SECTION 9.4:
ARCS & CHORDS

Standards:

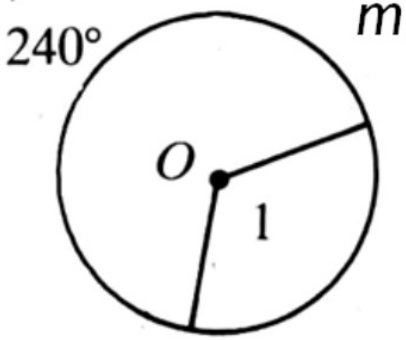
7.0 - Students prove and use theorems involving the properties of parallel lines cut by a transversal, the properties of quadrilaterals, and the properties of circles.

21.0 - Students prove and solve problems regarding relationships among chords, secants, tangents, inscribed angles, and inscribed and circumscribed polygons of circles.

WARMUP #4

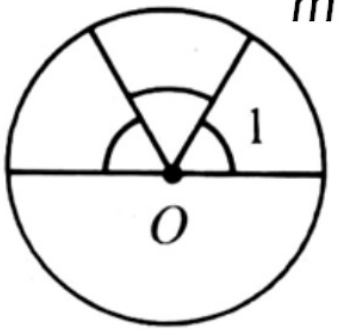
1-4: Find the measure of each numbered angle. O is the center of the circle.

1)



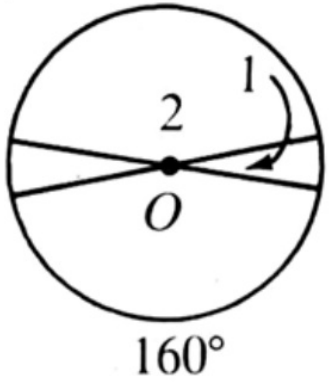
$m(\angle 1) = 120$

2)



$m(\angle 1) = 60$

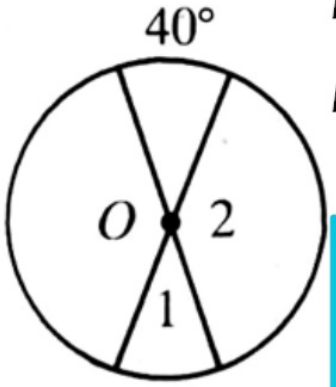
3)



$m(\angle 1) = 20$

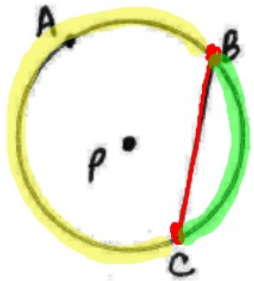
$m(\angle 2) = 160$

4)



$m(\angle 1) = 40$

$m(\angle 2) = 140$



$\overline{BC}$ cuts off 2 arcs: $\overset{\frown}{BC}$ and $\overset{\frown}{BAC}$

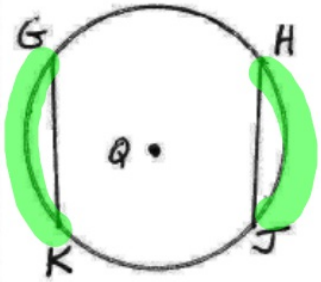
The minor arc, $\overset{\frown}{BC}$, is the arc of chord $\overline{BC}$

Fill in the blanks in your notes

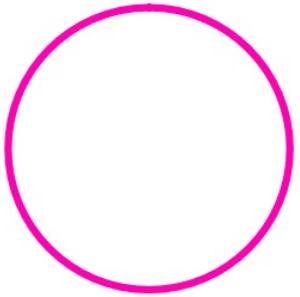
THEOREM

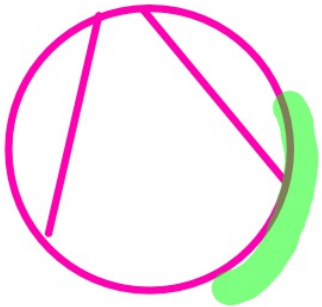
In the same circle or in congruent circles:

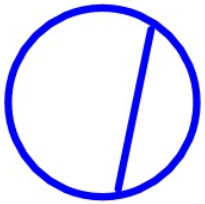
1. **congruent arcs have congruent chords**
2. **Congruent chords have congruent arcs**



