7. $\int_1^e \left(\frac{x^2-1}{x}\right)dx =$

(A) $e-\frac{1}{e}$ (B) e^2-e (C) $\frac{e^2}{2}-e+\frac{1}{2}$ (D) e^2-2 (E) $\frac{e^2}{2}-\frac{3}{2}$

11. If f is a linear function and $0<a<b$, then $\int_a^b f''(x)\,dx =$

(A) 0 (B) 1 (C) $\frac{ab}{2}$ (D) $b-a$ (E) $\frac{b^2-a^2}{2}$

15. If $F(x)=\int_0^x \sqrt{t^3+1}\,dt$, then $F'(2)=$

(A) –3 (B) –2 (C) 2 (D) 3 (E) 18

20. What are all values of k for which $\int_{-3}^{k} x^2 dx = 0$?

(A) –3 (B) 0 (C) 3 (D) –3 and 3 (E) –3, 0, and 3

7. E $\int_1^e \frac{x^2-1}{x}\,dx = \int_1^e x - \frac{1}{x}\,dx = \left(\frac{1}{2}x^2 - \ln x\right)\Big|_1^e = \left(\frac{1}{2}e^2 - 1\right) - \left(\frac{1}{2} - 0\right) = \frac{1}{2}e^2 - \frac{3}{2}$

11. A Since f is linear, its second derivative is zero. The integral gives the area of a rectangle with zero height and width $(b-a)$. This area is zero.

15 D By the Fundamental Theorem of Calculus, $F'(x) = \sqrt{x^3+1}$, thus $F'(2) = \sqrt{2^3+1} = \sqrt{9} = 3$.

20. A $\int_{-3}^{k} x^2\,dx = \frac{1}{3}x^3\Big|_{-3}^{k} = \frac{1}{3}\left(k^3 - (-3)^3\right) = \frac{1}{3}\left(k^3 + 27\right) = 0$ only when $k = -3$.

82. If f is a continuous function and if $F'(x) = f(x)$ for all real numbers x, then $\int_1^3 f(2x)\,dx =$

(A) $2F(3) - 2F(1)$

(B) $\frac{1}{2}F(3) - \frac{1}{2}F(1)$

(C) $2F(6) - 2F(2)$

(D) $F(6) - F(2)$

(E) $\frac{1}{2}F(6) - \frac{1}{2}F(2)$

82. If $f(x) = g(x) + 7$ for $3 \le x \le 5$, then $\int_3^5 [f(x) + g(x)]\,dx =$

(A) $2\int_3^5 g(x)\,dx + 7$

(B) $2\int_3^5 g(x)\,dx + 14$

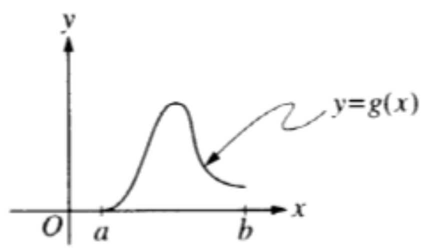
(C) $2\int_3^5 g(x)\,dx + 28$

(D) $\int_3^5 g(x)\,dx + 7$

(E) $\int_3^5 g(x)\,dx + 14$

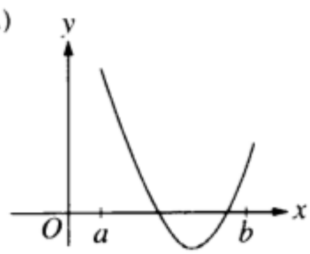
82. E Since F is an antiderivative of f, $\int_1^3 f(2x)\,dx = \frac{1}{2}F(2x)\Big|_1^3 = \frac{1}{2}\big(F(6) - F(2)\big)$

82. B $\int_3^5 \left[f(x) + g(x)\right]dx = \int_3^5 \left[2g(x) + 7\right]dx = 2\int_3^5 g(x)\,dx + (7)(2) = 2\int_3^5 g(x)\,dx + 14$

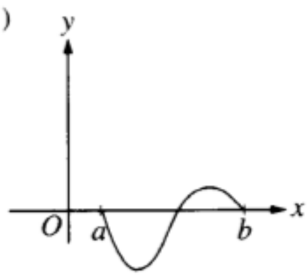


88. Let $g(x)=\int_a^x f(t)\,dt$, where $a \le x \le b$. The figure above shows the graph of g on $[a,b]$. Which of the following could be the graph of f on $[a,b]$?

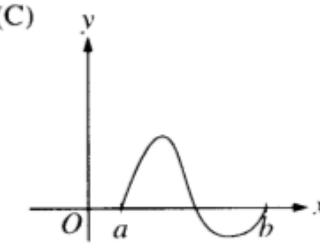
(A)



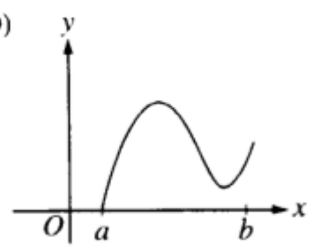
(B)



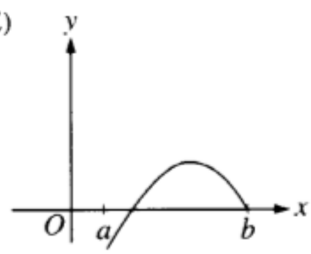
(C)



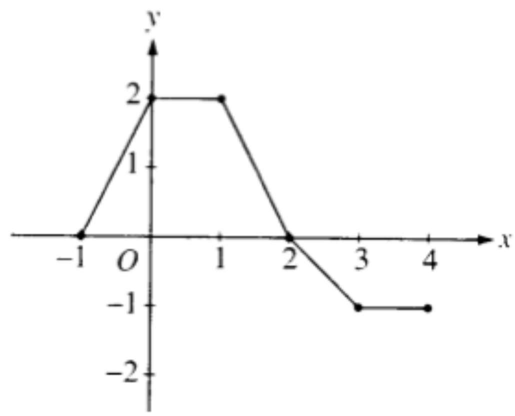
(D)



(E)



88. C From the given information, f is the derivative of g. We want a graph for f that represents the slopes of the graph g. The slope of g is zero at a and b. Also the slope of g changes from positive to negative at one point between a and b. This is true only for figure (C).



2. The graph of a piecewise-linear function f, for $-1 \le x \le 4$, is shown above. What is the value of $\int_{-1}^{4} f(x)\,dx$?

