

Old text p. 138

39) $f(x) = -x^3 + x^2 - 2$

a) LC test: $\lim_{x \rightarrow -\infty} f(x) = -\infty / \lim_{x \rightarrow \infty} f(x) = \infty$

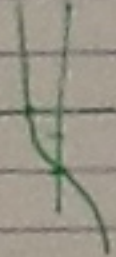
b) p.a. $\pm 1, \pm 2$

$$\begin{array}{r|rrrr} -1 & -1 & 1 & 0 & -2 \\ & & 1 & -2 & 2 \\ \hline & -1 & 2 & -2 & 0 \end{array}$$

$-x^2 + 2x + 2 \rightarrow x = \frac{-2 \pm \sqrt{4 - 4(-1)(2)}}{-2} = \frac{-2 \pm \sqrt{4 + 8}}{-2} = \frac{-2 \pm 2\sqrt{3}}{-2} = 1 \pm \sqrt{3}$

zeros: $x = -1, 1 \pm i$

c) -d)



40) $g(x) = 2x^3 + 4x^2$

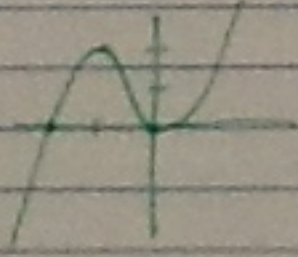
a) LC test: $\lim_{x \rightarrow -\infty} f(x) = -\infty / \lim_{x \rightarrow \infty} f(x) = \infty$

b) $2x^2(x+2) = 0$

$x = 0, -2$

c) -d)

$f(-1) = 2$



42) $h(x) = 3x^2 - x^4$

a) LC test: $\lim_{x \rightarrow -\infty} f(x) = -\infty / \lim_{x \rightarrow \infty} f(x) = -\infty$

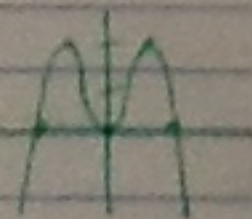
b) $x^2(3 - x^2) = 0$

$x = 0, \pm\sqrt{3}$

c) -d)

$f(-1) = 2$

$f(1) = 2$



62) $f(x) = 2x^3 + 11x^2 - 21x - 90$
 factor: $(x+6)$

$$\begin{array}{r|rrrr} -6 & 2 & 11 & -21 & -90 \\ & & -12 & 6 & 40 \\ \hline & 2 & -1 & -15 & 0 \end{array}$$

$$\begin{array}{l} 2x^2 - x - 15 \\ 2x^2 - 6x + 5x - 15 \\ \hline 2x(x-3) + 5(x-3) \\ (2x+5)(x-3) \\ x = -5/2, 3 \end{array}$$

$$f(x) = (x+6)(2x+5)(x-3)$$

 zeros: $x = -6, -5/2, 3$

63) $f(x) = x^4 - 4x^3 - 7x^2 + 22x + 24$
 factors: $(x+2)(x-3)$

$$\begin{array}{r|rrrrr} -2 & 1 & -4 & -7 & 22 & 24 \\ & & -2 & 12 & -10 & -24 \\ \hline 3 & 1 & -6 & 5 & 12 & 0 \\ & & 3 & -9 & -12 & \\ \hline & 1 & -3 & -4 & 0 & \end{array}$$

$$\begin{array}{l} x^2 - 3x - 4 \\ (x-4)(x+1) \\ x = 4, -1 \end{array}$$

$$f(x) = (x+2)(x-3)(x-4)(x+1)$$

 zeros: $x = -2, 3, 4, -1$

7) $3x^2 + 1 = 0$
 $x = \pm \frac{\sqrt{3}i}{3}$

8) $x^2 - 2x + 10 = 0$

$$x = \frac{2 \pm \sqrt{4 - 4(1)(10)}}{2} = \frac{2 \pm \sqrt{-36}}{2} = \frac{2 \pm 6i}{2} = 1 \pm 3i$$