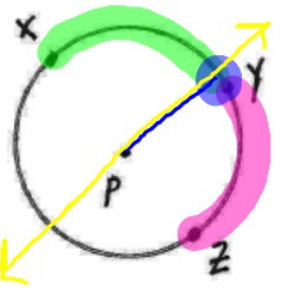
1. If $\overset{\frown}{GK} \cong \overset{\frown}{HJ}$, then $\overline{GK} \cong \overline{HJ}$
2. If $\overline{GK} \cong \overline{HJ}$, then $\overset{\frown}{GK} \cong \overset{\frown}{HJ}$





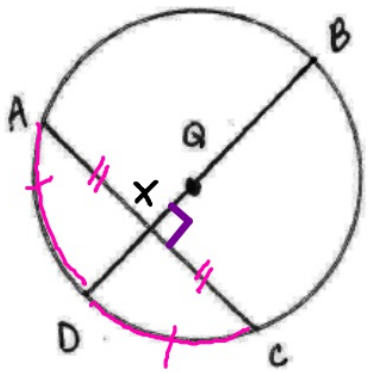


A point Y is called the midpoint of $\overset{\frown}{XYZ}$ if $\overset{\frown}{XY} \cong \overset{\frown}{YZ}$. Any line, segment, or ray that contains Y bisects $\overset{\frown}{XYZ}$



Fill in the blanks in your notes

THEOREM



A diameter that is perpendicular to a chord bisects the chord and its arc.

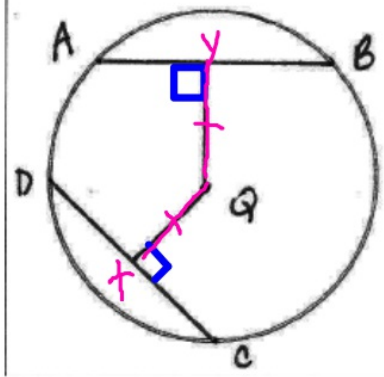
If $\overline{DB} \perp \overline{AC}$ then $\overline{AX} \cong \overline{XC}$ and $\overset{\frown}{AD} \cong \overset{\frown}{DC}$

Fill in the blanks in your notes

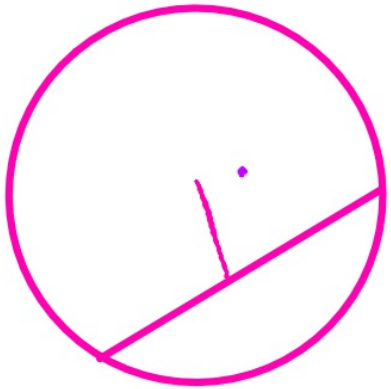
THEOREM

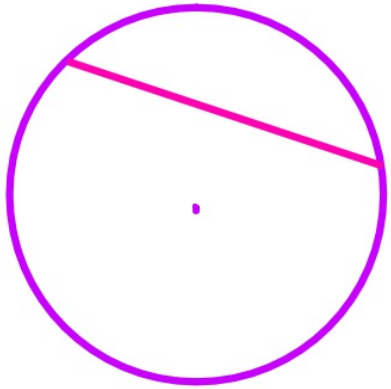
In the same circle or in congruent circles:

1. **chords equally distant from the center are ≅**
2. **≅ chords are equally distant from the center.**



1. **QX = QY then $\overline{AB} \cong \overline{DC}$**
2. **If $\overline{AB} \cong \overline{DC}$, then XQ = YQ**

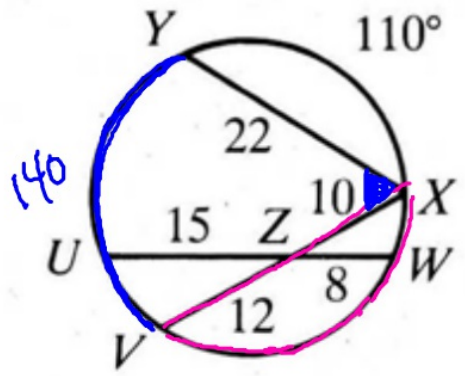




Complete.