(A) 1 (B) 2.5 (C) 4 (D) 5.5 (E) 8

3. $\int_{1}^{2} \frac{1}{x^2}\,dx =$

(A) $-\frac{1}{2}$ (B) $\frac{7}{24}$ (C) $\frac{1}{2}$ (D) 1 (E) $2\ln 2$

5. $\int_{0}^{x} \sin t\,dt =$

(A) $\sin x$ (B) $-\cos x$ (C) $\cos x$ (D) $\cos x - 1$ (E) $1 - \cos x$

2. B $\int_{-1}^{4} f(x)dx = \int_{-1}^{2} f(x)dx + \int_{2}^{4} f(x)dx$

= Area of trapezoid(1) – Area of trapezoid(2) = $4 - 1.5 = 2.5$

3. C $\int_{1}^{2} \frac{1}{x^2}dx = \int_{1}^{2} x^{-2}dx = -x^{-1}\Big|_{1}^{2} = \frac{1}{2}$

5. E $\int_{0}^{x} \sin t\, dt = -\cos t\Big|_{0}^{x} = -\cos x - (-\cos 0) = -\cos x + 1 = 1 - \cos x$

25. The closed interval $[a,b]$ is partitioned into n equal subintervals, each of width Δx, by the numbers $x_0, x_1, \ldots, x_n$ where $a = x_0 < x_1 < x_2 < \cdots < x_{n-1} < x_n = b$. What is $\lim_{n\to\infty} \sum_{i=1}^{n} \sqrt{x_i}\,\Delta x$?

(A) $\frac{2}{3}\left(b^{\frac{3}{2}} - a^{\frac{3}{2}}\right)$

(B) $b^{\frac{3}{2}} - a^{\frac{3}{2}}$

(C) $\frac{3}{2}\left(b^{\frac{3}{2}} - a^{\frac{3}{2}}\right)$

(D) $b^{\frac{1}{2}} - a^{\frac{1}{2}}$

(E) $2\left(b^{\frac{1}{2}} - a^{\frac{1}{2}}\right)$

82. If $0 \le x \le 4$, of the following, which is the greatest value of x such that $\int_0^x (t^2 - 2t)\,dt \ge \int_2^x t\,dt$?

(A) 1.35 (B) 1.38 (C) 1.41 (D) 1.48 (E) 1.59

88. Let $f(x) = \int_0^{x^2} \sin t\,dt$. At how many points in the closed interval $\left[0, \sqrt{\pi}\right]$ does the instantaneous rate of change of f equal the average rate of change of f on that interval?

(A) Zero
(B) One
(C) Two
(D) Three
(E) Four

25. A This is the limit of a right Riemann sum of the function $f(x)=\sqrt{x}$ on the interval $[a,b]$, so

$$\lim_{n\to\infty}\sum_{i=1}^{n}\sqrt{x_i}\,\Delta x=\int_a^b\sqrt{x}\,dx=\frac{2}{3}x^{\frac{3}{2}}\Big|_a^b=\frac{2}{3}\left(b^{\frac{3}{2}}-a^{\frac{3}{2}}\right)$$

82. B $\int_0^x (t^2-2t)\,dt \ge \int_2^x t\,dt$; $\frac{1}{3}x^3-x^2\ge\frac{1}{2}x^2-2$. Using the calculator, the greatest x value on the interval $[0,4]$ that satisfies this inequality is found to occur at $x=1.3887$.

88. C $f(x)=\int_0^{x^2}\sin t\,dt$; $f'(x)=2x\sin(x^2)$; For the average rate of change of f we need to determine $f(0)$ and $f(\sqrt{\pi})$. $f(0)=0$ and $f\left(\sqrt{\pi}\right)=\int_0^{\pi}\sin t\,dt=2$. The average rate of change of f on the interval is $\frac{2}{\sqrt{\pi}}$. See how many points of intersection there are for the graphs of $y=2x\sin(x^2)$ and $y=\frac{2}{\sqrt{\pi}}$ on the interval $\left[0,\sqrt{\pi}\right]$. There are two.

1. $\int_0^1 \sqrt{x}(x+1)\,dx =$

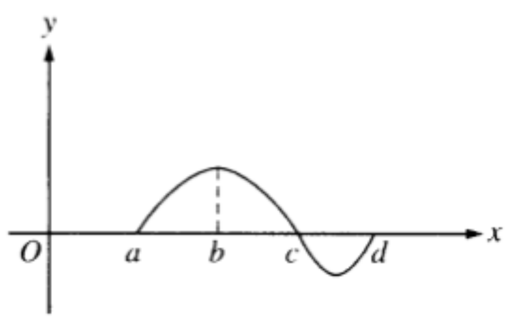
(A) 0 (B) 1 (C) $\frac{16}{15}$ (D) $\frac{7}{5}$ (E) 2

9. If $\int_{-1}^{1} e^{-x^2}\,dx = k$, then $\int_{-1}^{0} e^{-x^2}\,dx =$

(A) $-2k$ (B) $-k$ (C) $-\frac{k}{2}$ (D) $\frac{k}{2}$ (E) $2k$

1. $\int_1^2 x^{-3}\,dx =$

(A) $-\frac{7}{8}$ (B) $-\frac{3}{4}$ (C) $\frac{15}{64}$ (D) $\frac{3}{8}$ (E) $\frac{15}{16}$



22. The graph of f is shown in the figure above. If $g(x) = \int_a^x f(t)\,dt$, for what value of x does $g(x)$ have a maximum?

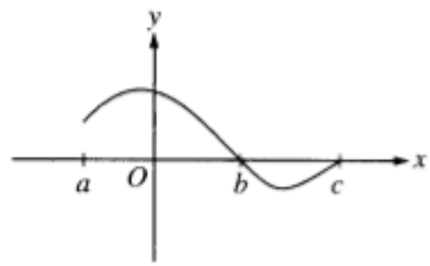
(A) a
(B) b
(C) c
(D) d
(E) It cannot be determined from the information given.

1. C $\int_0^1 \sqrt{x}(x+1)\,dx = \int_0^1 x^{\frac{3}{2}} + x^{\frac{1}{2}}\,dx = \frac{2}{5}x^{\frac{5}{2}} + \frac{2}{3}x^{\frac{3}{2}}\Big|_0^1 = \frac{16}{15}$

9. D Since e^{-x^2} is even, $\int_{-1}^{0} e^{-x^2}\,dx = \frac{1}{2}\int_{-1}^{1} e^{-x^2}\,dx = \frac{1}{2}k$

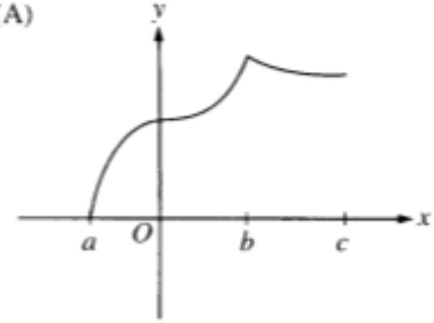
1. D $\int_1^2 x^{-3}\,dx = -\frac{1}{2}x^{-2}\Big|_1^2 = -\frac{1}{2}\left(\frac{1}{4}-1\right) = \frac{3}{8}$.

22. C $g'(x) = f(x)$. The only critical value of g on (a,d) is at $x = c$. Since g' changes from positive to negative at $x = c$, the absolute maximum for g occurs at this relative maximum.

