29. $f(x) = x^3 + 2x^2$; $g(x) = 3x^2 - 1$. $D = \mathbb{R}$ for both f and g.

$(f + g)(x) = (x^3 + 2x^2) + (3x^2 - 1) = x^3 + 5x^2 - 1,\ D = \mathbb{R}$.

$(f - g)(x) = (x^3 + 2x^2) - (3x^2 - 1) = x^3 - x^2 + 1,\ D = \mathbb{R}$.

$(fg)(x) = (x^3 + 2x^2)(3x^2 - 1) = 3x^5 + 6x^4 - x^3 - 2x^2,\ D = \mathbb{R}$.

$\left(\frac{f}{g}\right)(x) = \frac{x^3 + 2x^2}{3x^2 - 1},\ D = \left\{x \mid x \neq \pm\frac{1}{\sqrt{3}}\right\}$ since $3x^2 - 1 \neq 0$.

32. $f(x) = x - 2$; $g(x) = x^2 + 3x + 4$. $D = \mathbb{R}$ for both f and g, and hence for their composites.

(a) $(f \circ g)(x) = f(g(x)) = f(x^2 + 3x + 4) = (x^2 + 3x + 4) - 2 = x^2 + 3x + 2$.

(b) $(g \circ f)(x) = g(f(x)) = g(x - 2) = (x - 2)^2 + 3(x - 2) + 4 = x^2 - 4x + 4 + 3x - 6 + 4 = x^2 - x + 2$.

(c) $(f \circ f)(x) = f(f(x)) = f(x - 2) = (x - 2) - 2 = x - 4$.

(d) $(g \circ g)(x) = g(g(x)) = g(x^2 + 3x + 4) = (x^2 + 3x + 4)^2 + 3(x^2 + 3x + 4) + 4$

$$= (x^4 + 9x^2 + 16 + 6x^3 + 8x^2 + 24x) + 3x^2 + 9x + 12 + 4$$

$$= x^4 + 6x^3 + 20x^2 + 33x + 32$$

33. $f(x) = 1 - 3x$; $g(x) = \cos x$. $D = \mathbb{R}$ for both f and g, and hence for their composites.

(a) $(f \circ g)(x) = f(g(x)) = f(\cos x) = 1 - 3\cos x$.

(b) $(g \circ f)(x) = g(f(x)) = g(1 - 3x) = \cos(1 - 3x)$.

(c) $(f \circ f)(x) = f(f(x)) = f(1 - 3x) = 1 - 3(1 - 3x) = 1 - 3 + 9x = 9x - 2$.

(d) $(g \circ g)(x) = g(g(x)) = g(\cos x) = \cos(\cos x)$ [Note that this is *not* $\cos x \cdot \cos x$.]

34. $f(x) = \sqrt{x},\ D = [0, \infty)$; $g(x) = \sqrt[3]{1 - x},\ D = \mathbb{R}$.

(a) $(f \circ g)(x) = f(g(x)) = f\left(\sqrt[3]{1 - x}\right) = \sqrt{\sqrt[3]{1 - x}} = \sqrt[6]{1 - x}$.

The domain of $f \circ g$ is $\{x \mid \sqrt[3]{1 - x} \geq 0\} = \{x \mid 1 - x \geq 0\} = \{x \mid x \leq 1\} = (-\infty, 1]$.

(b) $(g \circ f)(x) = g(f(x)) = g(\sqrt{x}) = \sqrt[3]{1 - \sqrt{x}}$.

The domain of $g \circ f$ is $\{x \mid x$ is in the domain of f and $f(x)$ is in the domain of $g\}$. This is the domain of f, that is, $[0, \infty)$.

(c) $(f \circ f)(x) = f(f(x)) = f(\sqrt{x}) = \sqrt{\sqrt{x}} = \sqrt[4]{x}$. The domain of $f \circ f$ is $\{x \mid x \geq 0$ and $\sqrt{x} \geq 0\} = [0, \infty)$.

(d) $(g \circ g)(x) = g(g(x)) = g\left(\sqrt[3]{1 - x}\right) = \sqrt[3]{1 - \sqrt[3]{1 - x}}$, and the domain is $(-\infty, \infty)$.

