8.2

(a) X = the number of students who pass. The probability of passing varies from student to student, so X is not binomially distributed.

(b) X = the number of correct answers in 10. Again, the trials are not independent, since there is intervention and the student learns between responses. The successes are dependent on previous work. X is not binomially distributed.

(c) X = number of times the substance dissolves completely. In general, as temperature increases, so does solubility, so we expect that the probability of dissolving is higher for the later trials. X is not binomially distributed.

8.8

(a) $n = 10$ and $p = 0.25$

If X = the number of employed women in a sample of 10 who have never been married, then

(b) $P(X = 2) = \text{binompdf}(10, 0.25, 2) = 0.2815$

(c) $P(X \leq 2) = \text{binomcdf}(10, 0.25, 2) = 0.52559$

8.20

(a) Since X counts the number of the 200 independent people seeking nutritious food, and since there are two possible responses, either they do or do not seek nutritious food, and since the probability that they do is the same (0.40) for each person, we can say that X is binomially distributed.

(b) $\mu_X = (0.4)(200) = 80$, and $\sigma_X = \sqrt{(200)(0.4)(0.6)} = 6.928$

(c) We observe that $np = 80$, and $nq = 120$, both of which are greater than 10, so we can say

$$P(75 < X < 85) \approx \text{normalcdf}(75,\ 85,\ 80,\ 6.928) = 0.5295$$

8.30

(a) Let X represent the number of truthful people that test "truthful" by the lie detector. Then X~B(12, 0.8).

$$P(X = 12) = \text{binompdf}(12,\ 0.8,\ 12) = 0.069$$

$$\begin{aligned} P(\text{At least one deceptive}) &= 1 - P(\text{All Truthful}) \\ &= 1 - P(X = 12) \\ &= 0.931 \end{aligned}$$

(b) $\mu = 12(0.2) = 2.4$, and $\sigma = \sqrt{12(0.2)(0.8)} = 1.386$

(c) Let Y represent the number of truthful people that test "deceptive" by the lie detector. Then Y~B(12, 0.2).

$$P(Y < 2.4) = 0.558$$

8.34

(a) Since X counts the number of successful detections in 20 independent trials, and since there are only two possible outcomes, detect or not detect, for each trial, and since the probability of detection is the same, 0.99, for each trial, X is binomially distributed with $n = 20$ and $p = 0.99$.

(b)

$$P(X = 20) = \text{binompdf}(20,\ 0.99,\ 20) = 0.818$$

$$\begin{aligned} P(\text{At least one not detected}) &= 1 - P(\text{All Detected}) \\ &= 1 - P(X = 20) \\ &= 0.182 \end{aligned}$$

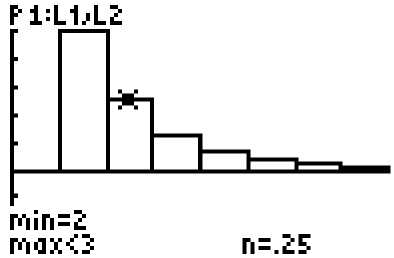
(c) $\mu_X = (0.99)(20) = 19.8$, and $\sigma_X = \sqrt{(20)(0.99)(0.01)} = 0.445$

8.38

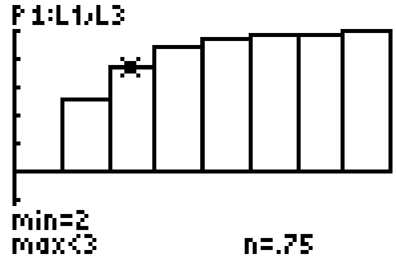
Since X counts the number of independent trials until the first success (prime number) and the probability of success is 0.5 for each trial, X is geometric.

X (L1)	P(X) (L2)	L3 (2)
1	.5	------
2	.25	
3	.125	
4	.0625	
5	.03125	
6	.01563	
7	.00781	

L2(1)=.5



pdf of X



cdf of X

$$P(X \le n) = 1 - q^n = 1 - (0.5)^7 = 0.9922, \text{ or}$$

$$\sum_{i=1}^{\infty} p_i = \lim_{n \to \infty} 1 - q^n = 1$$

8.42

(a) $P(X > 10) = \left(\dfrac{11}{12}\right)^{10} = 0.4189$

(b) $P(X > 10) = 1 - P(X \le 10) = 1 - \text{geometcdf}(1/12, 10) = 0.4189$

8.46

(a) $P(\text{No one will win}) = 2\left(\dfrac{1}{8}\right) = \dfrac{1}{4}$

(b) $P(\text{Success}) = P(\text{Someone wins}) = 1 - P(\text{No one wins}) = \dfrac{3}{4}$

(c) X = number of times the players toss until someone wins. X is geometric since the probability of success is the same for each independent toss of the coins, and we are counting the number of trials until the first success.

(d)

X	pdf(X)	cdf(X) 3
1	.75	.75
2	.1875	.9375
3	.04688	.98438
4	.01172	.99609
5	.00293	.99902
6	7.3E-4	.99976
7	1.8E-4	.99994

(e) $P(X \le 2) = 0.9375$

(f) $P(X > 4) = 1 - 0.99609 = 0.00391$

(g) $E(X) = \mu_X = \dfrac{1}{0.75} = \dfrac{4}{3}$

8.50

(a) Let X represent the number of children a couple has until they have exactly one son. Then X is a geometric random variable with parameter $p = 0.5$. Then $E(X) = 1/0.5 = 2$.

(b) If the expected number of children is 2 and one of them is a boy, then the expected number of girls is 1.

(c) For the question in part (a) Assign the digits from 0 to 1: 0 = girl, and 1 = boy. Generate random integers from 0 to 1 keeping track of the number of trial until the first 1 appears. Average the number of trials for each "run" and that will approximate the expected number of children.

For the question in part (b) follow the same procedure, but record the number of girls ($= n - 1$) for each run. Average this number to approximate the expected number of girls.

8.64

(a) Recall that approximately 68% of all sample means will fall within one standard deviation of the target mean. Then if we let *X* represent the number of sample means out of 5 that fall within one standard deviation, we can see that *X*~B(5, 0.68).
So,
$P(X = 4) = \text{binompdf}(5,\ 0.68,\ 4) = 0.342$

(b) Now let *Y* represent the number of samples taken until the sample mean is greater than 2 standard deviations above the mean. Then the probability of "success" is 0.02275, and *Y* is geometric. Then
$P(Y = 4) = \text{geometpdf}(0.02275,\ 4) = 0.0212$

8.65

If *X*~B(*n, p*)

$$\begin{aligned} P(X \geq 1) &= 1 - P(X = 0) \\ &= 1 - \binom{n}{0} p^0 (1-p)^n \\ &= 1 - (1-p)^n \end{aligned}$$