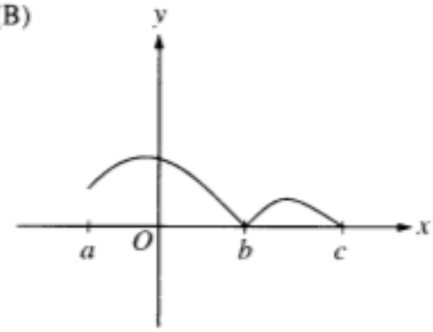


88. Let $f(x) = \int_a^x h(t)\,dt$, where h has the graph shown above. Which of the following could be the graph of f?

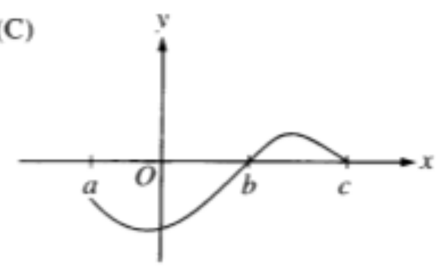
(A)



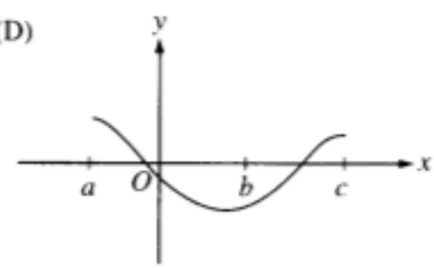
(B)



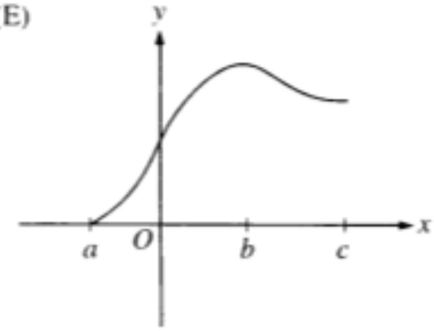
(C)



(D)



(E)



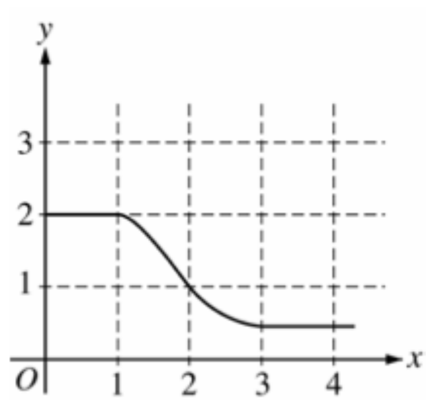
88. E $f(x)=\int_a^x h(x)dx \Rightarrow f(a)=0,$ therefore only (A) or (E) are possible. But $f'(x)=h(x)$ and therefore f is differentiable at $x=b$. This is true for the graph in option (E) but not in option (A) where there appears to be a corner in the graph at $x=b$. Also, Since h is increasing at first, the graph of f must start out concave up. This is also true in (E) but not (A).

3. If $\int_a^b f(x)\,dx = a + 2b$, then $\int_a^b (f(x)+5)\,dx =$

(A) $a+2b+5$ (B) $5b-5a$ (C) $7b-4a$ (D) $7b-5a$ (E) $7b-6a$

18. $\int_0^{\frac{\pi}{4}} \frac{e^{\tan x}}{\cos^2 x}\,dx$ is

(A) 0 (B) 1 (C) $e-1$ (D) e (E) $e+1$



78. The graph of f is shown in the figure above. If $\int_1^3 f(x)\,dx = 2.3$ and $F'(x) = f(x)$, then $F(3) - F(0) =$

(A) 0.3 (B) 1.3 (C) 3.3 (D) 4.3 (E) 5.3

3. C $\int_a^b (f(x)+5)\,dx = \int_a^b f(x)dx + 5\int_a^b 1\,dx = a + 2b + 5(b-a) = 7b - 4a$

18. C $\int_0^{\frac{\pi}{4}} \frac{e^{\tan x}}{\cos^2 x}dx$ is of the form $\int e^u du$ where $u = \tan x$..

$$\int_0^{\frac{\pi}{4}} \frac{e^{\tan x}}{\cos^2 x}dx = e^{\tan x}\Big|_0^{\frac{\pi}{4}} = e^1 - e^0 = e - 1$$

78. D $F(3) - F(0) = \int_0^3 f(x)dx = \int_0^1 f(x)dx + \int_1^3 f(x)dx = 2 + 2.3 = 4.3$

(Count squares for $\int_0^1 f(x)dx$)

33. Which of the following is equal to $\int_0^{\pi} \sin x\, dx$?

(A) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x\, dx$ (B) $\int_0^{\pi} \cos x\, dx$ (C) $\int_{-\pi}^{0} \sin x\, dx$

(D) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin x\, dx$ (E) $\int_{\pi}^{2\pi} \sin x\, dx$

37. If $f(x) = \begin{cases} x & \text{for } x \le 1 \\ \dfrac{1}{x} & \text{for } x > 1, \end{cases}$ then $\int_0^{e} f(x) dx =$

(A) 0 (B) $\frac{3}{2}$ (C) 2 (D) e (E) $e + \frac{1}{2}$

41. Let $f(x) = \int_{-2}^{x^2-3x} e^{t^2}\, dt$. At what value of x is $f(x)$ a minimum?

(A) For no value of x (B) $\frac{1}{2}$ (C) $\frac{3}{2}$ (D) 2 (E) 3

1. $\int_1^2 (4x^3 - 6x)\, dx =$

(A) 2
(B) 4
(C) 6
(D) 36
(E) 42

33. A The value of this integral is 2. Option A is also 2 and none of the others have a value of 2. Visualizing the graphs of $y = \sin x$ and $y = \cos x$ is a useful approach to the problem.

37. B $\int_0^e f(x)\,dx = \int_0^1 x\,dx + \int_1^e \frac{1}{x}\,dx = \frac{1}{2} + \ln e = \frac{3}{2}$

41. C $f'(x) = (2x-3)e^{\left(x^2-3x\right)^2}$; $f' < 0$ for $x < \frac{3}{2}$ and $f' > 0$ for $x > \frac{3}{2}$.

Thus f has its absolute minimum at $x = \frac{3}{2}$.

