

**50. Sawtooth Function** Draw the graph of  $f(x) = x - [x]$ . At which points is  $f$  discontinuous? Is it left- or right-continuous at those points?

In Exercises 51–54, sketch the graph of  $f(x)$ . At each point of discontinuity, state whether  $f$  is left- or right-continuous.

$$51. f(x) = \begin{cases} x^2 & \text{for } x \leq 1 \\ 2 - x & \text{for } x > 1 \end{cases}$$

$$52. f(x) = \begin{cases} x + 1 & \text{for } x < 1 \\ \frac{1}{x} & \text{for } x \geq 1 \end{cases}$$

$$53. f(x) = \begin{cases} \frac{x^2 - 3x + 2}{|x - 2|} & x \neq 2 \\ 0 & x = 2 \end{cases}$$

$$54. f(x) = \begin{cases} x^3 + 1 & \text{for } -\infty < x \leq 0 \\ -x + 1 & \text{for } 0 < x < 2 \\ -x^2 + 10x - 15 & \text{for } x \geq 2 \end{cases}$$

55. Show that the function

$$f(x) = \begin{cases} \frac{x^2 - 16}{x - 4} & x \neq 4 \\ 10 & x = 4 \end{cases}$$

has a removable discontinuity at  $x = 4$ .

**56. GU** Define  $f(x) = x \sin \frac{1}{x} + 2$  for  $x \neq 0$ . Plot  $f(x)$ . How should  $f(0)$  be defined so that  $f$  is continuous at  $x = 0$ ?

In Exercises 57–59, find the value of the constant ( $a$ ,  $b$ , or  $c$ ) that makes the function continuous.

$$57. f(x) = \begin{cases} x^2 - c & \text{for } x < 5 \\ 4x + 2c & \text{for } x \geq 5 \end{cases}$$

$$58. f(x) = \begin{cases} 2x + 9x^{-1} & \text{for } x \leq 3 \\ -4x + c & \text{for } x > 3 \end{cases}$$

$$59. f(x) = \begin{cases} x^{-1} & \text{for } x < -1 \\ ax + b & \text{for } -1 \leq x \leq \frac{1}{2} \\ x^{-1} & \text{for } x > \frac{1}{2} \end{cases}$$

60. Define

$$g(x) = \begin{cases} x + 3 & \text{for } x < -1 \\ cx & \text{for } -1 \leq x \leq 2 \\ x + 2 & \text{for } x > 2 \end{cases}$$

Find a value of  $c$  such that  $g(x)$  is

(a) left-continuous

(b) right-continuous

In each case, sketch the graph of  $g(x)$ .

61. Define  $g(t) = \tan^{-1} \left( \frac{1}{t-1} \right)$  for  $t \neq 1$ . Answer the following questions, using a plot if necessary.

(a) Can  $g(1)$  be defined so that  $g(t)$  is continuous at  $t = 1$ ?

(b) How should  $g(1)$  be defined so that  $g(t)$  is left-continuous at  $t = 1$ ?

62. Each of the following statements is *false*. For each statement, sketch the graph of a function that provides a counterexample.

(a) If  $\lim_{x \rightarrow a} f(x)$  exists, then  $f(x)$  is continuous at  $x = a$ .

(b) If  $f(x)$  has a jump discontinuity at  $x = a$ , then  $f(a)$  is equal to either  $\lim_{x \rightarrow a^-} f(x)$  or  $\lim_{x \rightarrow a^+} f(x)$ .

In Exercises 63–66, draw the graph of a function on  $[0, 5]$  with the given properties.

63.  $f(x)$  is not continuous at  $x = 1$ , but  $\lim_{x \rightarrow 1^+} f(x)$  and  $\lim_{x \rightarrow 1^-} f(x)$  exist and are equal.

64.  $f(x)$  is left-continuous but not continuous at  $x = 2$  and right-continuous but not continuous at  $x = 3$ .

