

Mastery Checklist

Quadratic, Exponential & Geometric Models

In order to prove Mastery for this concept you must be able complete the following all by **yourself**. No help from Notes, Partners or Teacher. Use all other problems to practice and test yourself with the following:

- Complete # 4 & 5 on "LT: 3D" worksheet.
- Complete # 2, 3 & 4 on "Geometric Series & Exponential Application Problems" worksheet.
- Create a Mind Map for the Modeling problems.

LT:3D

- 1.) A city council is planning to use 200yd of fencing to enclose a park for physically challenged citizens. The park will be adjacent to the community center and will have two rectangular areas connected by a bridge crossing a creek that is 10yd from the building. The area next to the community center can have a length no greater than the length of the building, which is 75yd, but the area across the creek may have any dimensions. NO fencing will be used next to the creek. What is the total area of the largest park they may enclose? Be sure to use a drawing.
- 2.) The area of the opening of the opening of a rectangular window is to be 143 square feet. If the length is to be 2 feet more than the width, what are the dimensions?
- 3.) A farmer has 2000 yards of fence to enclose rectangular field. What are the dimensions of the rectangle that encloses the most area?
- 4.) A farmer with 4000meters of fencing wants to enclose a rectangular plot that borders on a river. If the farmer does not fence along the river, what is the largest area that can be enclosed?
- 5.) A parabolic arch has a span of 120feet and a maximum height of 25 feet. Choose a suitable rectangular coordinate axes and find the equation of the parabola. Then calculate the height of the arch at points 10 feet and 20 feet from one end.

Answers:

1.) 4800yd^2

2.) dimensions are 11ft by 13ft

3.) 250,000 square yards

4.) $2,000,000\text{m}^2$

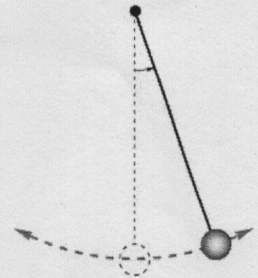
5.) $y = -\frac{5}{720}x^2 + \frac{5}{6}x$ or $y = -\frac{5}{720}x(x - 120)$; when $x=10$ ft, $Y \approx 7.63\text{ft}$; when $x=20$ ft, $y \approx 13.89\text{ft}$

Geometric Series Application Problems

1. Salary Increases: Suppose you have just been hired at an annual salary of \$18,000 and expect to receive annual increases of 5%. What will your salary be when you begin your fifth year?

2. Pendulum Swings Initially, a pendulum swings through an arc of 2 feet. On each successive swing (left or right), the length of the arc is 0.9 of the previous length.

- (a) What is the length of the arc after 10 swings?
- (b) On which swing is the length of the arc less than 1 foot?
- (c) After 15 swings, what total length will the pendulum have swung?



3. The Rapper Ice Cream has signed a 7 – year music contract after his album, “The Fast and the Frozen” made double platinum and his single, “Too hot to handle, too cold to hold” stayed #1 on the Billboard charts for 6 months. His starting salary is \$2,000,000 per year and management gives him the following options with regard to his salary over the 7 years.

- Option A. A bonus of \$100,000 each year
- Option B. An annual increase in his yearly salary of 4.5% per year beginning after 1 year
- Option C. An annual increase in his yearly salary of \$95,000 per year beginning after 1 year

As his financial consultant, “Cause you’re ice cold” (i.e. beast) with the “flow of digits” (i.e. math) which option will give him the most money at the end of 7 years? The least?

Exponential Application Problems

4. Interest Rate: You work at the bank an create new types of accounts that customers can start. The annual interest at the bank is 4%.

(a) Setup the calculations for the following accounts of \$3,000 over 10 years. Assume that no money is taken out.

- Account A: Interest is compounded yearly (i.e. Interest is calculated a the end of each year)
- Account B: Interest is compounded quarterly (i.e. Interest is calculated every quarter of a year.)
- Account C: Interest is compounded monthly
- Account D: Interest is compounded continuously

(b) If a customer had Account D and wanted to have at least \$10,000 in the account after 18 years, how much would they have to put in initially?

