

4.3 Determinants

A **determinant** is a real number associated with a square matrix.

For a 2×2 matrix:

$$\det \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

1 Find the determinant of $\begin{bmatrix} -5 & 7 \\ -2 & 9 \end{bmatrix} = \begin{vmatrix} -5 & 7 \\ -2 & 9 \end{vmatrix} = -45 + 14 = -31$

For a 3×3 matrix:

$$\det \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = aei + bfg + cdh - gec - hfa - idb$$

2 Find the determinant of $\begin{bmatrix} 1 & 3 & -5 \\ 0 & -2 & 4 \\ -7 & 8 & 9 \end{bmatrix}$

$$= \begin{vmatrix} 1 & 3 & -5 \\ 0 & -2 & 4 \\ -7 & 8 & 9 \end{vmatrix} = -18 - 84 + 0 + 70 - 32 + 0 - 102 + 38 = -64$$