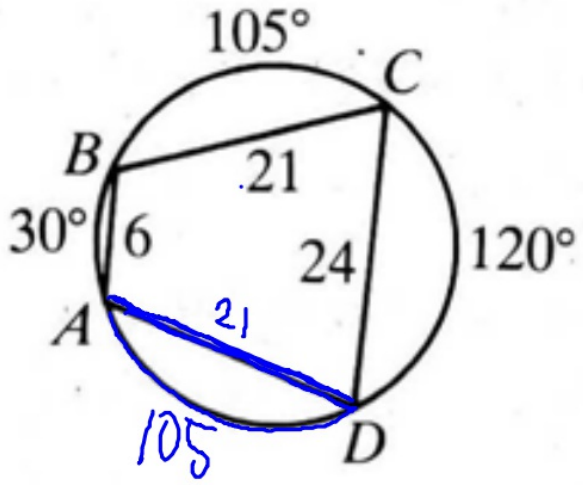
1. $m\overset{\frown}{VWX} =$ 110°

$m\overset{\frown}{VUY} =$ 140°



110

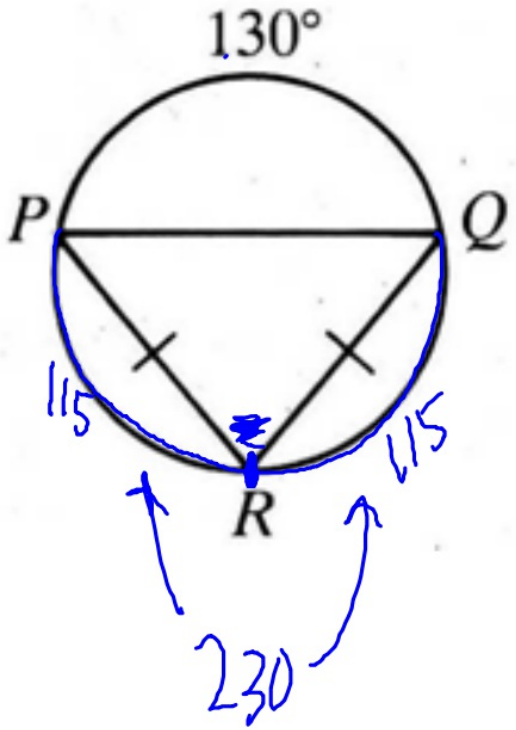
2. $AD =$ 21



120
+30
+105
255

3. $m\widehat{PR} =$ 115

$m\widehat{RQ} =$ 115



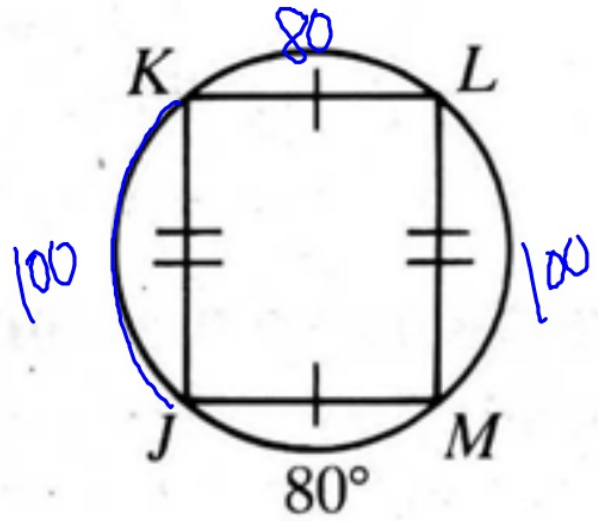
360
−130
230

115

115

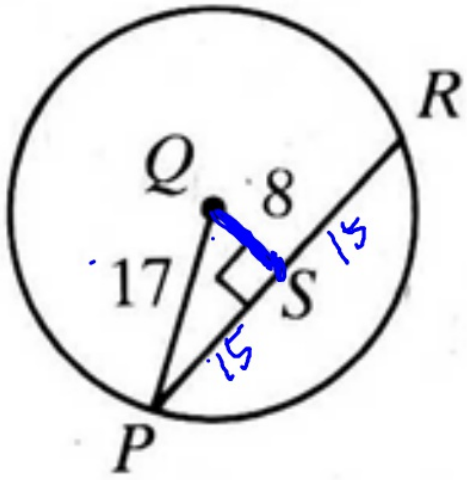
230

4. $m\overset{\frown}{JK} =$ 100, $m\overset{\frown}{LM} =$ 100,
$m\overset{\frown}{KL} =$ 80



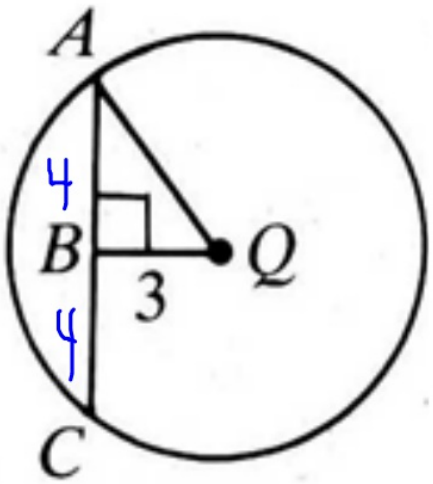
In the diagrams that follow, Q is the center of each