1. C $\int_1^2 (4x^3 - 6x)\,dx = (x^4 - 3x^2)\Big|_1^2 = (16-12)-(1-3) = 6$

41. $\frac{d}{dx}\int_0^x \cos(2\pi u)\,du$ is

(A) 0 (B) $\frac{1}{2\pi}\sin x$ (C) $\frac{1}{2\pi}\cos(2\pi x)$ (D) $\cos(2\pi x)$ (E) $2\pi\cos(2\pi x)$

3. If p is a polynomial of degree n, $n > 0$, what is the degree of the polynomial $Q(x) = \int_0^x p(t)\,dt$?

(A) 0 (B) 1 (C) $n-1$ (D) n (E) $n+1$

7. $\int_0^1 x^3 e^{x^4}\,dx =$

(A) $\frac{1}{4}(e-1)$ (B) $\frac{1}{4}e$ (C) $e-1$ (D) e (E) $4(e-1)$

41. D Answer follows from the Fundamental Theorem of Calculus.

3. E $Q'(x) = p(x) \Rightarrow$ degree of Q is $n+1$

7. A $\int_0^1 x^3 e^{x^4}\,dx = \frac{1}{4}\int_0^1 e^{x^4}(4x^3dx) = \frac{1}{4}e^{x^4}\Big|_0^1 = \frac{1}{4}(e-1)$

17. $\int_2^3 \frac{3}{(x-1)(x+2)}\,dx =$

(A) $-\frac{33}{20}$ (B) $-\frac{9}{20}$ (C) $\ln\left(\frac{5}{2}\right)$ (D) $\ln\left(\frac{8}{5}\right)$ (E) $\ln\left(\frac{2}{5}\right)$

31. $\int_0^2 \sqrt{4-x^2}\,dx =$

(A) $\frac{8}{3}$ (B) $\frac{16}{3}$ (C) π (D) 2π (E) 4π

42. If $\int_1^4 f(x)\,dx = 6$, what is the value of $\int_1^4 f(5-x)\,dx$?

(A) 6 (B) 3 (C) 0 (D) –1 (E) –6

28. $\int_1^{500}\left(13^x - 11^x\right)dx + \int_2^{500}\left(11^x - 13^x\right)dx =$

(A) 0.000 (B) 14.946 (C) 34.415 (D) 46.000 (E) 136.364

17. D Use partial fractions:

$$\int_2^3 \frac{3}{(x-1)(x+1)}\,dx = \int_2^3 \left(\frac{1}{x-1} - \frac{1}{x+2}\right)dx = \left(\ln|x-1| - \ln|x+2|\right)\Big|_2^3 = \ln 2 - \ln 5 - \ln 1 + \ln 4 = \ln\frac{8}{5}$$

31. C This integral gives $\frac{1}{4}$ of the area of the circle with center at the origin and radius = 2.

$$\frac{1}{4}\left(\pi \cdot 2^2\right) = \pi$$

42. A Let $5 - x = u$, $dx = -du$, substitute

$$\int_1^4 f(5-x)\,dx = \int_4^1 f(u)(-du) = \int_1^4 f(u)\,du = \int_1^4 f(x)\,dx = 6$$

28. B $\int_1^{500} (13^x - 11^x)\,dx + \int_2^{500} (11^x - 13^x)\,dx = \int_1^{500} (13^x - 11^x)\,dx - \int_2^{500} (13^x - 11^x)\,dx$

$$= \int_1^2 (13^x - 11^x)\,dx = \left(\frac{13^x}{\ln 13} - \frac{11^x}{\ln 11}\right)\Bigg|_1^2 = \frac{13^2 - 13}{\ln 13} - \frac{11^2 - 11}{\ln 11} = 14.946$$

39. If $\int_1^{10} f(x)\,dx = 4$ and $\int_{10}^{3} f(x)\,dx = 7$, then $\int_1^{3} f(x)\,dx =$

(A) -3 (B) 0 (C) 3 (D) 10 (E) 11

2. $\int_0^1 x\left(x^2+2\right)^2 dx =$

(A) $\frac{19}{2}$ (B) $\frac{19}{3}$ (C) $\frac{9}{2}$ (D) $\frac{19}{6}$ (E) $\frac{1}{6}$

14. If $F(x) = \int_1^{x^2} \sqrt{1+t^3}\,dt$, then $F'(x) =$

(A) $2x\sqrt{1+x^6}$ (B) $2x\sqrt{1+x^3}$ (C) $\sqrt{1+x^6}$

(D) $\sqrt{1+x^3}$ (E) $\int_1^{x^2} \frac{3t^2}{2\sqrt{1+t^3}}\,dt$

39. E $\int_3^{10} f(x)\,dx = -\int_{10}^{3} f(x)\,dx;\ \int_1^3 f(x)\,dx = \int_1^{10} f(x)\,dx - \int_3^{10} f(x)\,dx = 4-(-7)=11$

2. D $\int_0^1 x\left(x^2+2\right)^2 dx = \frac{1}{2}\int_0^1 \left(x^2+2\right)^2 (2x\,dx) = \frac{1}{2}\cdot\frac{1}{3}\left(x^2+2\right)^3\Big|_0^1 = \frac{1}{6}\left(3^3-2^3\right) = \frac{19}{6}$

14 A Use the Fundamental Theorem of Calculus: $\sqrt{1+(x^2)^3}\cdot\frac{d(x^2)}{dx} = 2x\sqrt{1+x^6}$

28. $\int_1^4 |x-3|\,dx =$

(A) $-\frac{3}{2}$ (B) $\frac{3}{2}$ (C) $\frac{5}{2}$ (D) $\frac{9}{2}$ (E) 5

38. For $x > 0$, $\int \left(\frac{1}{x}\int_1^x \frac{du}{u}\right) dx =$

(A) $\frac{1}{x^3} + C$

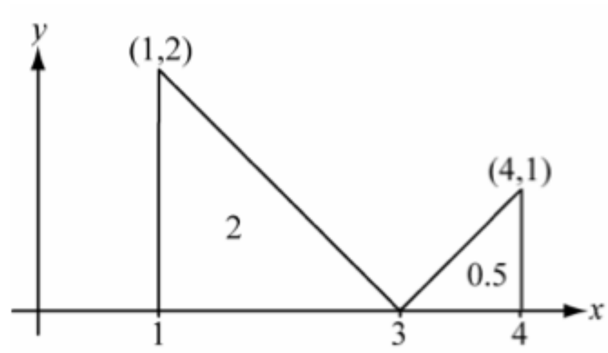
(B) $\frac{8}{x^4} - \frac{2}{x^2} + C$

(C) $\ln(\ln x) + C$

(D) $\frac{\ln(x^2)}{2} + C$

(E) $\frac{(\ln x)^2}{2} + C$

28. C The graph is a V with vertex at $x = 3$. The integral gives the sum of the areas of the two triangles that the V forms with the horizontal axis for x from 1 to 4. These triangles have areas of 2 and 0.5 respectively.



