p. 653

1) $a_n = \frac{2}{3}n$

$a_1 = \frac{2}{3}(1) = \frac{2}{3}$

$a_2 = \frac{2}{3}(2) = \frac{4}{3}$

$a_3 = \frac{2}{3}(3) = 2$

$a_4 = \frac{2}{3}(4) = \frac{8}{3}$

$a_5 = \frac{2}{3}(5) = \frac{10}{3}$

$\frac{2}{3}, \frac{4}{3}, 2, \frac{8}{3}, \frac{10}{3}$

2) $a_n = 4^{n-1}$

$a_1 = 4^{1-1} = 4^0 = 1$

$a_2 = 4^{2-1} = 4^1 = 4$

$a_3 = 4^{3-1} = 4^2 = 16$

$a_4 = 4^{4-1} = 4^3 = 64$

$a_5 = 4^{5-1} = 4^4 = 256$

1, 4, 16, 64, 256

3) $a_1 = -1$ and $a_n = 2(a_{n-1}) - 12$

a_n: current term; a_{n-1}: previous term

$a_1 = -1$

$a_2 = 2(a_1) - 12 = 2(-1) - 12 = -14$

$a_3 = 2(a_2) - 12 = 2(-14) - 12 = -40$

$a_4 = 2(a_3) - 12 = 2(-40) - 12 = -92$

$a_5 = 2(a_4) - 12 = 2(-92) - 12 = -196$

$-1, -14, -40, -92, -196$

4) $a_n = n^2 - 2n$

$a_1 = 1^2 - 2(1) = -1$

$a_2 = 2^2 - 2(2) = 0$

$a_3 = 3^2 - 2(3) = 3$

$a_4 = 4^2 - 2(4) = 8$

$a_5 = 5^2 - 2(5) = 15$

$-1, 0, 3, 8, 15$

5) $8, 11, 14, 17, 20, \ldots\ldots$

+3 +3 +3 +3

Arithmetic, $d = 3$

$a_n = a_1 + (n-1)(d)$

$a_n = 8 + (n-1)(3)$

$a_n = 8 + 3n - 3$

$a_n = 3n + 5$

6) $-2, -8, -18, -32, -50, \ldots\ldots$

hmmmm.....

if you divide them all by -2

$1, 4, 9, 16, 25, \ldots\ldots$

So, $a_n = (-2)n^2$

(or see next page)