5. The value of a car depreciates continuously with a annual rate of 18.6%. The car was initially worth \$18,500:

- (a) Create a function called $v(t)$ = that illustrates this relationship
- (b) Using the formula from (a) what will the car be worth in 18 months?
- (c) Estimate with a calculator the amount of years it will take for the car to depreciate to half of its original value.

Answers

1. \$21,879.11

2. (a) 0.77 ft. remember units!
(b) 8th
(c) 15.88 ft.

3. Option B results in the most overall gain over the 7 years of \$16,038,304; Option A results in the least \$14,700,000

4. (a) Account A $CurrentMoney = 3000(1 + .04)^{10}$

$$Account\ B\ CurrentMoney = 3000\left(1 + \frac{.04}{4}\right)^{10 \cdot 4}$$

$$Account\ C\ CurrentMoney = 3000\left(1 + \frac{.04}{12}\right)^{10 \cdot 12}$$

$$Account\ D\ CurrentMoney = 3000e^{.04 \cdot 10}$$

(b) $10,000 = Pe^{(.04)(18)}$ Solving for P gives approx. 4867.52

5. (a) $v(t) = 18500e^{(-0.186)t}$

(b) 18 months is 1.5 years (remember the year and the rate have to be the same time units)

$$v(t) = 18500e^{(-0.186)(1.5)} = \$13,995.99$$

(c) $9,250 = 18,500e^{(-0.186)t}$ $t = 3.72\text{years}$

LT:5C Application Problems

61. Find x so that x , $x + 2$, and $x + 3$ are consecutive terms of a geometric sequence.

62. Find x so that $x - 1$, x , and $x + 2$ are consecutive terms of a geometric sequence.

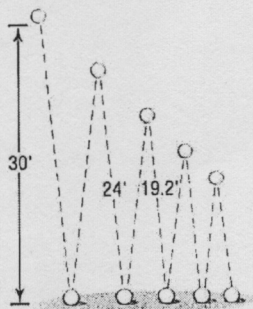
63. **Salary Increases** Suppose that you have just been hired at an annual salary of \$18,000 and expect to receive annual increases of 5%. What will your salary be when you begin your fifth year?

64. **Equipment Depreciation** A new piece of equipment cost a company \$15,000. Each year, for tax purposes, the company depreciates the value by 15%. What value should the company give the equipment after 5 years?

65. **Pendulum Swings** Initially, a pendulum swings through an arc of 2 feet. On each successive swing, the length of the arc is 0.9 of the previous length.

- What is the length of the arc after 10 swings?
- On which swing is the length of the arc first less than 1 foot?
- After 15 swings, what total length will the pendulum have swung?
- When it stops, what total length will the pendulum have swung?

66. **Bouncing Balls** A ball is dropped from a height of 30 feet. Each time it strikes the ground, it bounces up to 0.8 of the previous height.



- What height will the ball bounce up to after it strikes the ground for the third time?
- How high will it bounce after it strikes the ground for the n th time?
- How many times does the ball need to strike the ground before its bounce is less than 6 inches?
- What total distance does the ball travel before it stops bouncing?

Critical Thinking You are interviewing for a job and receive two offers:

- \$20,000 to start, with guaranteed annual increases of 6% for the first 5 years
- \$22,000 to start, with guaranteed annual increases of 3% for the first 5 years

Which offer is best if your goal is to be making as much as possible after 5 years? Which is best if your goal is to make as much money as possible over the contract (5 years)?

68. **Critical Thinking** Which of the following choices, A or B, results in more money?