38. E $\int\left(\frac{1}{x}\int_1^x \frac{du}{u}\right)dx = \int \frac{1}{x}\ln x\,dx = \int \ln x\left(\frac{dx}{x}\right)$. This is $\int u\,du$ with $u = \ln x$, so the value is

$\frac{(\ln x)^2}{2} + C$

10. If $\int_0^k \left(2kx - x^2\right) dx = 18$, then $k =$

(A) -9 (B) -3 (C) 3 (D) 9 (E) 18

14. $\int_0^{\frac{\pi}{2}} \frac{\cos\theta}{\sqrt{1+\sin\theta}} d\theta =$

(A) $-2\left(\sqrt{2}-1\right)$ (B) $-2\sqrt{2}$ (C) $2\sqrt{2}$

(D) $2\left(\sqrt{2}-1\right)$ (E) $2\left(\sqrt{2}+1\right)$

17. $\int_0^1 (3x-2)^2 \, dx =$

(A) $-\frac{7}{3}$ (B) $-\frac{7}{9}$ (C) $\frac{1}{9}$ (D) 1 (E) 3

19. $\int_2^3 \frac{x}{x^2+1} dx =$

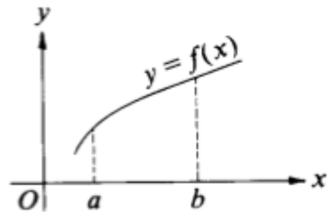
(A) $\frac{1}{2}\ln\frac{3}{2}$ (B) $\frac{1}{2}\ln 2$ (C) $\ln 2$ (D) $2\ln 2$ (E) $\frac{1}{2}\ln 5$

10. C $18 = \left(kx^2 - \frac{1}{3}x^3\right)\Big|_0^k = \frac{2}{3}k^3 \Rightarrow k^3 = 27, \text{ so } k = 3$

14. D $\int_0^{\frac{\pi}{2}} (1+\sin\theta)^{-1/2}(\cos\theta\, d\theta) = 2(1+\sin\theta)^{1/2}\Big|_0^{\frac{\pi}{2}} = 2\left(\sqrt{2}-1\right)$

17. D $\int_0^1 (3x-2)^2\, dx = \frac{1}{3}\int_0^1 (3x-2)^2(3\, dx) = \frac{1}{3}\cdot\frac{1}{3}(3x-2)^3\Big|_0^1 = \frac{1}{9}(1-(-8)) = 1$

19. B $\int_2^3 \frac{x}{x^2+1}dx = \frac{1}{2}\int_2^3 \frac{2x\,dx}{x^2+1} = \frac{1}{2}\ln\left(x^2+1\right)\Big|_2^3 = \frac{1}{2}(\ln 10 - \ln 5) = \frac{1}{2}\ln 2$



27. If f is the continuous, strictly increasing function on the interval $a \le x \le b$ as shown above, which of the following must be true?

I. $\int_a^b f(x)\,dx < f(b)(b-a)$

II. $\int_a^b f(x)\,dx > f(a)(b-a)$

III. $\int_a^b f(x)\,dx = f(c)(b-a)$ for some number c such that $a < c < b$

(A) I only (B) II only (C) III only (D) I and III only (E) I, II, and III

40. If the substitution $u = \frac{x}{2}$ is made, the integral $\int_2^4 \frac{1-\left(\frac{x}{2}\right)^2}{x}\,dx =$

(A) $\int_1^2 \frac{1-u^2}{u}\,du$

(B) $\int_2^4 \frac{1-u^2}{u}\,du$

(C) $\int_1^2 \frac{1-u^2}{2u}\,du$

(D) $\int_1^2 \frac{1-u^2}{4u}\,du$

(E) $\int_2^4 \frac{1-u^2}{2u}\,du$

27. E Each of the right-hand sides represent the area of a rectangle with base length $(b-a)$.

I. Area under the curve is less than the area of the rectangle with height $f(b)$.

II. Area under the curve is more than the area of the rectangle with height $f(a)$.

III. Area under the curve is the same as the area of the rectangle with height $f(c)$, $a < c < b$.
Note that this is the Mean Value Theorem for Integrals.

40. A $u=\frac{x}{2},\ du=\frac{1}{2}dx$; when $x=2$, $u=1$ and when $x=4$, $u=2$

$$\int_2^4 \frac{1-\left(\frac{x}{2}\right)^2}{x}\,dx=\int_1^2 \frac{1-u^2}{2u}\cdot 2\,du=\int_1^2 \frac{1-u^2}{u}\,du$$

40. Let f be a continuous function on the closed interval $[0,2]$. If $2 \le f(x) \le 4$, then the greatest possible value of $\int_0^2 f(x)\,dx$ is

(A) 0 (B) 2 (C) 4 (D) 8 (E) 16

42. $\dfrac{d}{dx}\int_2^x \sqrt{1+t^2}\,dt =$

(A) $\dfrac{x}{\sqrt{1+x^2}}$ (B) $\sqrt{1+x^2}-5$ (C) $\sqrt{1+x^2}$

(D) $\dfrac{x}{\sqrt{1+x^2}}-\dfrac{1}{\sqrt{5}}$ (E) $\dfrac{1}{2\sqrt{1+x^2}}-\dfrac{1}{2\sqrt{5}}$

3. $\int_1^2 \dfrac{x+1}{x^2+2x}\,dx =$

(A) $\ln 8-\ln 3$ (B) $\dfrac{\ln 8-\ln 3}{2}$ (C) $\ln 8$ (D) $\dfrac{3\ln 2}{2}$ (E) $\dfrac{3\ln 2+2}{2}$

23. $\lim\limits_{h\to 0}\dfrac{\int_1^{1+h}\sqrt{x^5+8}\,dx}{h}$ is

(A) 0 (B) 1 (C) 3 (D) $2\sqrt{2}$ (E) nonexistent

40. D $\int_0^2 f(x)\,dx \le \int_0^2 4\,dx = 8$

42. C This is a direct application of the Fundamental Theorem of Calculus: $f'(x) = \sqrt{1+x^2}$

3. B $\int_1^2 \frac{x+1}{x^2+2x}\,dx = \frac{1}{2}\int_1^2 \frac{(2x+2)\,dx}{x^2+2x} = \frac{1}{2}\ln\left|x^2+2x\right|\Big|_1^2 = \frac{1}{2}(\ln 8 - \ln 3)$

23. C $\lim_{h\to 0}\frac{\int_1^{1+h}\sqrt{x^5+8}\,dx}{h} = \lim_{h\to 0}\frac{F(1+h)-F(1)}{h} = F'(1)$ where $F'(x) = \sqrt{x^5+8}$. $F'(1)=3$

Alternate solution by L'Hôpital's Rule: $\lim_{h\to 0}\frac{\int_1^{1+h}\sqrt{x^5+8}\,dx}{h} = \lim_{h\to 0}\frac{\sqrt{(1+h)^5+8}}{1} = \sqrt{9} = 3$

22. $\int_1^2 \frac{x^2-1}{x+1}\,dx =$

(A) $\frac{1}{2}$ (B) 1 (C) 2 (D) $\frac{5}{2}$ (E) $\ln 3$

24. If $\int_{-2}^{2}\left(x^7+k\right)dx = 16$, then $k =$

(A) -12 (B) -4 (C) 0 (D) 4 (E) 12

27. $\int_0^3 |x-1|\,dx =$

(A) 0 (B) $\frac{3}{2}$ (C) 2 (D) $\frac{5}{2}$ (E) 6

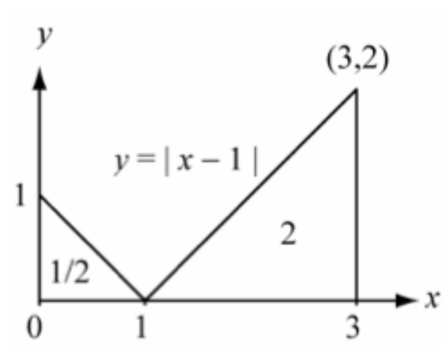
32. $\int_0^{\frac{\pi}{3}} \sin(3x)\,dx =$