7) $1000, 200, 40, 8, \frac{8}{5}, \ldots\ldots$

$\cdot(\frac{1}{5})$ $\cdot(\frac{1}{5})$ $\cdot(\frac{1}{5})$

Geo. $r = \frac{1}{5}$

$a_n = a_1 \cdot r^{n-1}$

$a_n = 1000\left(\frac{1}{5}\right)^{n-1}$

or

$a_n = 5000\left(\frac{1}{5}\right)^{n}$

8) $437, 393, 349, 305, 261, \ldots\ldots\ldots\ldots$

-44 -44 -44 -44

Arithmetic $d = -44$

$a_n = a_1 + (n-1)(d)$

$a_n = 437 + (n-1)(-44)$

$a_n = 437 - 44n + 44$

$a_n = 481 - 44n$

6) $-2, -8, -18, -32, -50, \ldots\ldots$

$-6 \quad -10 \quad -14 \quad -18$

$-4 \quad -4 \quad -4$ ← 2nd order differences are constant

quadratic $(y = ax^2 + bx + c)$

$y = ax^2 + bx + c$

$(1, -2)$ [1] $-2 = a + b + c$

$(2, -8)$ [2] $-8 = 4a + 2b + c$

$(3, -18)$ [3] $-18 = 9a + 3b + c$

[2-1] $-8 = 4a + 2b + c$
$-(-2 = a + b + c)$

$-6 = 3a + b$

[2-3] $-8 = 4a + 2b + c$
$-(-18 = 9a + 3b + c)$

$10 = -5a - b$

$-6 = 3a + b$
$10 = -5a - b$

$4 = -2a$

$a = -2$

$-6 = 3(-2) + b$
$-6 = -6 + b$
$b = 0$

$-2 = a + b + c$
$-2 = -2 + 0 + c$
$c = 0$

$a_n = -2n^2$

OR

use graphing calculator

List 1	List 2
1	-2
2	-8
3	-18

Stat ⇒ quadratic regression

$a = -2$
$b = 0$
$c = 0$

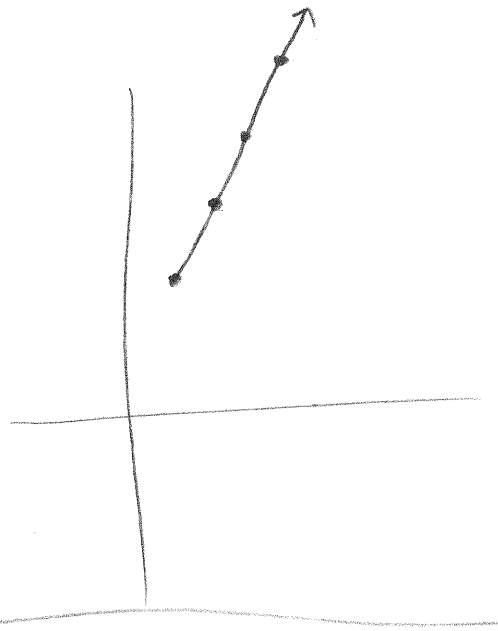
$y = -2x^2 + 0x + 0$

$y = -2x^2$

$a_n = -2n^2$

9)

n	mile marker
0	18
1	73
2	128
3	183
4	238



the graph is linear with a slope of 55

10) $\sum_{k=1}^{4} (-14-2k)$

$k=1 \quad k=2 \quad k=3 \quad k=4$

$(-16) + (-18) + (-20) + (-22)$

$= -76$

11) $\sum_{k=1}^{4} \left(\frac{k}{k+2}\right)$

$k=1 \quad k=2 \quad k=3 \quad k=4$

$\frac{1}{3} + \frac{2}{4} + \frac{3}{5} + \frac{4}{6}$

$= \frac{21}{10}$

12) $\sum_{k=1}^{5} (-1)^k (k^2-2)$

$k=1 \quad k=2 \quad k=3 \quad k=4 \quad k=5$

$1 + 2 + (-7) + 14 + (-23)$

$= -13$

13) $\sum_{k=1}^{5} \frac{1}{2}$

* special formula *

$\sum_{k=1}^{n} k = nk$

$5\left(\frac{1}{2}\right) = \frac{5}{2}$

14) $\sum_{k=1}^{40} k^2$

** special formula **

$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$

$= \frac{40(41)(81)}{6}$

$= 22,140$

15) $\sum_{k=1}^{15} k$

*** special formula ***

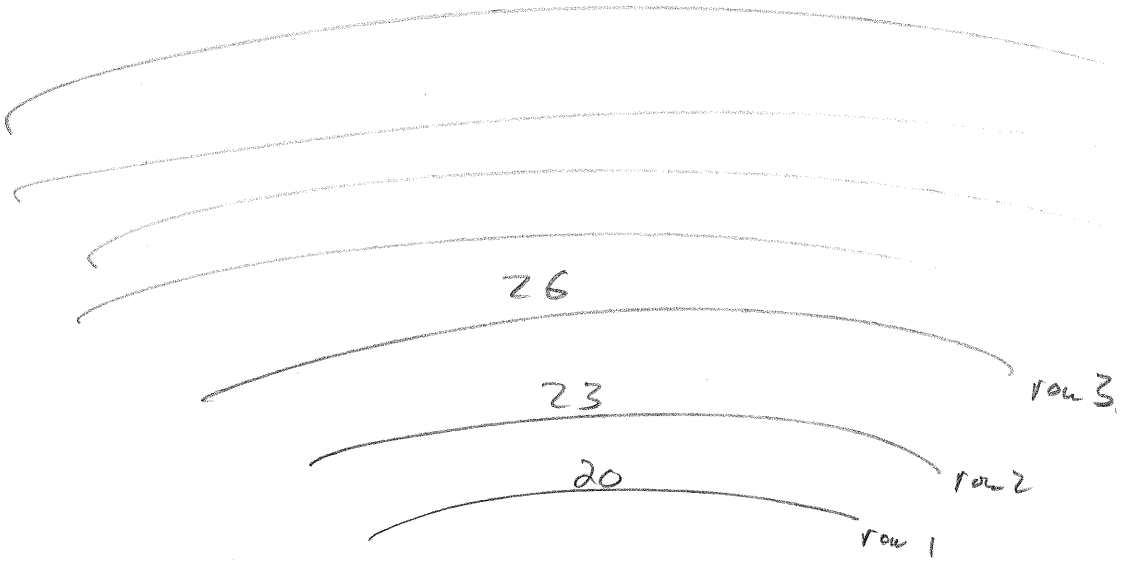
$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$

$= \frac{15(16)}{2}$

$= 120$

16) Next page

16)



20, 23, 26, ...

arithmetic, $d = 3$

$a_n = a_1 + (n-1)(d)$

$a_n = 20 + (n-1)(3)$

$a_n = 20 + 3n - 3$

$a_n = 17 + 3n$ rule

$a_{12} = 17 + 3(12)$

$= 17 + 36$

$a_{12} = 53$

There are 53 seats in the 12th row

$S_{12} = 12\left(\frac{a_1 + a_{12}}{2}\right)$

$S_{12} = 12\left(\frac{20 + 53}{2}\right)$

$= 12\left(\frac{73}{2}\right)$

$= 438$

There are a total of 438 seats in the first 12 rows

17) 10.00, 10.11, 10.22, 10.33, ...