- To receive \$1000 on day 1, \$999 on day 2, \$998 on day 3, with the process to end after 1000 days
- To receive \$1 on day 1, \$2 on day 2, \$4 on day 3, for 19 days

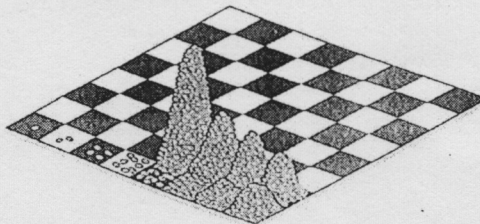
69. **Critical Thinking** You have just signed a 7-year professional football league contract with a beginning salary of \$2,000,000 per year. Management gives you the following options with regard to your salary over the 7 years.

- A bonus of \$100,000 each year
- An annual increase of 4.5% per year beginning after 1 year
- An annual increase of \$95,000 per year beginning after 1 year

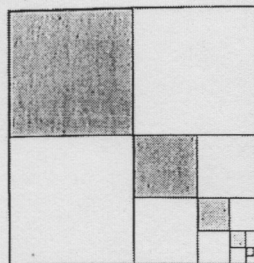
Which option provides the most money over the 7-year period? Which the least? Which would you choose? Why?

70. **A Rich Man's Promise** A rich man promises to give you \$1000 on September 1, 2001. Each day thereafter he will give you $\frac{9}{10}$ of what he gave you the previous day. What is the first date on which the amount you receive is less than 1¢? How much have you received when this happens?

71. **Grains of Wheat on a Chess Board** In an old fable, a commoner who had just saved the king's life was told he could ask the king for any just reward. Being a shrewd man, the commoner said, "A simple wish, sire. Place one grain of wheat on the first square of a chessboard, two grains on the second square, four grains on the third square, continuing until you have filled the board. This is all I seek." Compute the total number of grains needed to do this to see why the request, seemingly simple, could not be granted. (A chessboard consists of $8 \times 8 = 64$ squares.)



72. Look at the figure below. What fraction of the square is eventually shaded if the indicated shading process continues indefinitely?



9.
$$\frac{-1 + \sum_{n=0}^{14} 2^n}{0, 14, 1} = -32767$$

50.
$$\frac{\sum_{n=0}^{15} (2 \cdot (3/5)^n)}{n, n, 0, 15, 1} = 4.998569445$$

51. $\frac{3}{2}$ 52. 6 53. 16 54. 9 55. $\frac{8}{5}$ 56. $\frac{4}{7}$ 57. $\frac{20}{3}$ 58. 12

59. $\frac{18}{5}$ 60. $\frac{8}{3}$ 61. -4 62. 2 63. \$21,879.11 64. \$6655.58

65. (a) 0.77 ft (b) 8th (c) 15.88 ft (d) 20 ft

66. (a) 15.36 ft (b) $30(0.8)^n$ (c) After the 19th strike (d) 270 ft

77. A: \$25,250 per year in 5th year, \$112,742 total; B: \$24,761 per year in 5th year, \$116,801 total 68. Option B results in more money
 A: \$500,500 versus B: \$524,287). 69. Option 2 results in the most: \$16,038,304; Option 1 results in the least: \$14,700,000.

70. December 20, 2001 (111 days); \$9999.92 71. 1.845×10^{19} 72. $\frac{1}{3}$ 73. 10 74. 20 75. \$72.67 per share 76. \$39.64

77. Yes. A constant sequence is both arithmetic and geometric. For example, 3, 3, 3, ... is an arithmetic sequence with $a_1 = 3$ and $d = 0$
 and is a geometric sequence with $a_1 = 3$ and $r = 1$.

*** Problem (Blue Problem):

Use the big dry-erase board to complete the problem below. Make sure show all your work and your thoughts through the process. Do not erase unless Ms. Mirzaian checks it, remember to write your name on the board.

To create a college fund, a parent makes a sequence of 18 yearly deposits of \$1000 each in a savings account on which interest is compounded annually at 3.2%. Find the amount in the bank account after the 5 years.

