NOTES 11/21/08

UPPER AND LOWER BOUNDS

A *positive* integer, a, is the <u>*upper bound*</u> of the real zeros of a polynomial function $f(x)$ if $f(x) \div (x-a)$ results in a polynomial function with *nonnegative* coefficients and remainder.

A *negative* integer, b, is the <u>*lower bound*</u> of the real zeros of a polynomial function $f(x)$ if $f(x) \div (x-b)$ results in a polynomial function that has coefficients and remainder with alternating signs.

*****Note: A coefficient of 0 may be positive or negative as needed to fit the pattern for upper or lower bound.***

LOWER BOUND $\leq$ REAL ZEROS $\leq$ UPPER BOUND

END BEHAVIOR OF A FUNCTION

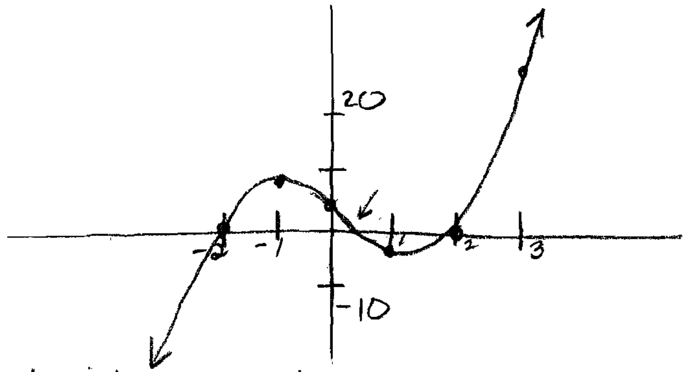
The <u>*end behavior*</u> of a polynomial function is the behavior of the function's graph as x approaches positive infinity ($x \to +\infty$) or negative infinity ($x \to -\infty$).

Examples:

① $f(x) = 2x^3 - x^2 - 8x + 4$

upper bound 3
lower bound -2

	x	2	-1	-8	4
	1	2	1	-7	-3
zero	2	2	3	-2	0
U.B.	3	2	5	7	25
	-1	2	-3	-5	9
L.B. Zero	-2	2	-5	2	0



$-2 \leq$ real zeros ≤ 3

$\frac{p}{q} = \frac{\pm(1,2,4)}{\pm(1,2)}$ possible rational zero: $\frac{1}{2}$

end behavior: evaluate $f(x)$ at -10 = $-$; $f(x)$ at 10 = $+$
as $x \to -\infty$, $f(x) \to -\infty$; as $x \to +\infty$, $f(x) \to +\infty$

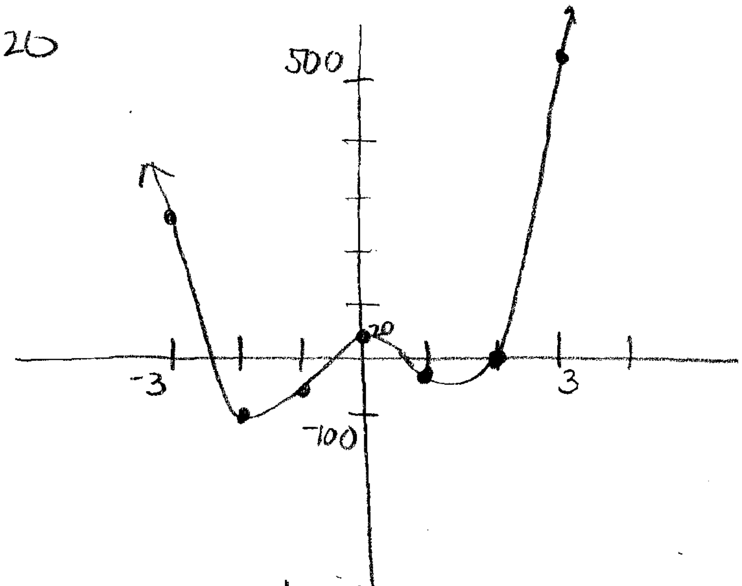
$$\begin{array}{r|rrrr} \frac{1}{2} & 2 & -1 & -8 & 4 \\ & & 1 & 0 & -4 \\ \hline & 2 & 0 & -8 & 0 \end{array}$$

$f(x) = 12x^4 + 4x^3 - 65x^2 + 8x + 20$

	12	4	-65	8	20
1	12	16	-49	-41	-21
zero 2	12	28	-9	-10	0 ☺
UB 3	12	40	55	173	539
-1	12	-8	-57	65	-45
-2	12	-20	-25	58	-96
LB -3	12	-32	31	-85	275

$-3 \le$ real zeros ≤ 3

$\frac{p}{q} = \frac{\pm(1, 2, 4, 5, 10, 20)}{\pm(1, 2, 3, 4, 6, 12)}$



possible rational zeros:

$\pm\left(\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{6}, \frac{1}{12}, \frac{2}{3}, \frac{5}{6}, \frac{5}{12}\right)$

$-5/2$

$-\frac{5}{2}$	12	28	-9	-10
		-30	5	10
	12	-2	-4	0

$6x^2 - x - 2 = 0$

$(3x - 2)(2x + 1)$

$2/3, \; -\frac{1}{2}$