+.11 +.11 +.11

arithmetic $d = .11$

$a_n = a_1 + (n-1)(d)$

$a_n = 10.00 + (n-1)(.11)$

$= 10.00 + .11n - .11$

$a_n = 9.89 + .11n$

$a_8 = 9.89 + .11(8)$

$a_8 = 9.89 + .88$

$a_8 = 10.77$

18) $-5, -13, -21, -29, \ldots\ldots$

-8 -8 -8

arithmetic $d = -8$

$a_n = a_1 + (n-1)(d)$

$a_n = (-5) + (n-1)(-8)$

$a_n = -5 - 8n + 8$

$a_n = 3 - 8n$

$a_8 = 3 - 8(8)$

$a_8 = 3 - 64$

$a_8 = -61$

19) $a_2 = 57.5$ and $a_5 = 80$

—, 57.5, —, —, 80, —

$a_n = a_1 + (n-1)d$

$80 = 57.5 + (5-2)d$

$22.5 = 3d$

$d = 7.5$

$80 = a_1 + (5-1)(7.5)$

$80 = a_1 + 30$

$a_1 = 50$

$a_n = 42.5 + 7.5n$

$a_8 = 42.5 + (7.5)(8)$

$a_8 = 42.5 + 60$

$a_8 = 102.5$

20) $a_{10} = 141$ and $a_{13} = 186$

$\_, \_, \_, \_, \_, \_, \_, \_, \_, 141, \_, \_, 186, \_$

$a_n = a_1 + (n-1)(d)$

$186 = 141 + (13-10)(d)$

$45 = 3d$

$d = 15$

$141 = a_1 + (10-1)(15)$

$141 = a_1 + 135$

$a_1 = 6$

$a_n = a_1 + (n-1)d$

$= 6 + (n-1)(15)$

$= 6 + 15n - 15$

$a_n = 15n - 9$

$a_8 = 15(8) - 9$

$= 120 - 9$

$a_8 = 111$

21)

$-23, \_, \_, -89$

$a_n = a_1 + (n-1)d$

$-89 = -23 + (4-1)d$

$-66 = 3d$

$-22 = d$

$-23, -45, -67, -89$

-49 and -67

#22) $31, \_, \_, \_, 79$

$a_n = a_1 + (n-1)(d)$

$79 = 31 + (5-1)d$

$48 = 4d$

$d = 12$

31, 43, 55, 67, 79

43 55 67

#23) $40 + 30 + 20 + 10 + \ldots\ldots\ldots$ S_{10}

$a_n = 50 - 10n$

$a_{10} = 50 - 10(10)$

$a_n = 50 - 100$

$a_n = -50$

$S_{10} = 10\left(\frac{a_1 + a_{10}}{2}\right)$

$S_{10} = 10\left(\frac{40 + (-50)}{2}\right)$

$= 10\left(\frac{-10}{2}\right)$

$S_{10} = -50$

#24) $\sum_{k=5}^{8} 4k$

$k=5$ $k=6$ $k=7$ $k=8$

$20 + 24 + 28 + 32$

$= 104$

25) $\sum_{k=1}^{11} (0.5k + 5.5)$

$6 + 6.5 + 7 + 7.5 + \cdots\cdots + 11$

$S_{11} = 11\left(\frac{a_1 + a_{11}}{2}\right) = 11\left(\frac{6+11}{2}\right) = 93.5$

26) $-6 - 1 + 4 + 9 + \cdots$ $\quad S_{14}$

$a_n = 5n - 11$

$a_{14} = 5(14) - 11$

$a_{14} = 70 - 11$

$a_{14} = 59$

$S_{14} = 14\left(\frac{a_1 + a_{14}}{2}\right)$

$= 14\left(\frac{(-6) + (59)}{2}\right)$

$= 14\left(\frac{53}{2}\right) = 371$

27) $1 + 1.5 + 2 + 2.5 + 3 + 3.5 + \cdots\cdots$

$a_n = 0.5n + 0.5$

$a_{52} = 0.5(52) + 0.5)$

$a_{52} = 26.5$

you deposit \$26.50 on the 52nd week

$S_{52} = 52\left(\frac{a_1 + a_{52}}{2}\right)$

$= 52\left(\frac{1 + 26.5}{2}\right)$

$= 715$

you have deposited a total of \$715 after 52 weeks