7.1 nth Roots and Rational Exponents

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$\sqrt[n]{a}$ is read as the "*n*th root of *a*"

a is the *radicand* and *n* is the *index (or root)*

$\sqrt[4]{16}$ is read as the 4th root of 16

Ex. 1 Find the real *n*th roots:

$n = 3$, $a = -343$ $\quad \sqrt[3]{-343} = -7$

$n = 4$, $a = 81$ $\quad \sqrt[4]{81} = 3$

2nd $\sqrt[3]{x}$ 2nd $\sqrt[x]{y}$

$\boxed{y^x}$ $\boxed{\wedge}$

when n is even, there is only 1 positive principal root.

Ex.2 Evaluate:

$\sqrt[3]{-512} = -8$ $\quad (-8)^3 = -512$

$\sqrt[6]{4096} = 4$

Ex.3 Solve for *x*. (all solutions)

$6x^4 = 3700$

$\sqrt[4]{x^4} = \sqrt[4]{\left(\frac{3700}{6}\right)}$

$x \approx \pm 4.98$

$\sqrt[3]{(x+1)^3} = \sqrt[3]{18}$

$x+1 = \sqrt[3]{18}$

$x = \sqrt[3]{18} - 1 \approx 1.62$

Definition of Rational Exponent

If $b > 0$, $n > 0$, and m and n are integers, then

$$b^{m/n} = \sqrt[n]{b^m} = \left(\sqrt[n]{b}\right)^m$$

Ex.4 **Write using rational exponents.**

$\sqrt{6} = 6^{1/2}$

$\sqrt[2]{6^1}$

$\left(\sqrt[8]{80}\right)^2 = 80^{2/8} = 80^{1/4}$

Ex.5 **Evaluate without a calculator.**

$64^{5/6} = \left(\sqrt[6]{64}\right)^5$

$2^5 = 32$

$(-32)^{-3/5} = \dfrac{1}{(-32)^{3/5}}$

$= \dfrac{1}{\left(\sqrt[5]{-32}\right)^3} = \dfrac{1}{-8}$

$(-2)^3$

Ex.6 **Simplify to a single power of *x*.**

$$\frac{\sqrt[4]{x^1} \cdot \sqrt[5]{x^2}}{\sqrt[2]{x^1}} = \frac{x^{1/4} \cdot x^{2/5}}{x^{1/2}} = \frac{x^{\frac{5+8}{20}}}{x^{\frac{10}{20}}}$$

$$= \frac{x^{13/20}}{x^{10/20}} = x^{3/20}$$

Ex.7 **Find *x*.**

$9\sqrt[3]{\sqrt[4]{3}} = 3^x$

$3^2 \cdot \sqrt[3]{\left(3^{1/4}\right)^1}$

$3^2 \cdot \left(3^{1/4}\right)^{1/3} = 3^{2+\frac{1}{12}} = 3^{\frac{25}{12}}$