✦ **5.** $PR =$? 30

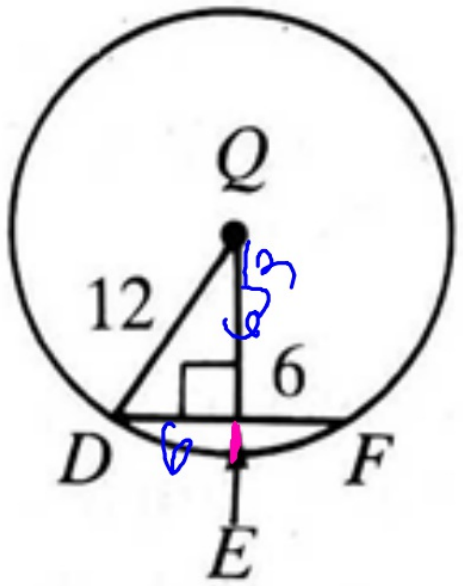


6. $AC = 8$

$AQ =$? 5

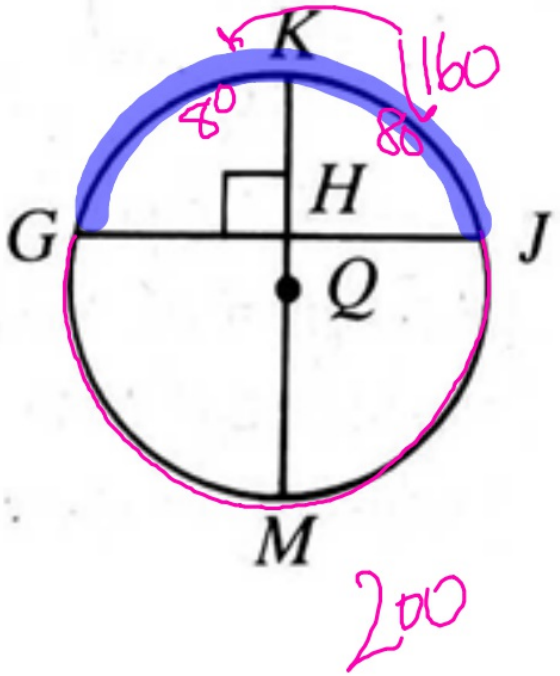


7. $QE =$? $6\sqrt{3}$



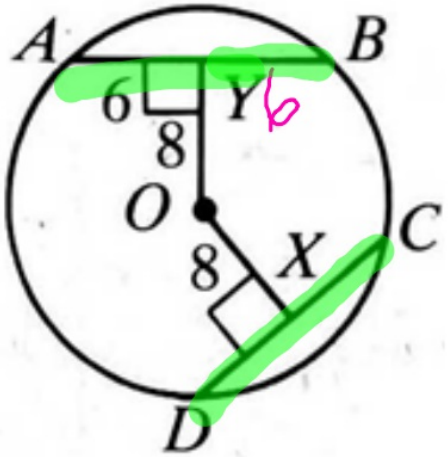
8. $m\widehat{GMJ} = 200$

$m\widehat{GK} =$ 80



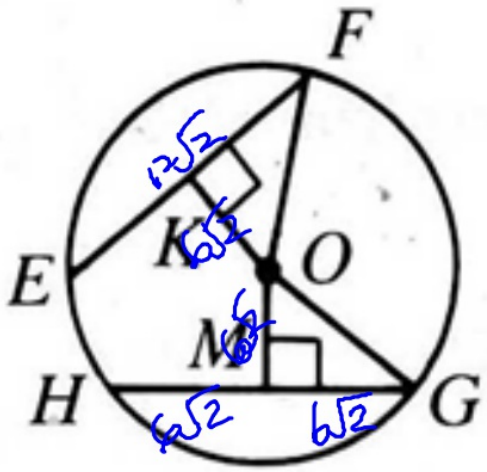
In the diagrams that follow, O is the center of the circle. Cor