9) $f(x) = x^3 - 2x^2 - 21x - 18$
 factors: $\pm 1, \pm 2, \pm 3, \pm 6, \pm 9, \pm 18$

$$\begin{array}{r|rrrr} -1 & 1 & -2 & -21 & -18 \\ & & -1 & 3 & 18 \\ \hline & 1 & -3 & -18 & 0 \end{array}$$

$$\begin{array}{l} x^2 - 3x - 18 = 0 \\ (x-6)(x+3) = 0 \\ x = 6, -3 \end{array}$$

zeros: $-1, -3, 6$

95) $f(x) = x^3 - 10x^2 + 17x - 8$
 PCZ: $\pm 1, \pm 2, \pm 4, \pm 8$

$$\begin{array}{r|rrrr} 1 & 1 & -10 & 17 & -8 \\ & & 1 & -9 & 8 \\ \hline & 1 & -9 & 8 & 0 \end{array} \quad \text{zeros: } 1, 1, 8$$

$$x^2 - 9x + 8 = 0$$

$$(x-8)(x-1) = 0$$

$$x = 8, 1$$

96) $f(x) = x^4 + x^3 - 11x^2 + x - 12$
 PCZ: $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$

$$\begin{array}{r|rrrrr} 3 & 1 & 1 & -11 & 1 & -12 \\ & & 3 & 12 & 4 & 12 \\ \hline & 1 & 4 & 1 & 4 & 0 \end{array}$$

PCZ: $\pm 1, \pm 2, \pm 4$

$$\begin{array}{r|rrrr} -4 & 1 & 4 & 1 & 4 \\ & & -4 & 0 & -4 \\ \hline & 1 & 0 & 1 & 0 \end{array} \quad \text{zeros: } x = 3, -4, \pm i$$

$$x^2 + 1 = 0$$

$$x = \pm i$$

98) zeros: $2, -3, 1-2i, 1+2i$ (not given)

$$\begin{aligned} f(x) &= (x-2)(x+3)(x-(1-2i))(x-(1+2i)) \\ &= (x^2+x-6) \left(x^2 - x(1+2i) - x(1-2i) + (1-2i)(1+2i) \right) \end{aligned}$$

$$= (x^2+x-6)(x^2-x-2xi-x+2xi+1+4)$$

$$= (x^2+x-6)(x^2-2x+5)$$

$$= x^4 - 2x^3 + 5x^2 + x^3 - 2x^2 + 5x - 6x^2 + 12x - 30$$

$$= x^4 - x^3 - 3x^2 + 17x - 30$$

$$100) h(x) = -x^3 + 2x^2 - 10x + 32$$

$$\text{zero: } -4i$$

Solution:

$$(x+4i)(x-4i) = x^2 + 16$$

$$\begin{array}{r} x^2 + 0x + 16 \overline{) -x^3 + 2x^2 - 10x + 32} \\ \underline{-(x^3 + 0x^2 - 10x)} \\ 2x^2 + 10x + 32 \\ \underline{-(2x^2 + 0x + 32)} \\ 0 \end{array}$$

$$-x + 2 = 0$$

$$x = 2$$

$$\text{zeros: } \pm 4i, 2$$

$$103) f(x) = x^3 + 4x^2 - 6x$$

$$= x(x^2 + 4x - 6)$$

$$f(x) = x(x+5)(x-1)$$

$$\text{zeros: } x = 0, -5, 1$$

$$104) g(x) = x^3 - 7x^2 + 36$$

$$\text{prz: } \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 9, \pm 12, \pm 18, \pm 36$$

$$\begin{array}{r|rrrr} -2 & 1 & -7 & 0 & 36 \\ & & -2 & 18 & -36 \\ \hline & 1 & -9 & 18 & 0 \end{array}$$

