Algebra 2H Extra Notes Section 3-4

Factoring by Grouping

ex. 1 $x^2y^2 - 3x^2 - 4y^2 + 12$

$(x^2y^2 - 3x^2) + (-4y^2 + 12)$

$x^2(y^2 - 3) - 4(y^2 - 3) = (x^2 - 4)(y^2 - 3) = (x-2)(x+2)(y^2-3)$

$a \quad c \quad -b \quad c \quad = (a-b)c$

ex. 2 $4x^4 + 4x^3 - 8x^2 - 8x$

$(4x^4 + 4x^3) + (-8x^2 - 8x)$

$4x^3(x+1) - 8x(x+1) = (4x^3 - 8x)(x+1)$

$4x(x^2 - 2)(x+1)$

Factoring Cubes

cube roots — prime

sum of cubes: $a^3 \oplus b^3 = (a \oplus b)(a^2 - ab + b^2)$

difference of cubes: $a^3 \ominus b^3 = (a \ominus b)(a^2 + ab + b^2)$

ex. 3 $x^3 + 64$

$(x)^3 + (4)^3 = (x+4)(x^2 - 4x + 16)$

$a \quad b \qquad\qquad 4^2$

ex. 4 $512x^6 - y^3$

$(8x^2)^3 - (y)^3 = (8x^2 - y)(64x^4 + 8x^2y + y^2)$

$a \quad b$

Roots: $\sqrt[3]{x} \times \sqrt[3]{y} = \sqrt[3]{64}$

$x^{1/3} = 64$ (1:3)

Examples: factor completely

ex. 5 $30x^4 + 5x^2 - 60$

$-72x^4 = \text{prod}$

$5(6x^4 + x^2 - 12) = 5(2x^2 + 3)(3x^2 - 4)$

$\text{Sum } x^2 = -8x^2, 9x^2$ ($9x^2$, $-8x^2$)

ex. 6 $25x^8 - 30x^4 + 9$

$(5x^4)^2 - 2(15x^4) + 3^2 = (5x^4 - 3)^2$

ex. 7 $3x^{20n} - 48$

$= 3(x^{20n} - 16) = 3(x^{10n} + 4)(x^{10n} - 4)$

$3(x^{10n} + 4)(x^{5n} - 2)(x^{5n} + 2)$

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Extra Examples Section 3-4 and 3-5

1 **Find all real solutions.**

$8x^4 - 30x^2 + 7 = 0$ $\quad x^2 =$

prod $= 56x^4$
sum $= -30x^2$
$-28x^2, -2x^2$

$(4x^2 - 1)(2x^2 - 7) = 0$ ($-2x^2$, $-28x^2$)

$4x^2 - 1 = 0$ $\quad 2x^2 - 7 = 0$
$4x^2 = 1$ $\quad 2x^2 = 7$
$x^2 = \frac{1}{4}$ $\quad x^2 = \frac{7}{2}$
$x = \pm\frac{1}{2}$ $\quad x = \pm\frac{\sqrt{14}}{2}$ ($\sqrt{x^2} = \frac{\sqrt{7}\sqrt{2}}{\sqrt{2}\sqrt{2}}$)

2 **Find all real solutions.**

$64x^3 = 27$

$64x^3 - 27 = 0$ (1 real, 2 complex)

$(4x - 3)(16x^2 + 12x + 9) = 0$

$x = \frac{3}{4}$ $\quad b^2 - 4ac$ $\quad 12^2 - 4(16)9 < 0$

$\sqrt[3]{x^3} = \sqrt[3]{\frac{27}{64}}$ $\quad x = \frac{3}{4}$

3

Factor p(x) given that p(a) = 0.

$p(x) = 2x^3 + 7x^2 - 33x - 18$; a = ~~+16~~ -6

-6 (a) | 2 $\quad$ 7 $\quad$ −33 $\quad$ −18
$\quad\quad$ −12 $\quad$ 30 $\quad$ 18
$\quad$ 2 $\quad$ −5 $\quad$ −3 $\quad$ | 0

$(x+6)(2x^2 - 5x - 3) = (x+6)(2x+1)(x-3)$

Extra Example ***Section 3-6*** Oct 15

If 1/3 is a zero of $f(x) = 9x^5 - 9x^4 - 13x^3 - x^2 - 22x + 8$,

a) find all zeros: $2, \frac{1}{3}, \pm i, -\frac{4}{3}$

$\frac{1}{3}$ | 9 -9 -13 -1 -22 8
3 -2 -5 -2 -8
9 -6 -15 -6 -24 | 0

$\div 3\ (9x^4 - 6x^3 - 15x^2 - 6x - 24)$
$3x^4 - 2x^3 - 5x^2 - 2x - 8$

2 | 3 -2 -5 -2 -8
6 8 6 8
3 4 3 4 | ☺

$(3x^3 + 4x^2) + (3x + 4)$
$x^2(3x+4) + 1(3x+4)$
$(x^2+1)(3x+4) = 0$
$\pm i, -4/3$

$\frac{c}{b} = \pm(8, 4, 2, 1)$
$\pm(3, 1)$

$\frac{c}{b} = \pm\left(\frac{8}{3}, 8, \frac{4}{3}, 4, \frac{2}{3}, 2, \frac{1}{3}, 1\right)$

b) write f(x) in factored form with linear factors

$f(x) = (3x-1)(x-2)(3x+4)(x-i)(x+i)$

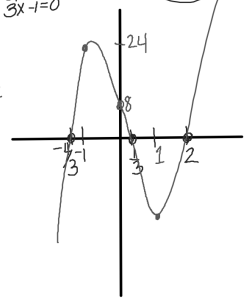
$x = 1/3$ (2) $x = -4/3$ i $-i$

$3x = 1$
$3x - 1 = 0$

$x^2 - i^2$
$(x^2 + 1)$

c) graph f(x)

x	y
-1	$(-4)(-3)(1)(2) = 24$
0	8
1	$2(-1)(7)(2) = -28$



Extra Example Sec 3-6 Oct. 16

Write the simplest polynomial function with integer coefficients that has the zeros $-\sqrt{2}$, $3i$, and -2/3 .

$\sqrt{2}$ $-3i$ $x = -\frac{2}{3}$ $3x = -2$ $3x+2 = 0$

$$P(x) = (x+\sqrt{2})(x-\sqrt{2})(x-3i)(x+3i)(3x+2)$$

$$= (x^2-2)(x^2-9i^2)$$

$$= (x^2-2)(x^2+9) \quad (i^2 = -1)$$

$$= (x^4+7x^2-18)(3x+2) = 3x^5+2x^4+21x^3+14x^2-54x-36$$