(A) -2 (B) $-\frac{2}{3}$ (C) 0 (D) $\frac{2}{3}$ (E) 2

22. A $\int_1^2 \frac{x^2-1}{x+1}\,dx = \int_1^2 \frac{(x+1)(x-1)}{x+1}\,dx = \int_1^2 (x-1)\,dx = \frac{1}{2}(x-1)^2\Big|_1^2 = \frac{1}{2}$

24. D $16 = \int_{-2}^{2}(x^7+k)\,dx = \int_{-2}^{2} x^7\,dx + \int_{-2}^{2} k\;dx = 0 + \big(2-(-2)\big)k = 4k \Rightarrow k = 4$

27. D The graph is a V with vertex at $x=1$. The integral gives the sum of the areas of the two triangles that the V forms with the horizontal axis for x from 0 to 3. These triangles have areas of 1/2 and 2 respectively.



32. D $\int_0^{\pi/3} \sin(3x)\,dx = -\frac{1}{3}\cos(3x)\Big|_0^{\pi/3} = -\frac{1}{3}\left(\cos\pi - \cos 0\right) = \frac{2}{3}$

38. If $\int_1^2 f(x-c)\,dx = 5$ where c is a constant, then $\int_{1-c}^{2-c} f(x)\,dx =$

(A) $5+c$ (B) 5 (C) $5-c$ (D) $c-5$ (E) -5

2. $\int_0^3 (x+1)^{1/2}\,dx =$

(A) $\frac{21}{2}$ (B) 7 (C) $\frac{16}{3}$ (D) $\frac{14}{3}$ (E) $-\frac{1}{4}$

5. $\int_{-1}^{2} \frac{|x|}{x}\,dx$ is

(A) -3 (B) 1 (C) 2 (D) 3 (E) nonexistent

12. If n is a known positive integer, for what value of k is $\int_1^k x^{n-1}\,dx = \frac{1}{n}$?

(A) 0 (B) $\left(\frac{2}{n}\right)^{1/n}$ (C) $\left(\frac{2n-1}{n}\right)^{1/n}$

(D) $2^{1/n}$ (E) 2^n

38. B Let $z = x - c$. Then $5 = \int_1^2 f(x-c)\,dx = \int_{1-c}^{2-c} f(z)\,dz$

2. D $\int_0^3 (x+1)^{\frac{1}{2}}\,dx = \frac{2}{3}(x+1)^{\frac{3}{2}}\Big|_0^3 = \frac{2}{3}\left(4^{\frac{3}{2}} - 1^{\frac{3}{2}}\right) = \frac{2}{3}(8-1) = \frac{14}{3}$

5. B $\int_{-1}^2 \frac{|x|}{x}\,dx = \int_{-1}^0 -1\,dx + \int_0^2 dx = -1 + 2 = 1$

12. D $\frac{1}{n} = \int_1^k x^{n-1}\,dx = \frac{x^n}{n}\Big|_1^k \Rightarrow \frac{1}{n} = \frac{k^n}{n} - \frac{1}{n};\ \ \frac{k^n}{n} = \frac{2}{n} \Rightarrow k = 2^{\frac{1}{n}}$

21. $\int_0^1 (x+1)e^{x^2+2x}\,dx =$

(A) $\frac{e^3}{2}$ (B) $\frac{e^3-1}{2}$ (C) $\frac{e^4-e}{2}$ (D) e^3-1 (E) e^4-e

25. $\int_0^{\pi/4} \tan^2 x\,dx =$

(A) $\frac{\pi}{4}-1$ (B) $1-\frac{\pi}{4}$ (C) $\frac{1}{3}$ (D) $\sqrt{2}-1$ (E) $\frac{\pi}{4}+1$

27. $\int_0^{1/2} \frac{2x}{\sqrt{1-x^2}}\,dx =$

(A) $1-\frac{\sqrt{3}}{2}$ (B) $\frac{1}{2}\ln\frac{3}{4}$ (C) $\frac{\pi}{6}$ (D) $\frac{\pi}{6}-1$ (E) $2-\sqrt{3}$

30. $\int_1^2 \frac{x-4}{x^2}\,dx =$

(A) $-\frac{1}{2}$ (B) $\ln 2-2$ (C) $\ln 2$ (D) 2 (E) $\ln 2+2$

21. B $\int_0^1 (x+1)e^{x^2+2x}\,dx = \frac{1}{2}\int_0^1 e^{x^2+2x}((2x+2)\,dx) = \frac{1}{2}\left(e^{x^2+2x}\right)\Big|_0^1 = \frac{1}{2}\left(e^3 - e^0\right) = \frac{e^3-1}{2}$

25. B $\int_0^{\frac{\pi}{4}} \tan^2 x\,dx = \int_0^{\frac{\pi}{4}} (\sec^2 x - 1)\,dx = (\tan x - x)\Big|_0^{\pi/4} = 1 - \frac{\pi}{4}$

27. E $\int_0^{1/2} \frac{2x}{\sqrt{1-x^2}}\,dx = -\int_0^{1/2} \left(1-x^2\right)^{-\frac{1}{2}}(-2x\,dx) = -2\left(1-x^2\right)^{\frac{1}{2}}\Big|_0^{\frac{1}{2}} = 2-\sqrt{3}$

30. B $\int_1^2 \frac{x-4}{x^2}\,dx = \int_1^2 \left(\frac{1}{x} - 4x^{-2}\right)dx = \left(\ln x + \frac{4}{x}\right)\Big|_1^2 = (\ln 2 + 2) - (\ln 1 + 4) = \ln 2 - 2$

40. If n is a non-negative integer, then $\int_0^1 x^n\,dx = \int_0^1 (1-x)^n\,dx$ for

(A) no n
(B) n even, only
(C) n odd, only
(D) nonzero n, only
(E) all n

10. $\int_0^1 \frac{x^2}{x^2+1}\,dx =$

(A) $\frac{4-\pi}{4}$
(B) $\ln 2$
(C) 0
(D) $\frac{1}{2}\ln 2$
(E) $\frac{4+\pi}{4}$

20. If F and f are continuous functions such that $F'(x) = f(x)$ for all x, then $\int_a^b f(x)\,dx$ is

(A) $F'(a) - F'(b)$
(B) $F'(b) - F'(a)$
(C) $F(a) - F(b)$
(D) $F(b) - F(a)$
(E) none of the above

40. E One solution technique is to evaluate each integral and note that the value is $\frac{1}{n+1}$ for each.

Another technique is to use the substitution $u = 1 - x$; $\int_0^1 (1-x)^n\,dx = \int_1^0 u^n(-du) = \int_0^1 u^n\,du$.

Integrals do not depend on the variable that is used and so $\int_0^1 u^n\,du$ is the same as $\int_0^1 x^n\,dx$.