65.  $f(x)$  has a removable discontinuity at  $x = 1$ , a jump discontinuity at  $x = 2$ , and

$$\lim_{x \rightarrow 3^-} f(x) = -\infty, \quad \lim_{x \rightarrow 3^+} f(x) = 2$$

66.  $f(x)$  is right- but not left-continuous at  $x = 1$ , left- but not right-continuous at  $x = 2$ , and neither left- nor right-continuous at  $x = 3$ .

In Exercises 67–80, evaluate using substitution.

$$67. \lim_{x \rightarrow -1} (2x^3 - 4)$$

$$68. \lim_{x \rightarrow 2} (5x - 12x^{-2})$$

$$69. \lim_{x \rightarrow 3} \frac{x + 2}{x^2 + 2x}$$

$$70. \lim_{x \rightarrow \pi} \sin \left( \frac{x}{2} - \pi \right)$$

$$71. \lim_{x \rightarrow \frac{\pi}{4}} \tan(3x)$$

$$72. \lim_{x \rightarrow \pi} \frac{1}{\cos x}$$

$$73. \lim_{x \rightarrow 4} x^{-5/2}$$

$$74. \lim_{x \rightarrow 2} \sqrt{x^3 + 4x}$$

$$75. \lim_{x \rightarrow -1} (1 - 8x^3)^{3/2}$$

$$76. \lim_{x \rightarrow 2} \left( \frac{7x + 2}{4 - x} \right)^{2/3}$$

$$77. \lim_{x \rightarrow 3} 10^{x^2 - 2x}$$

$$78. \lim_{x \rightarrow -\frac{\pi}{2}} 3^{\sin x}$$

$$79. \lim_{x \rightarrow 4} \sin^{-1} \left( \frac{x}{4} \right)$$

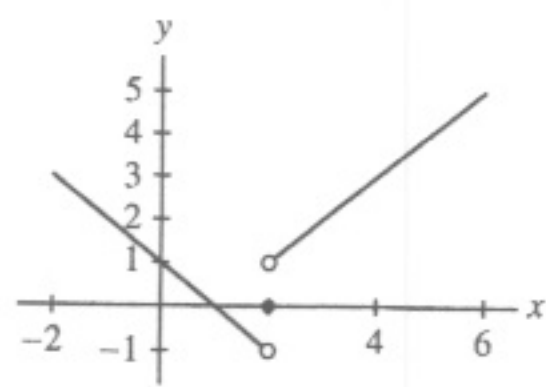
$$80. \lim_{x \rightarrow 0} \tan^{-1}(e^x)$$

81. Suppose that  $f(x)$  and  $g(x)$  are discontinuous at  $x = c$ . Does it follow that  $f(x) + g(x)$  is discontinuous at  $x = c$ ? If not, give a counterexample. Does this contradict Theorem 1 (i)?

82. Prove that  $f(x) = |x|$  is continuous for all  $x$ . *Hint:* To prove continuity at  $x = 0$ , consider the one-sided limits.