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BLUE Problem

Approach

$$\text{year 1 } (1,000) \times 0.032$$

$$\text{year 2 } (1,000 + 1,000 \cdot 0.032) \cdot 0.032$$
$$1000(1 + 0.032) \cdot 0.032$$

\therefore since it is deposited every year

$$\sum_{t=1}^5 1000 (1 + 0.032)^{t-1}$$

\uparrow a_1 \uparrow r

In order to find the sum of a series

$$S_5 = \frac{1000 (1 - (1 + 0.032)^5)}{1 - (1 + 0.032)}$$

$$S_5 = \frac{1000 (1 - (1.032)^5)}{-0.032}$$

$$S_5 = \$5330$$

*** Problem (Green Problem):

Use the big dry-erase board to complete the problem below. Make sure show all your work and your thoughts through the process. Do not erase unless Ms. Mirzaian checks it, remember to write your name on the board.

A bungee jumper always rebounds 60% of the distance fallen. A bungee jump is made using a cord that stretches to 200ft. After jumping and then rebounding 9 times, how far has a bungee jumper traveled upward?

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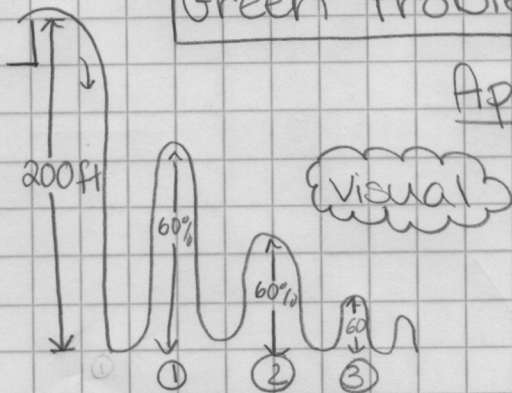
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Green Problem

Approach:



$$a_1 = 200 \text{ ft}$$

$$r = 60\% \rightarrow 0.6$$

Plan:

* Make a general term for the n^{th} rebounding time

$$a_n = a_1 r^{n-1}$$

1st term

common ratio

← but n^{th} rebound so I can't count the first one.

Execute

$$a_n = 200 (0.6)^9$$

$a_n = 2.016 \text{ ft.}$ is the height of the rebound.

*** Problem(Pink Problem)

Use the big dry-erase board to complete the problem below. Make sure show all your work and your thoughts through the process. Do not erase unless Ms. Mirzaian checks it.

A Ping-pong ball is dropped from a height of 16ft and always rebounds $\frac{1}{4}$ of the distance fallen.

- a.) How high does it rebound the 6th time?
- b.) Find the total sum of the rebound heights of the ball.

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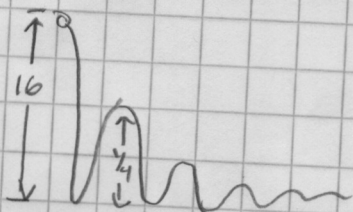
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- a.) How high does it rebound the 6th time?
- b.) Find the total sum of the rebound heights of the ball.

Pink Problem

Approach



a) How high does it rebound the 6th term

$$a_6 = 16(0.25)^{5 \leftarrow n-1}$$

\uparrow \uparrow
 a_1 r

$$a_6 = 0.015625 \text{ ft}$$

b.)

$$\sum_{n=1}^6 16(0.25)^{n-1}$$

\uparrow \uparrow
 a_1 r

$$S_6 = \frac{16(1-0.25^6)}{1-0.25}$$

$$S_6 = \frac{16(15.99)}{0.75} = 21.328$$

$$S_6 \approx 21.3 \text{ ft.}$$



Find a pair of numbers with a sum of 26
and a product that is a maximum. Find
the maximum product.

Approach

One number = x
second number = y

$x + y = 26$
sum

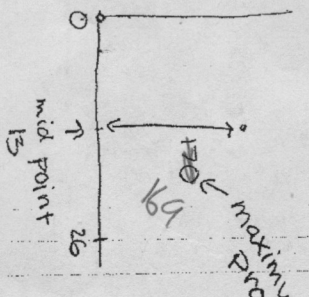
- * need to put in terms of one variable
- * use the two terms and multiply them to find product.
- * Graph to find max amount.