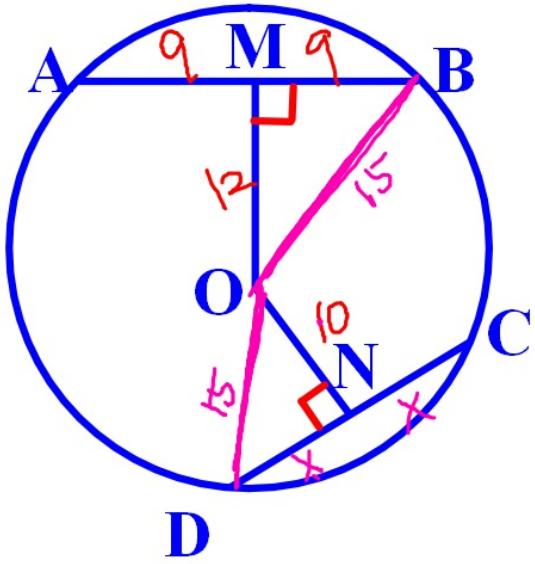
9 $DC =$ 12



10. $HG = 12\sqrt{2}; OM = 6\sqrt{2};$

$EF = 12\sqrt{2}; OK =$?$6\sqrt{2}$





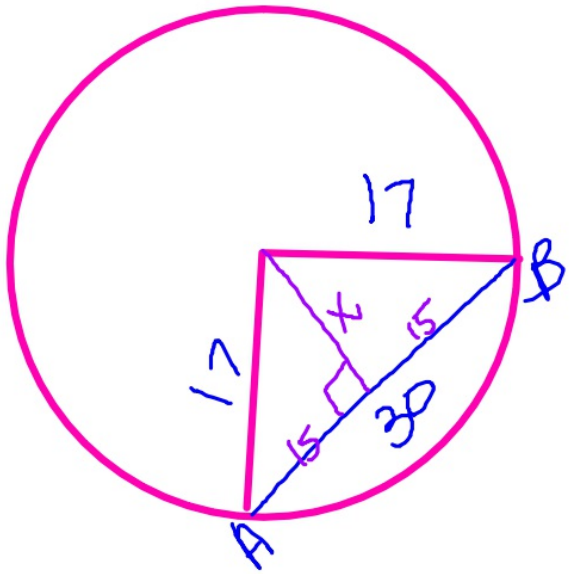
AB=18; OM = 12
ON = 10; CD = ?

$10^2 + x^2 = 15^2$

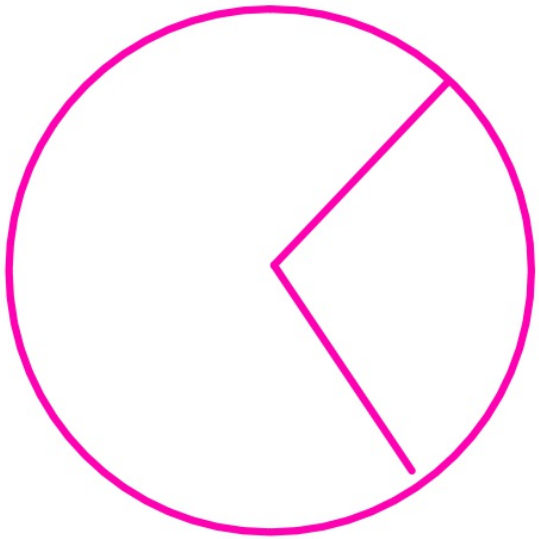
$\sqrt{125} = 5\sqrt{5}$

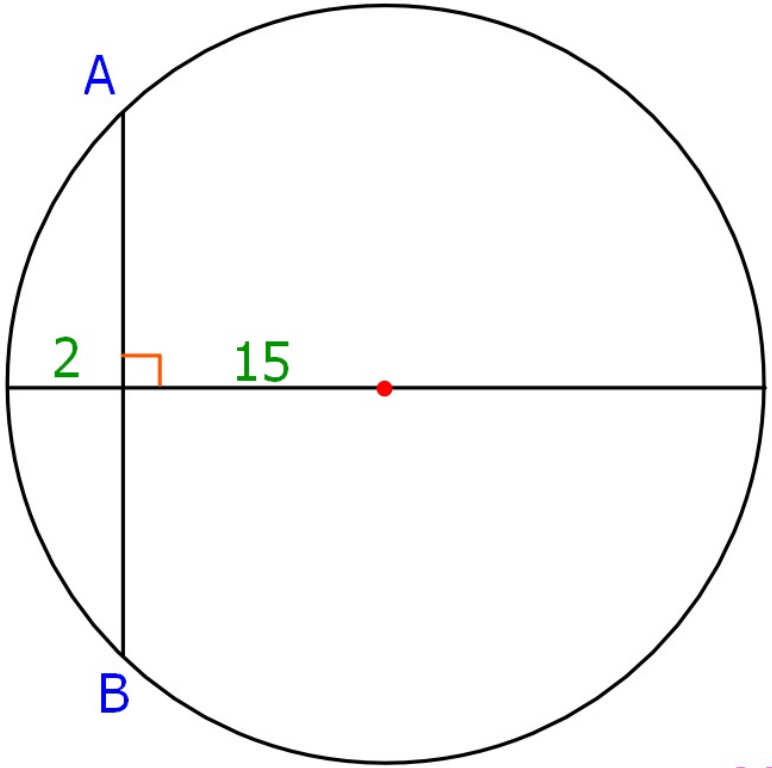
- $10\sqrt{5}$

Sketch a $\odot O$ with radius of length 17 and chord $\overline{AB}$ of length 30. How far is the chord from O? 8