10. A $\int_0^1 \frac{x^2}{x^2+1}\,dx = \int_0^1 \frac{x^2+1-1}{x^2+1}\,dx = \int_0^1 \left(\frac{x^2+1}{x^2+1} - \frac{1}{x^2+1}\right)dx = \left(x - \tan^{-1} x\right)\Big|_0^1 = 1 - \frac{\pi}{4} = \frac{4-\pi}{4}$

20. D By the Fundamental Theorem of Calculus, $\int_a^b f(x)\,dx = F(b) - F(a)$ where $F'(x) = f(x)$.

4. $\int_0^8 \frac{dx}{\sqrt{1+x}} =$

(A) 1 (B) $\frac{3}{2}$ (C) 2 (D) 4 (E) 6

26. $\int_0^1 \sqrt{x^2 - 2x + 1}\, dx$ is

(A) -1

(B) $-\frac{1}{2}$

(C) $\frac{1}{2}$

(D) 1

(E) none of the above

29. $\int_{\pi/4}^{\pi/2} \frac{\cos x}{\sin x} dx =$

(A) $\ln\sqrt{2}$ (B) $\ln\frac{\pi}{4}$ (C) $\ln\sqrt{3}$ (D) $\ln\frac{\sqrt{3}}{2}$ (E) $\ln e$

4. D $\int_0^8 \frac{dx}{\sqrt{1+x}} = 2\sqrt{1+x}\Big|_0^8 = 2(3-1) = 4$

26. C $\int_0^1 \sqrt{x^2-2x+1}\,dx = \int_0^1 |x-1|\,dx = \int_0^1 -(x-1)\,dx = -\frac{1}{2}(x-1)^2\Big|_0^1 = \frac{1}{2}$

Alternatively, the graph of the region is a right triangle with vertices at (0,0), (0,1), and (1,0). The area is $\frac{1}{2}$.

29. A $\int_{\pi/4}^{\pi/2} \frac{\cos x}{\sin x}\,dx = \ln(\sin x)\Big|_{\pi/4}^{\pi/2} = \ln 1 - \ln\frac{1}{\sqrt{2}} = \ln\sqrt{2}$

No Calculator

21. If $f(x) = \begin{cases} x^2 + 2 & \text{for } 0 \le x \le 2 \\ 8 - x & \text{elsewhere} \end{cases}$ then $\int_0^7 f(x)\,dx$ is a number between

(A) 0 and 10

(B) 10 and 20

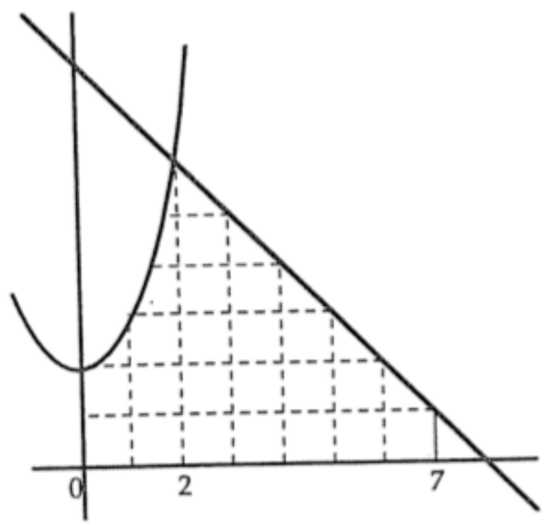
(C) 20 and 30

(D) 30 and 40

(E) 40 and 50

21. C p. 8

With a reasonably careful graph, it is possible to obtain an estimate of the definite integral by counting the squares under the graph of f(x) on the interval [0,7].



Alternatively, having determined that the change in the function definition occurs at $x = 2$,

evaluate $\int_0^7 f(x)\,dx$.

This is done in two parts, as

$$\int_0^2 (x^2+2)\,dx + \int_2^7 (8-x)\,dx = \left[\frac{x^3}{3} + 2x\right|_0^2 + \left[8x - \frac{x^2}{2}\right|_2^7$$

$$= \left[\frac{8}{3} + 4\right] - 0 + \left[56 - \frac{49}{2}\right] - (10-2)$$

$$= \frac{145}{6} \approx 24$$

2. Consider the polynomial function f with the following properties:

i) $\int_{1}^{3} f(x)\,dx = \frac{5}{2}$, ii) $\int_{1}^{5} f(x)\,dx = 10$.

(a) Find the average value of the function f over the interval [1, 3].

(b) Find the value of $\int_{3}^{5} [2f(x) + 6]\,dx$

(c) If $f(x) = ax + b$, determine a and b.

Calculator Active

2. p. 18

(a) The average (mean) value of a function f on the interval $[a,b]$ is defined to be:

$$M = \frac{1}{b-a}\int_a^b f(x)\,dx.$$

For this function, that gives $M = \frac{1}{2}\int_1^3 f(x)\,dx = \frac{1}{2}\cdot\frac{5}{2} = \frac{5}{4}$.

(b) $$2\int_3^5 f(x)\,dx + \int_3^5 6\,dx = 2\left[\int_1^5 f(x)\,dx - \int_1^3 f(x)\,dx\right] + \int_3^5 6\,dx$$
$$= 2\left[10 - \frac{5}{2}\right] + 6\cdot 2 = 27$$

(c) $$\int_1^3 (ax+b)\,dx = \frac{ax^2}{2} + bx\Big]_1^3 = \left[\frac{9a}{2} + 3b\right] - \left[\frac{a}{2} + b\right] = 4a + 2b = \frac{5}{2}$$

$$\int_1^5 (ax+b)\,dx = \frac{ax^2}{2} + bx\Big]_1^5 = \left[\frac{25a}{2} + 5b\right] - \left[\frac{a}{2} + b\right] = 12a + 4b = 10$$

This produces two equations in two unknowns: $\begin{cases} 8a + 4b = 5 \\ 12a + 4b = 10 \end{cases}$

Subtracting, $4a = 5$; hence $a = \frac{5}{4}$ and $b = -\frac{5}{4}$.

No Calculator

2. $\int_{0}^{1} \sin \pi x \; dx =$

(A) $\frac{2}{\pi}$ (B) $\frac{1}{\pi}$ (C) 0 D) $-\frac{2}{\pi}$ (E) $-\frac{1}{\pi}$

No Calculator

8. $\int_{\pi/4}^{\pi/3} \frac{\sec^2 x}{\tan x} \, dx =$

(A) $\ln \sqrt{3}$ (B) $-\ln \sqrt{3}$ (C) $\ln \sqrt{2}$ (D) $\sqrt{3} - 1$ (E) $\ln \frac{\pi}{3} - \ln \frac{\pi}{4}$

2. A p. 23

$$\int_0^1 \sin(\pi x)\,dx = -\frac{1}{\pi}\cos(\pi x)\Big]_0^1 = -\frac{1}{\pi}(\cos\pi - \cos 0) = -\frac{1}{\pi}(-1-1) = \frac{2}{\pi}$$

This integration problem can also be done with a formal substitution.