$$x^2 - 9x + 18$$

$$(x-6)(x-3)$$

$$x = 6, 3$$

$$f(x) = (x+2)(x-6)(x-3)$$

$$\text{zeros: } x = -2, 6, 3$$

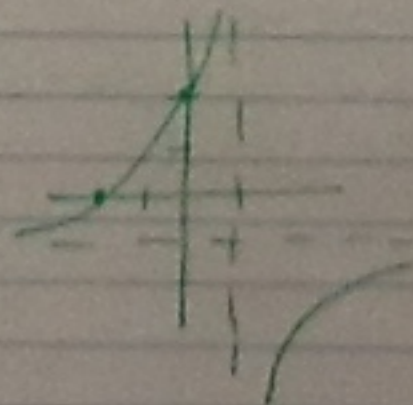
$$121) g(x) = \frac{2+x}{1-x}$$

$$\text{VA: } x = 1$$

$$\text{HA: } y = -1$$

$$x\text{-int: } -2$$

$$y\text{-int: } 2$$



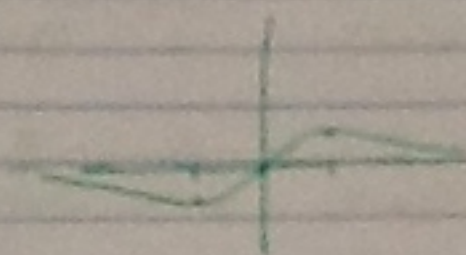
$$126) f(x) = \frac{x}{x^2+1}$$

VA: none

HA: $y=0$

x-int: $x=0$

y-int: $y=0$



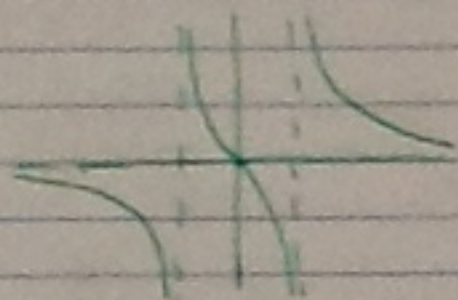
$$129) y = \frac{x}{x^2-1}$$

VA: $x=1$

HA: $y=0$

x-int: $x=0$

y-int: $y=0$



$$131) f(x) = \frac{2x^3}{x^2+1}$$

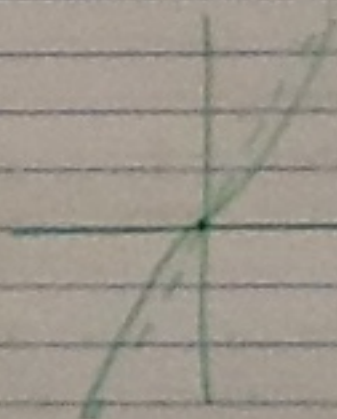
VA: none

SA: $y=2x$

$$\begin{array}{r} 2x \\ x^2+0x+1 \overline{) 2x^3+0x^2+0x+0} \\ \underline{-(2x^3+0x^2+2x)} \\ -2x \end{array}$$

x-int: $x=0$

y-int: $y=0$



$$133) f(x) = \frac{x^2+3x-10}{x+2}$$

VA: $x=-2$

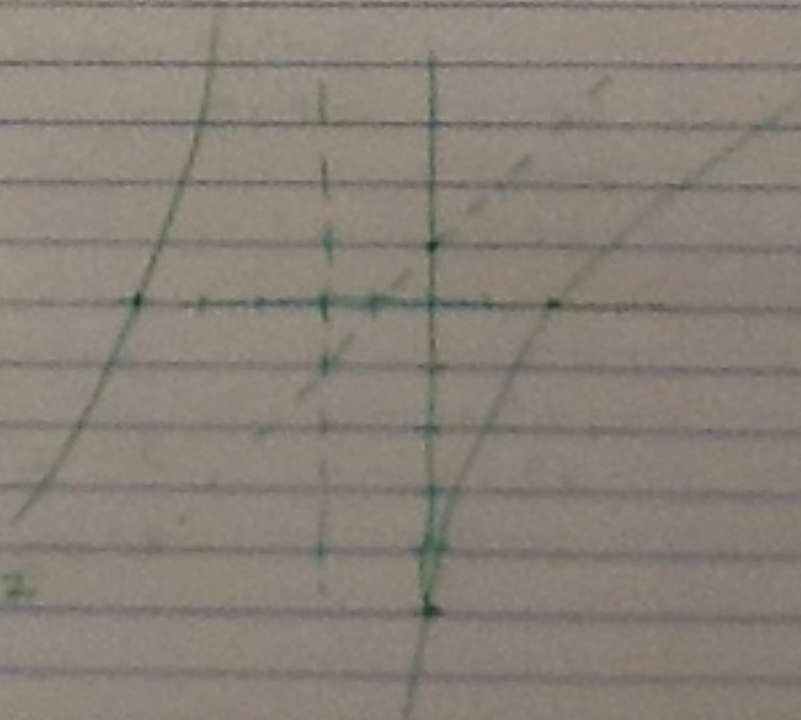
SA: $y=x+1$

$$\begin{array}{r} x+1 \\ x+2 \overline{) x^2+3x-10} \\ \underline{-(x^2+2x)} \\ x-10 \\ \underline{-(x+2)} \\ -12 \end{array}$$