Plan

$x + y = 26$
 $y = 26 - x$ ← and #
 x 1st #

The product.

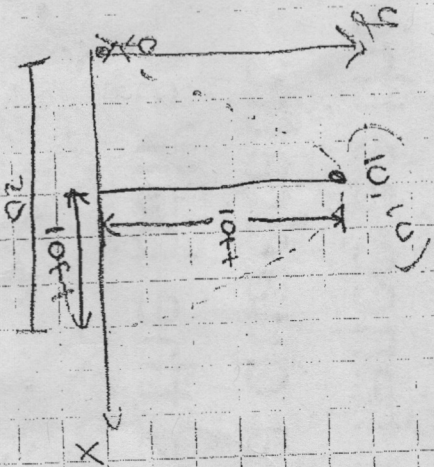
$f(x) = x(26 - x) = 26x - x^2 = -x^2 + 26x = -x(x - 26)$



$f(13) = -(13)^2 + 26(13)$
 $f(13) = -169 + 338$
 $f(13) = 169$

*

Suppose you throw a ball over a 10-ft fence. Barely clearing the fence the ball reaches it's highest point directly above the fence and lands 10ft from the fence. Using the fence as the axis of symmetry, write a quadratic function that models the ball's height.



$$f(x) = -a(x-0)(x-20)$$

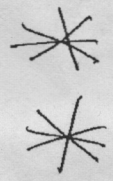
$$10 = -a(10-0)(10-20)$$

$$10 = -a(10)(-10)$$

$$\frac{10}{100} = \frac{a(100)}{100}$$

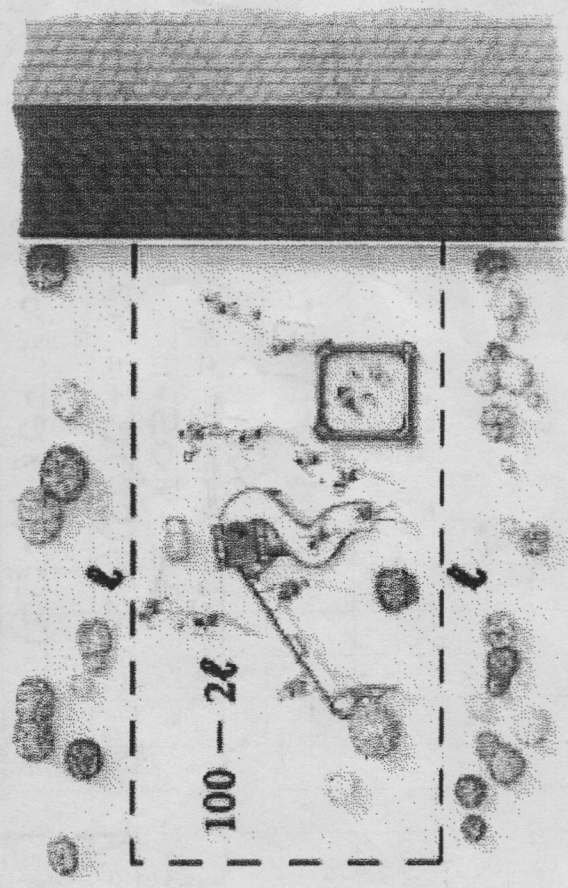
$$\frac{1}{10} = a$$

$$f(x) = -\frac{1}{10}x(x-20)$$



Multiple Choice. A town is planning a playground. It wants to fence in a rectangular space using an existing wall. What is the greatest area it can fence in using 100 ft of donated fencing?

