segment Addition Postulate.

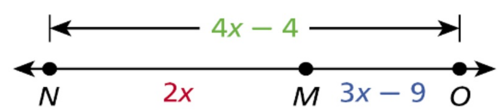
Angle Addition Postulate

If *D* is in the interior of $\angle ABC$, then ______



Example 3: Solving an Equation in Geometry

Write a justification for each step.



$NO = NM + MO$

$4x - 4 = 2x + (3x - 9)$

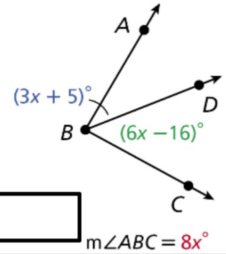
$4x - 4 = 5x - 9$

$-4 = x - 9$

$5 = x$

Check It Out! Example 3

Write a justification for each step.



Step	Justification
$m\angle ABC = m\angle ABD + m\angle DBC$	
$8x° = (3x + 5)° + (6x - 16)°$	
$8x = 9x - 11$	
$-x = -11$	
$x = 11$	

1. If A, B, C, and D are collinear, as shown in the figure, with $AC = BD$, then $AB = CD$. Complete the proof by writing the missing statements or reasons.

Given: $AC = BD$
Prove: $AB = CD$

Statements	Reasons
1.	1. Given
2. $AC = AB + BC$; $BD = BC + CD$	2.
3.	3. Substitution Property of Equality
4. $AB = CD$	4.

2. In the figure, X is the midpoint of $\overline{WY}$, and Y is the midpoint of $\overline{XZ}$. Explain how to prove $WX = YZ$.