37. $(f \circ g \circ h)(x) = f(g(h(x))) = f(g(x - 1)) = f(2(x - 1)) = 2(x - 1) + 1 = 2x - 1$

38. $(f \circ g \circ h)(x) = f(g(h(x))) = f(g(1 - x)) = f((1 - x)^2) = 2(1 - x)^2 - 1 = 2x^2 - 4x + 1$

39. $(f \circ g \circ h)(x) = f(g(h(x))) = f(g(x^3 + 2)) = f[(x^3 + 2)^2]$

$$= f(x^6 + 4x^3 + 4) = \sqrt{(x^6 + 4x^3 + 4) - 3} = \sqrt{x^6 + 4x^3 + 1}$$

41. Let $g(x) = x^2 + 1$ and $f(x) = x^{10}$. Then $(f \circ g)(x) = f(g(x)) = f(x^2 + 1) = (x^2 + 1)^{10} = F(x)$.

42. Let $g(x) = \sqrt{x}$ and $f(x) = \sin x$. Then $(f \circ g)(x) = f(g(x)) = f\left(\sqrt{x}\right) = \sin\left(\sqrt{x}\right) = F(x)$.

43. Let $g(x) = \sqrt[3]{x}$ and $f(x) = \dfrac{x}{1+x}$. Then $(f \circ g)(x) = f(g(x)) = f(\sqrt[3]{x}) = \dfrac{\sqrt[3]{x}}{1+\sqrt[3]{x}} = F(x)$.

47. Let $h(x) = x^2$, $g(x) = 3^x$, and $f(x) = 1 - x$. Then

$$(f \circ g \circ h)(x) = f(g(h(x))) = f(g(x^2)) = f\left(3^{x^2}\right) = 1 - 3^{x^2} = H(x).$$

50. (a) $f(g(1)) = f(6) = 5$

(b) $g(f(1)) = g(3) = 2$

(c) $f(f(1)) = f(3) = 4$

(d) $g(g(1)) = g(6) = 3$

(e) $(g \circ f)(3) = g(f(3)) = g(4) = 1$

(f) $(f \circ g)(6) = f(g(6)) = f(3) = 4$

51. (a) $g(2) = 5$, because the point $(2, 5)$ is on the graph of g. Thus, $f(g(2)) = f(5) = 4$, because the point $(5, 4)$ is on the graph of f.

(b) $g(f(0)) = g(0) = 3$

(c) $(f \circ g)(0) = f(g(0)) = f(3) = 0$

(d) $(g \circ f)(6) = g(f(6)) = g(6)$. This value is not defined, because there is no point on the graph of g that has x-coordinate 6.

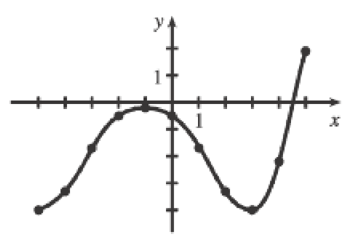
(e) $(g \circ g)(-2) = g(g(-2)) = g(1) = 4$

(f) $(f \circ f)(4) = f(f(4)) = f(2) = -2$

52. To find a particular value of $f(g(x))$, say for $x = 0$, we note from the graph that $g(0) \approx 2.8$ and $f(2.8) \approx -0.5$. Thus, $f(g(0)) \approx f(2.8) \approx -0.5$. The other values listed in the table were obtained in a similar fashion.

x	$g(x)$	$f(g(x))$
-5	-0.2	-4
-4	1.2	-3.3
-3	2.2	-1.7
-2	2.8	-0.5
-1	3	-0.2

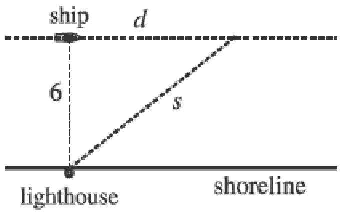
x	$g(x)$	$f(g(x))$
0	2.8	-0.5
1	2.2	-1.7
2	1.2	-3.3
3	-0.2	-4
4	-1.9	-2.2
5	-4.1	1.9



54. (a) The radius r of the balloon is increasing at a rate of 2 cm/s, so $r(t) = (2 \text{ cm/s})(t \text{ s}) = 2t$ (in cm).

(b) Using $V = \frac{4}{3}\pi r^3$, we get $(V \circ r)(t) = V(r(t)) = V(2t) = \frac{4}{3}\pi(2t)^3 = \frac{32}{3}\pi t^3$.
The result, $V = \frac{32}{3}\pi t^3$, gives the volume of the balloon (in cm^3) as a function of time (in s).

55. (a) From the figure, we have a right triangle with legs 6 and d, and hypotenuse s. By the Pythagorean Theorem, $d^2 + 6^2 = s^2 \quad \Rightarrow \quad s = f(d) = \sqrt{d^2 + 36}$.



(b) Using $d = rt$, we get $d = (30 \text{ km/hr})(t \text{ hr}) = 30t$ (in km). Thus, $d = g(t) = 30t$.

(c) $(f \circ g)(t) = f(g(t)) = f(30t) = \sqrt{(30t)^2 + 36} = \sqrt{900t^2 + 36}$. This function represents the distance between the lighthouse and the ship as a function of the time elapsed since noon.