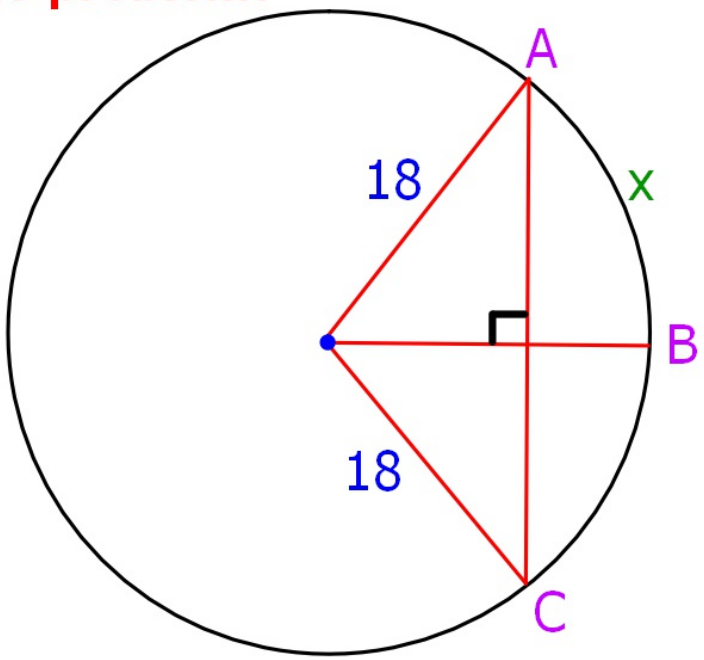
Sketch a $\odot R$ with radius of length $5\sqrt{3}$ and chord $\overline{XY}$ that is 5 units from R. Find the length of $\overline{XY}$.





AB = _____

Challenge problem:



AC = 24

Hint:

x = _____

A plane 12 cm from the center of a sphere intersects the sphere in a circle with diamete of 32. Find the diameter of the sphere.