Let $u = \tan x$.
Then $du = \sec^2 x\,dx$.

In addition, since this is a definite integral, we can change the limits of integration.

When $x = \frac{\pi}{4}$, then $u = \tan\frac{\pi}{4} = 1$.

When $x = \frac{\pi}{3}$, then $u = \tan\frac{\pi}{3} = \sqrt{3}$.

Hence $$\int_{\pi/4}^{\pi/3} \frac{\sec^2 x}{\tan x}\,dx = \int_1^{\sqrt{3}} \frac{du}{u} = \ln|u|\Big]_1^{\sqrt{3}} = \ln\sqrt{3} - \ln 1 = \ln\sqrt{3}.$$

10. $\int_0^2 \sqrt{x^2 - 4x + 4}\, dx$ is:

(A) 1

(B) –1

(C) –2

(D) 2

(E) none of the above

No Calculator

14. If $\int_2^4 f(x)\, dx = 6$, then $\int_2^4 (f(x) + 3)\, dx =$

(A) 3

(B) 6

(C) 9

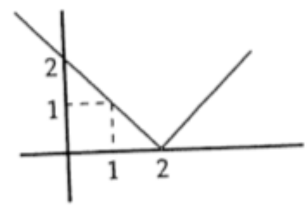
(D) 12

(E) 15

10. D p. 26

$$\int_0^2 \sqrt{x^2 - 4x + 4}\,dx = \int_0^2 \sqrt{(x-2)^2}\,dx = \int_0^2 |x-2|\,dx$$

To evaluate this integral, count squares in the graph at the right, or note that the area of the triangle is $\frac{1}{2}(2)(2) = 2$.



14. D p. 27

$$\int_2^4 (f(x) + 3)\,dx = \int_2^4 f(x)\,dx + \int_2^4 3\,dx$$

$$= 6 + 3 \cdot 2 = 12$$

8. Given: $5x^3 + 40 = \int_a^x f(t)\,dt$. The value of a is

(A) -2

(B) 2

(C) 1

(D) -1

(E) 0

Calculator Active

3. If $f(x) = 2x^2 - x^3$ and $g(x) = x^2 - 2x$, for what values of a and b is $\int_a^b f(x)\,dx > \int_a^b g(x)\,dx$?

I. $a = -1$ and $b = 0$ II. $a = 0$ and $b = 2$ III. $a = 2$ and $b = 3$

(A) I only

(B) II only

(C) I and II only

(D) I and III only

(E) I, II, III

8. A p. 34

Starting with $5x^3 + 40 = \int_a^x f(t)\,dt$, differentiate to obtain $15x^2 = f(x)$.

Then $\int_a^x 15t^2\,dt = 5t^3 \Big]_a^x = 5x^3 - 5a^3$

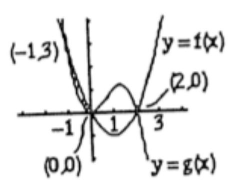
Since we are given that $5x^3 + 40 = \int_a^x f(t)\,dt$,

we can set $5x^3 + 40 = 5x^3 - 5a^3$.
Then $-5a^3 = 40$, so $a = -2$.

3. B p. 11

Here are two possible calculator solutions.

I. First, look at graphs of the functions f and g, and find those intervals where the graph of f is <u>above</u> the graph of g.



The quadratic function $f(x)$ is above the cubic function $g(x)$ when x is between -1 and 0, and then again when $x > 2$. Thus the integral of f will have a larger value than the integral of g on the intervals $[-1,0]$ and $[2,3]$.

II. Second, use the calculator to evaluate these definite integrals on the intervals $[a,b]$ indicated.

			$\int_a^b f(x)\,dx$	$\int_a^b g(x)\,dx$	
I.	$a = -1$	$b = 0$	.917	1.333	False
II.	$a = 0$	$b = 2$	1.333	−1.333	True
III.	$a = 2$	$b = 3$	−3.583	1.333	False

8. $\displaystyle\int_1^3 \frac{x}{x^2+1}\,dx =$

(A) $\ln 5$ (B) $\ln 10$ (C) $2\ln 2$ (D) $\frac{1}{2}\ln 5$ (E) $\ln\left(\frac{5}{2}\right)$

No Calculator

13. For what value of k, $k > 0$, does $\displaystyle\int_0^k (4kx - 5k)\,dx = k^2$?

(A) 1

(B) 2

(C) 3

(D) 4

(E) 5

8. Let $u = x^2 + 1$

$du = 2x\,dx \Rightarrow x\,dx = \frac{1}{2}\,du$

$$\int \frac{x}{x^2+1}\,dx = \frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\ln|u| = \frac{1}{2}\ln(x^2+1)\Big|_1^3 = \frac{1}{2}(\ln 10 - \ln 2)$$

Since $\ln 10 - \ln 2 = \ln(\frac{10}{2}) = \ln 5$, therefore, $\int_1^3 \frac{x}{x^2+1}\,dx = \frac{1}{2}\ln 5.$

The correct choice is (D).

13. If $\int_0^k (4kx - 5k)\,dx = k^2$, then $\frac{4kx^2}{2} - 5kx\Big|_0^k = k^2$, or

$$2kx^2 - 5kx\Big|_0^k = k^2$$

$$2k^3 - 5k^2 = k^2$$

$$2k^3 = 6k^2$$ (dividing both sides by k^2, since $k > ($

$$2k = 6$$

$$k = 3$$

The correct choice is (C).

18. $\dfrac{d}{dx}\displaystyle\int_x^0 \dfrac{du}{1+u^2} =$

(A) $\dfrac{1}{x^2+1}$

(B) $\dfrac{-1}{x^2+1}$

(C) x^2+1

(D) $-x^2+1$

(E) Arctan x

No Calculator

22. If $0 < k < \pi$, then $\displaystyle\int_0^k \cos(2x)\,dx = \dfrac{1}{2}$ when $k =$

(A) $\dfrac{\pi}{4}$

(B) $\dfrac{\pi}{2}$

(C) $\dfrac{\pi}{12}$

(D) $\dfrac{3\pi}{4}$

(E) $\dfrac{5\pi}{12}$

18. By the Fundamental Theorem of Calculus (version 1),

$$\frac{d}{dx}\int_x^0 \frac{du}{1+u^2} = 0 - \left(\frac{1}{1+x^2}\right) = \frac{-1}{1+x^2}$$

The correct choice is (B).

22. Let $u = 2x$

$du = 2\ dx \Rightarrow dx = \frac{1}{2}\ du$

$$\int \cos 2x\ dx = \frac{1}{2}\int \cos u\ du = \frac{1}{2}\sin 2x\Big|_0^k = \frac{1}{2}\sin 2k$$

Since $\frac{1}{2}\sin 2k = \frac{1}{2}$, then $\sin 2k = 1 \Rightarrow 2k = \frac{\pi}{2}$ and $\boxed{k = \frac{\pi}{4}}$

The correct choice is (A).