- (A) 1000 ft²
- (B) 1250 ft²
- (C) 2500 ft²
- (D) 10,000 ft²



$$\text{Area} = 2(100 - 2l)$$

$$\text{Area} = -2l^2 + 100l \quad \text{Area}$$

$$\text{Area} = -2l(l - 50)$$

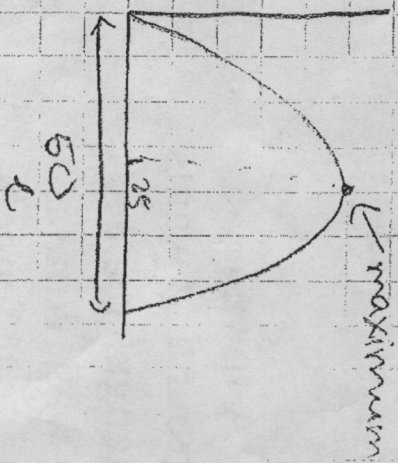
$$\text{Area} = -2(25)(25 - 50)$$

$$= (-50)(-25)$$

$$\text{Area} = \cancel{2500} (25)$$

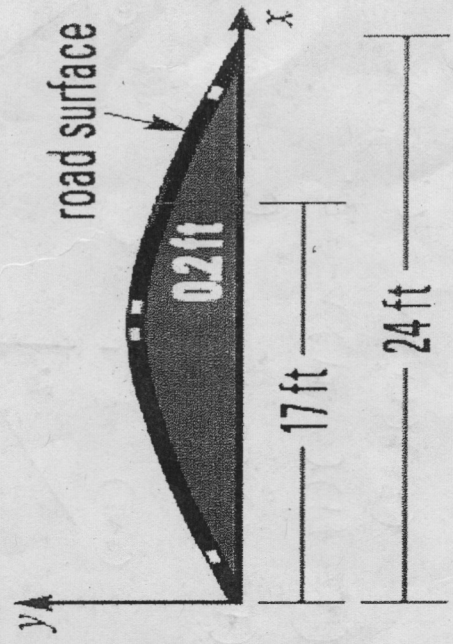
$$1250$$

B





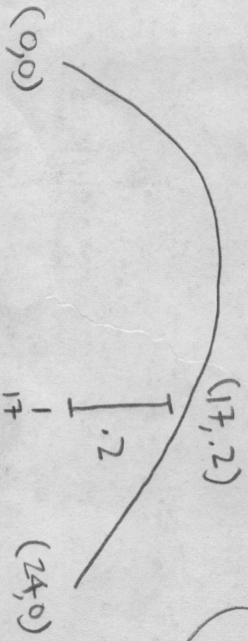
TRANSPORTATION The surfaces of some roads are shaped like parabolas to allow rain to run off to either side. (This is also true of football fields; see Exercise 52 on page 254.) Write a quadratic model for the surface of the road shown.



Not drawn to scale

► Source: Massachusetts Highway Department

a is negative because
 $-x^2$ upsidedown
 parabola



$$y = -a(x-0)(x-24)$$

$$y = -a(17-0)(17-24)$$

$$.2 = -a(17)(-7)$$

$$.2 = (-17a)(-7)$$

$$.2 = \frac{119a}{119}$$

$$\frac{.2}{119} = a$$

a is negative

$$y = -\frac{.2}{119}(x-0)(x-24)$$

$$y = -\frac{.2}{119}(17-0)(17-24)$$

$$y = \frac{.2}{119}(17)(-7)$$

$$y = -\frac{.2}{119}(17)(-7)$$

← To Check

**

Find two numbers with a difference of 10 and a product that is a minimum. Find the minimum product.