83. Use the result of Exercise 82 to prove that if  $g(x)$  is continuous, then  $f(x) = |g(x)|$  is also continuous.

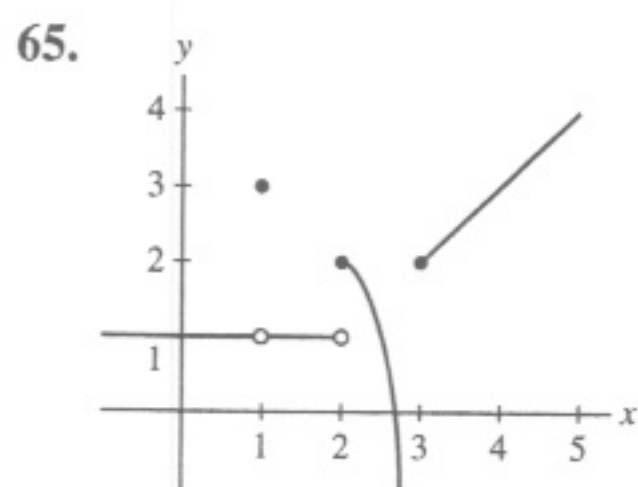
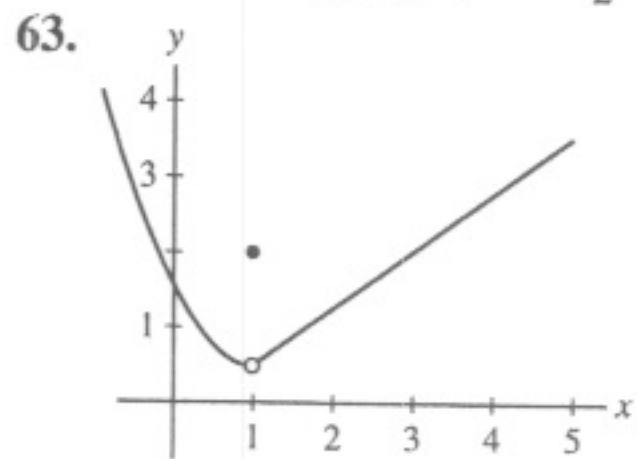
53. The function  $f$  is neither left- nor right-continuous at  $x = 2$ .



55.  $\lim_{x \rightarrow 4} \frac{x^2-16}{x-4} = \lim_{x \rightarrow 4} (x+4) = 8 \neq 10 = f(4)$

57.  $c = \frac{5}{3}$  59.  $a = 2$  and  $b = 1$

61. (a) No (b)  $g(1) = -\frac{\pi}{2}$

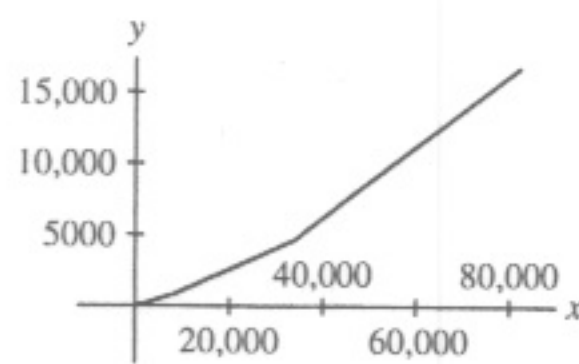


67. -6 69.  $\frac{1}{3}$  71. -1 73.  $\frac{1}{32}$  75. 27 77. 1000 79.  $\frac{\pi}{2}$

81. No. Take  $f(x) = -x^{-1}$  and  $g(x) = x^{-1}$

83.  $f(x) = |g(x)|$  is a composition of the continuous functions  $g(x)$  and  $|x|$

85. No.



87.  $f(x) = 3$  and  $g(x) = [x]$

### Section 2.5 Preliminary Questions

1.  $\frac{x^2-1}{\sqrt{x+3}-2}$

2. (a)  $f(x) = \frac{x^2-1}{x-1}$  (b)  $f(x) = \frac{x^2-1}{x-1}$  (c)  $f(x) = \frac{1}{x}$

3. The "simplify and plug-in" strategy is based on simplifying a function which is indeterminate to a continuous function. Once the simplification has been made, the limit of the remaining continuous function is obtained by evaluation.

### Section 2.5 Exercises

1.  $\lim_{x \rightarrow 6} \frac{x^2-36}{x-6} = \lim_{x \rightarrow 6} \frac{(x-6)(x+6)}{x-6} = \lim_{x \rightarrow 6} (x+6) = 12$

3. 0 5.  $\frac{1}{14}$  7. -1 9.  $\frac{11}{10}$  11. 2 13. 1 15. 2 17.  $\frac{1}{8}$

19.  $\frac{7}{17}$

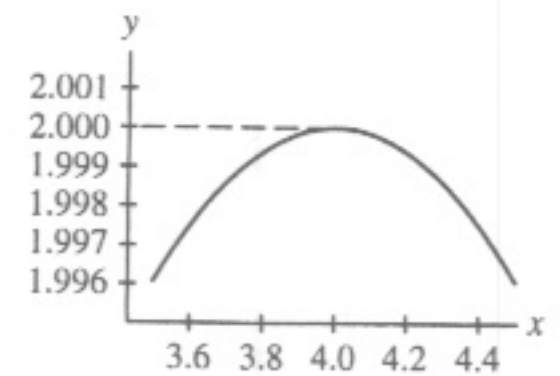
21. Limit does not exist.