Step 1

Step 2

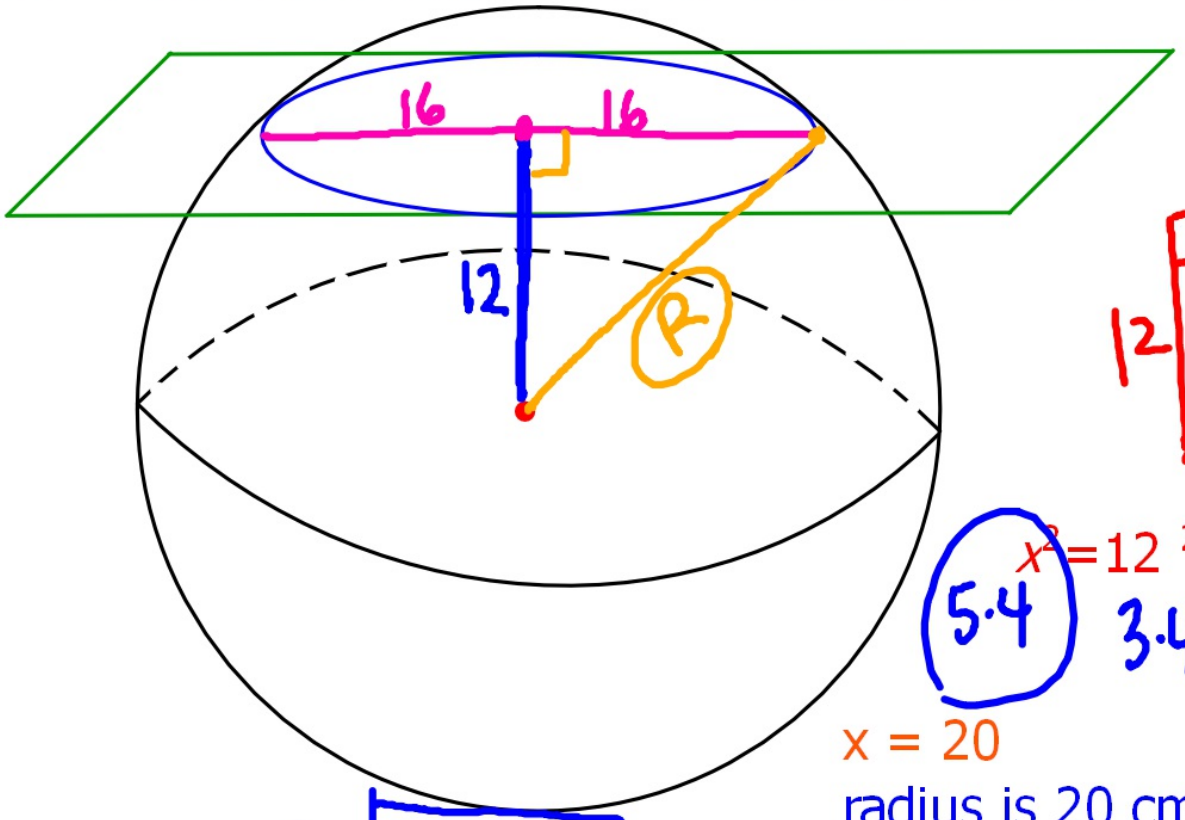
Step 3

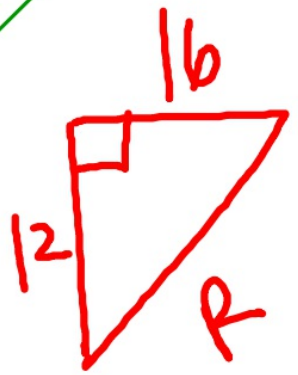
Step 4

Step 5

Step 6

ANSWER





$x^2 = 12^2 + 16^2$

5.4 3.4 4.4

$x^2 = \sqrt{144 + 256}$

$x^2 = \sqrt{400}$

x = 20

radius is 20 cm

diameter is 40 cm

HOMEWORK

HW 12.4

- Pg. 347 (WE): # 1-14, 17-22
- Pg. 349 (S-T-I): #1-3

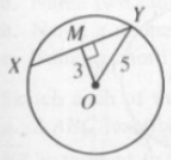
HW 12.4

Pg. 347 (WE): # 1-14, 17-22 and Pg. 349 (S-T-1): #1-3

Written Exercises

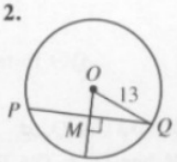
In the diagrams that follow, O is the center of the circle.

1.



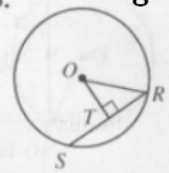
$XY = \underline{?}$

2.



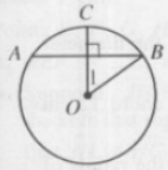
$PQ = 24$; $OM = \underline{?}$

3.



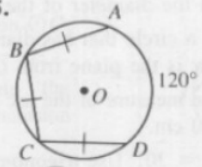
$OT = 9$; $RS = 18$
$OR = \underline{?}$

4.



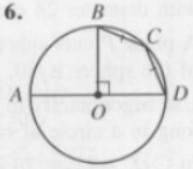
$m\overset{\frown}{ACB} = 110$;
$m\angle 1 = \underline{?}$

5.



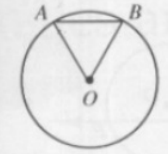
$m\overset{\frown}{BC} = \underline{?}$

6.



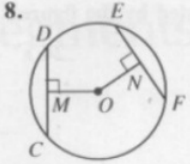
$m\overset{\frown}{CD} = \underline{?}$

7.



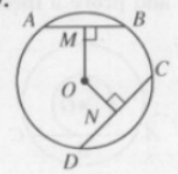
$m\angle AOB = 60$;
$AB = 24$; $OA = \underline{?}$

8.



$OM = ON = 7$;
$CM = 6$; $EF = \underline{?}$

9.



$AB = 18$; $OM = 12$;
$ON = 10$; $CD = \underline{?}$

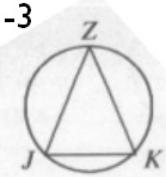
10. Sketch a circle with two noncongruent chords. Is the longer chord farther from the center or closer to the center than the shorter chord?

11. Sketch a circle O with radius 10 and chord $\overline{XY}$ 8 cm long. How far is the chord from O?

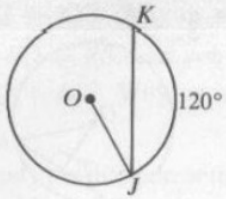
12. Sketch a circle Q with a chord $\overline{RS}$ that is 16 cm long and 2 cm from Q. What is the radius of $\odot Q$?