Approach

* put numbers in terms of one variable. number 1 = x
number 2 = y

* product means multiplication

* minimum means lowest point. (when graphed)

Plan

number 1
 $x - y = 10$

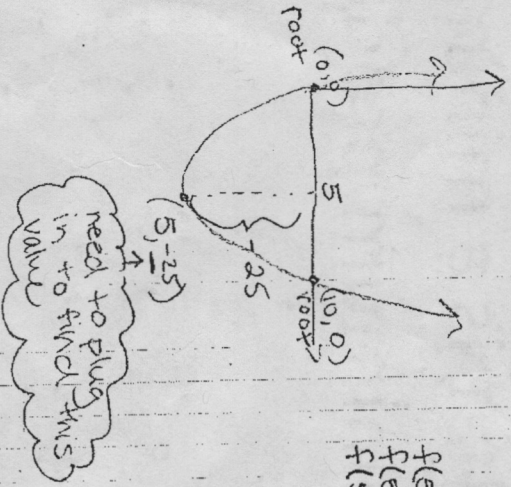
number 2
 $-y = 10 - x$

$y = -10 + x$ or $x - 10$

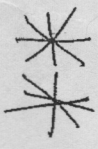
Product

$$f(x) = \text{number 1} \cdot \text{number 2} = x(x - 10)$$

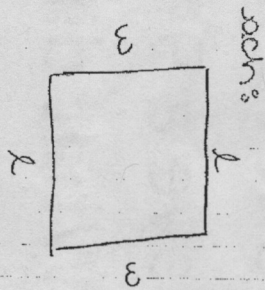
$$\begin{aligned} f(5) &= 5(5 - 10) \\ f(5) &= 5(-5) \\ f(5) &= -25 \end{aligned}$$



∴ lowest product is -25.



Suppose you want to frame a collage of pictures. You have a 9-ft strip of wood for the frame. What dimensions of the frame give you the maximum area for the collage? What is the maximum area?



- * perimeter = $2l + 2w$
- * 9 ft of wood
- * need area = lw
- * need to put dimensions in one term

* since we have 9 ft and is the perimeter

$$9 = 2l + 2w$$

$$\frac{9 - 2w}{2} = \frac{9l}{2}$$

$$\frac{9 - 2w}{2} = l$$

Plan

$$\text{Area} = l \times w$$

$$\text{Area} = \left(\frac{9-2w}{2}\right)(w)$$

$$\text{Area} = (4.5 - w)(w)$$

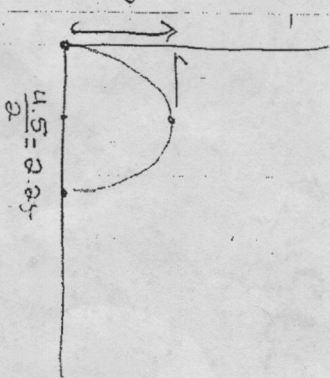
$$\text{Area} = 4.5w - w^2$$

$$\text{Area} = -w^2 + 4.5w$$

$$= -(w)(w - 4.5)$$

$$\text{Max Area} = -(0.25)(0.25 - 4.5)$$

$$\text{Max Area} = 5.0625$$



Mind Map

Quadratic/Exponential and Geometric Models

Task 1:

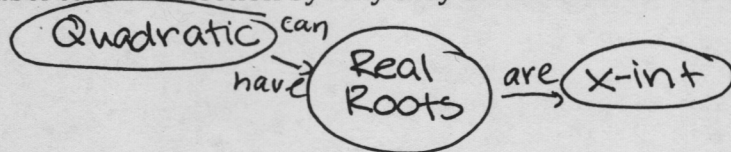
Review your notes/Quick Check/Unit 3,4, 5 Test and write down all key ideas/concepts/vocabulary/examples for LT: 3D, 4D & 5C

Task 2:

Group and Connect all your concepts together

Task 3:

Label each Connection by why they are connected. For example:



Task 4:

Add in examples and visuals where they fit.

Task 5:

Review Checklist below and make sure that your Mind Map contains everything you need.

- Contains all of these Concepts/Vocabulary
 - Exponential Models
 - Simple Interest
 - Area Problem
 - Quadratic Standard form
 - Compound Interest
 - Box Problems
 - Geometric Factored form
 - Repeated Growth/Decay
 - Bridge Problems
- All Connections are labeled
- Contains Example Problems
- Contains Visuals
- Review Mind Map on the Backside and see if you need to adjust or add anything.