x-int:

$$(x+5)(x-2) = 0 \rightarrow x = -5, 2$$

y-int: -5



$$131) \frac{3}{x^2+20x} = \frac{3}{x(x+20)} = \frac{A}{x} + \frac{B}{x+20}$$

$$140) \frac{x-8}{x^2-3x-28} = \frac{x-8}{(x-7)(x+4)} = \frac{A}{x-7} + \frac{B}{x+4}$$

$$141) \frac{3x-4}{x^3-5x^2} = \frac{3x-4}{x^2(x-5)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-5}$$

$$142) \frac{x-2}{x(x^2+2)^2} = \frac{A}{x} + \frac{Bx+C}{x^2+2} + \frac{Dx+E}{(x^2+2)^2}$$

$$143) \frac{4-x}{x^2+6x+8} = \frac{4-x}{(x+4)(x+2)} = \frac{A}{x+4} + \frac{B}{x+2}$$

$$4-x = A(x+2) + B(x+4)$$

$$\text{let } x = -4$$

$$\text{let } x = -2$$

$$8 = -2A$$

$$6 = 2B$$

$$A = -4$$

$$B = 3$$

$$\text{ans: } \frac{-4}{x+4} + \frac{3}{x+2}$$

$$147) \frac{x^2+2x}{x^3-x^2+x-1} = \frac{x^2+2x}{(x^2+1)(x-1)} = \frac{Ax+B}{x^2+1} + \frac{C}{x-1}$$

$$\begin{aligned} x^2+2x &= (Ax+B)(x-1) + C(x^2+1) \\ &= Ax^2 + Ax + Bx - B + Cx^2 + C \\ &= (A+C)x^2 + (A+B)x + (-B+C) \end{aligned}$$

$$A+C=1 \rightarrow A=1-C$$

$$-A+B=2$$

$$-B+C=0 \rightarrow C=B$$

$$-(1-C)+C=2$$

$$A=1-\frac{3}{2}$$

$$\text{ans: } \frac{-x+3}{2(x^2+1)} + \frac{3}{2(x-1)}$$

$$-1+C+C=2$$

$$2C=3$$

$$C=\frac{3}{2}$$

$$A=-\frac{1}{2}$$

$$B=\frac{3}{2}$$

$$148) \frac{4x-2}{3(x-1)^2} = \frac{A}{3(x-1)} + \frac{B}{(x-1)^2}$$

$$4x-2 = A(x-1) + 3B$$

~~$$\text{Let } x=0 \quad \text{Let } x=1$$~~

~~$$-2 = -A \quad 2 = 3B$$~~

~~$$A = 2$$~~

$$4x-2 = Ax - A + 3B$$

$$= Ax + (3B - A)$$

$$4 = A$$

$$3B - A = -2$$

$$3B - 4 = -2$$

$$3B = 2$$

$$B = \frac{2}{3}$$

$$\text{ans: } \frac{4}{3(x-1)} + \frac{2}{3(x-1)^2}$$

$$149) \frac{3x^3+4x}{(x^2+1)^2} = \frac{Ax+B}{x^2+1} + \frac{Cx+D}{(x^2+1)^2}$$

$$3x^3+4x = (Ax+B)(x^2+1) + Cx+D$$

$$= Ax^3 + Ax + Bx^2 + B + Cx + D$$

$$= Ax^3 + Bx^2 + (A+C)x + (B+D)$$

$$A = 3$$

$$B = 0$$

$$A+C=4 \rightarrow C=1 \quad \text{ans: } \frac{3x}{x^2+1} + \frac{x}{(x^2+1)^2}$$

$$B+D=0 \rightarrow D=0$$