13. Sketch a circle P with radius 5 cm and chord $\overline{AB}$ that is 2 cm from P. Find the length of $\overline{AB}$.

14. Given: $\overset{\frown}{JZ} \cong \overset{\frown}{KZ}$
Prove: $\angle J \cong \angle K$

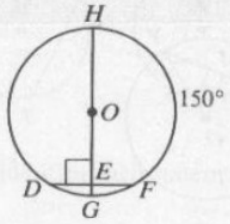


17.



If $OJ = 10$, $JK = \underline{?}$.

18.



If $OE = 8\sqrt{3}$, $HG = \underline{?}$.

19. A plane 5 cm from the center of a sphere intersects the sphere in a circle with diameter 24 cm. Find the diameter of the sphere.

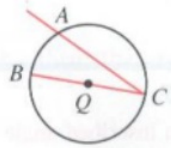
20. A plane P cuts sphere O in a circle that has diameter 20. If the diameter of the sphere is 30, how far is the plane from O?

21. Use trigonometry to find the measure of the arc cut off by a chord 12 cm long in a circle of radius 10 cm.

22. In $\odot O$, $m\overset{\frown}{RS} = 70$ and $RS = 20$. Use trigonometry to find the radius of $\odot O$.

Self-Test 1

1. Points A, B, and C lie on $\odot Q$.



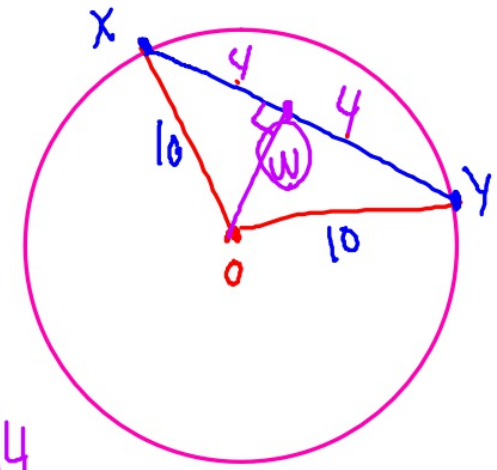
a. Name two radii of $\odot Q$.
b. Name a diameter of $\odot Q$.
c. Name a chord and a secant of $\odot Q$.

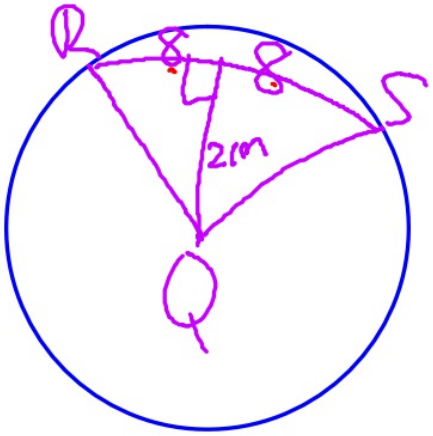
2. Sketch each of the following.
a. $\triangle ABC$ inscribed in $\odot O$ **b.** Quad. $LUMX$ circumscribed about $\odot Q$

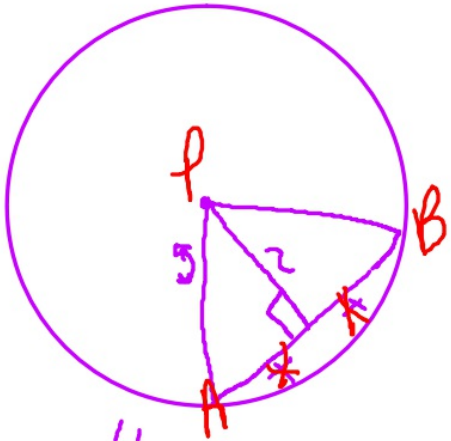
3. $\overline{NP}$ is tangent to $\odot O$ at P. If $NO = 25$ and $NP = 20$, find OP.

12. Sketch a circle Q with a chord RS that is 16 cm long and 2 cm from Q. What is the radius of $\odot Q$?

13. Sketch a circle P with radius 5 cm and chord $\overline{AB}$ that is 2 cm from P. Find the length of $\overline{AB}$.







$8^2+2^2=R^2$

$4+x^2=25$

$4^2+w^2=10^2$

$-16 \quad -16$

$\sqrt{w^2}=\sqrt{84} = 4\cdot21 = 2\sqrt{21}$

$\sqrt{x^2}=\sqrt{21}$

$x=\sqrt{21}$