• As  $h \rightarrow 0+$ ,  $\frac{\sqrt{h+2}-2}{h} \rightarrow -\infty$ .

• As  $h \rightarrow 0-$ ,  $\frac{\sqrt{h+2}-2}{h} \rightarrow \infty$ .

23. 2 25.  $\frac{1}{4}$  27. 1 29. 9 31.  $\frac{\sqrt{2}}{2}$  33.  $\frac{1}{2}$

35.  $\lim_{x \rightarrow 4} f(x) \approx 2.00$ ; to two decimal places, this matches the value of 2 obtained in Exercise 23.



37. 12 39. -1 41.  $\frac{4}{3}$  43.  $\frac{1}{4}$  45.  $2a$  47.  $-4 + 5a$  49.

51.  $\frac{1}{2\sqrt{a}}$  53.  $3a^2$  55.  $c = -1$  and  $c = 6$  57.  $c = 3$  59. +

### Section 2.6 Preliminary Questions

1.  $\lim_{x \rightarrow 0} f(x) = 0$ ; No

2. Assume that for  $x \neq c$  (in some open interval containing  $c$ ),

$$l(x) \leq f(x) \leq u(x)$$

and that  $\lim_{x \rightarrow c} l(x) = \lim_{x \rightarrow c} u(x) = L$ . Then  $\lim_{x \rightarrow c} f(x)$  exists and

$$\lim_{x \rightarrow c} f(x) = L.$$

3. (a)

### Section 2.6 Exercises

1. For all  $x \neq 1$  on the open interval  $(0, 2)$  containing  $x = 1$ ,  $l(x) \leq f(x) \leq u(x)$ . Moreover,

$$\lim_{x \rightarrow 1} l(x) = \lim_{x \rightarrow 1} u(x) = 2.$$

Therefore, by the Squeeze Theorem,

$$\lim_{x \rightarrow 1} f(x) = 2.$$

3.  $\lim_{x \rightarrow 7} f(x) = 6$

5. (a) not sufficient information (b)  $\lim_{x \rightarrow 1} f(x) = 1$

(c)  $\lim_{x \rightarrow 1} f(x) = 3$

7.  $\lim_{x \rightarrow 0} x^2 \cos \frac{1}{x} = 0$  9.  $\lim_{x \rightarrow 1} (x-1) \sin \frac{\pi}{x-1} = 0$

11.  $\lim_{t \rightarrow 0} (2^t - 1) \cos \frac{1}{t} = 0$

13.  $\lim_{t \rightarrow 2} (t^2 - 4) \cos \frac{1}{t-2} = 0$

15.  $\lim_{\theta \rightarrow \frac{\pi}{2}} \cos \theta \cos(\tan \theta) = 0$

17. 1 19. 3 21. 1 23. 0 25.  $\frac{2\sqrt{2}}{\pi}$  27. (b)  $L = 14$  29.

31.  $\frac{1}{5}$  33.  $\frac{7}{3}$  35.  $\frac{1}{25}$  37. 6 39.  $-\frac{3}{4}$  41.  $\frac{1}{2}$  43.  $\frac{6}{5}$  45.

47. 0 49. -1 53.  $-\frac{9}{2}$

55.  $\lim_{t \rightarrow 0+} \frac{\sqrt{1-\cos t}}{t} = \frac{\sqrt{2}}{2}$ ;  $\lim_{t \rightarrow 0-} \frac{\sqrt{1-\cos t}}{t} = -\frac{\sqrt{2}}{2}$

59. (a)

$x$	$c - .01$	$c - .001$	$c + .001$	$c + .01$
$\frac{\sin x - \sin c}{x - c}$	.999983	.99999983	.99999983	.999983

Here  $c = 0$  and  $\cos c = 1$ .