CHAPTER 8 Probability and Statistics
Solutions Key

ARE YOU READY?

1. Possible answer: 12 units^2
2. If there are 12 square units and each square is 50 square feet, then the triangle represents about $50 \times 12 = 600$ square feet. If Jo paints 400 square feet per hour, it will take him about $\frac{600}{400} = 1.5$ hours to paint the area.
3. The area of a triangle is $A = \frac{1}{2}bh$. Using the graph, the base is 5 units and the height is 5 units. So $A = \frac{1}{2}(5)(5) = 12.5$ units^2.
4. $4a^3 \cdot 28b^2 = 4(28)a^3b^2 = 112a^3b^2$
5. $3x^5y \cdot 21xy^2 = 3(21)(x^{5+1})(y^{1+2}) = 63x^6y^3$
6. $-m^3n^4 \cdot 10m^2n^5 = -10(m^{3+2})(n^{4+5}) = -10m^5n^9$
7. $(0.1)(0.9)^4 = (0.1)(0.6561) \approx 0.07$
8. $(0.6)^{4-4} = (0.6)^0 = 1$
9. $(0.25)^1(0.75)^3 = (0.25)(0.421875) \approx 0.11$
10. $(0.5)^{5-3}(0.5) = (0.5)^2(0.5) = (0.5)^3 \approx 0.13$

11.
$$\begin{array}{r} 0.343 \\ +\underline{0.601} \\ 0.944 \\ 0.403 \end{array}$$

12.
$$\begin{array}{r} +\underline{0.084} \\ 0.487 \\ 0.555 \end{array}$$

13.
$$\begin{array}{r} +\underline{0.191} \\ 0.746 \\ 0.756 \end{array}$$

14.
$$\begin{array}{r} +\underline{0.129} \\ 0.885 \end{array}$$

15. 2, 4, 4, 6, 9
mean: 5
median: 4
mode: 4

16. 1, 1, 1, 2, 2, 2
mean: 1.5
median: 1.5
mode: 1, 2

17. 1, 2, 3, 4, 5, 6
mean: 3.5
median: 3.5
mode: none

18. 3, 14, 14, 18, 18, 18, 20
mean: 15
median: 18
mode: 18

8-1 MEASURES OF CENTRAL TENDENCY AND VARIATION

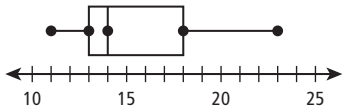
CHECK IT OUT!

1a. mean: $\frac{26}{4} = 6.5$
median: $\frac{6+8}{2} = 7$
mode: no mode

b. mean: $\frac{21}{5} = 4.2$
median: 5
mode: 2 and 6

2. expected value
$= 0(0.75) + 1(0.15) + 2(0.08) + 3(0.02)$
$= 0.37$
The expected number of accidents in one week is 0.37.

3. 11, 12, 12, 13, 13, 13, 14, 14, 14, 15, 17, 18, 18, 19, 22, 23
minimum: 11
maximum: 23
median: 14
first quartile: 13
third quartile: 18
IQR: 5



4. $\bar{x} = \frac{14}{10} = 1.4$

	0	**3**	**1**	**1**	**0**
$x - \bar{x}$	−1.4	1.6	−0.4	−0.4	−1.4
$(x - \bar{x})^2$	1.96	2.56	0.16	0.16	1.96
	5	**1**	**0**	**3**	**0**
$x - \bar{x}$	3.6	−0.4	−1.4	1.6	−1.4
$(x - \bar{x})^2$	12.96	0.16	1.96	2.56	1.96

$\sigma^2 = \frac{26.4}{10} = 2.64$; $\sigma = \sqrt{2.64} \approx 1.6$

5. The mean is about 5.4 and the standard deviation is about 4.5.
Two standard deviations is about $2(4.5) = 9$.
Values 9 units below the mean are negative and would not make sense in this problem. Values greater than 9 units away from the mean are outliers. So, 19, the number of scored runs, is an outlier.
The mean increases from ≈ 4.3 to ≈ 5.4, and the standard deviation increases from ≈ 2.3 to ≈ 4.5.

THINK AND DISCUSS

1. Possible answer: The mean increases by the constant.
2. Possible answer: The standard deviation is unchanged.
3. Possible answer: The standard deviation is multiplied by $\sqrt{2}$.
4. **Measures of Variability**
   - **Range**: the difference between the maximum and minimum value. For {1, 2, 3, 4, 5}, the range is 5 − 1, or 4.
   - **Variance**: the average of the squared differences from the mean. For {1, 2, 3, 4, 5}, the variance is $\frac{2^2 + 1^2 + 0^2 + 1^2 + 2^2}{5} = 2$
   - **Interquartile Range**: the difference between the third and first quartiles. For {1, 2, 3, 4, 5}, the interquartile range is 4.5 − 1.5, or 3.
   - **Standard Deviation**: the square root of the variance. For {1, 2, 3, 4, 5}, the variance is $\sqrt{2} \approx 1.4$

EXERCISES

GUIDED PRACTICE

1. variance

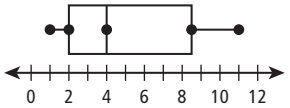
2. mean: $\frac{36}{6} = 6$
median: $\frac{6+7}{2} = 6.5$
mode: 7

3. mean: $\frac{43}{8} = 5.375$
median: $\frac{6+6}{2} = 6$
mode: 6

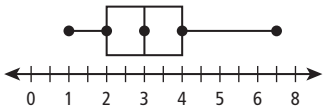
4. mean: $\frac{90}{5} = 18$
median: 18
mode: no mode

5. expected value
$= 0(0.9359) + 1(0.05) + 5(0.01) + 20(0.003) + 100(0.001) + 1000(0.0001)$
$= 0.36$
The expected value of the prize is \$0.36.

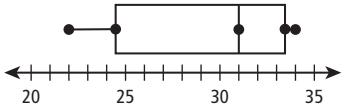
6. 1, 2, 2, 3, 5, 8, 9, 11
minimum: 1
maximum: 11
median: 4
first quartile: 2
third quartile: 8.5
IQR: 6.5



7. 1, 2, 2, 2, 4, 4, 4, 7
minimum: 1
maximum: 7
median: 3
first quartile: 2
third quartile: 4
IQR: 2



8. 22, 27, 31, 33, 34
minimum: 22
maximum: 34
median: 31
first quartile: 24.5
third quartile: 33.5
IQR: 9



9. $\bar{x} = \frac{20}{5} = 4$

	3	3	4	5	5
$x - \bar{x}$	−1	−1	0	1	1
$(x - \bar{x})^2$	1	1	0	1	1

$\sigma^2 = \frac{4}{5} = 0.8$ $\sigma = \sqrt{0.8} = 0.89$

10. $\bar{x} = \frac{112}{7} = 16$

	10	12	14	15	18	20	23
$x - \bar{x}$	−6	−4	−2	−1	2	4	7
$(x - \bar{x})^2$	36	16	4	1	4	16	49

$\sigma^2 = \frac{126}{7} = 18$ $\sigma = \sqrt{18} = 4.24$

11. $\bar{x} = \frac{147}{6} = 24.5$

	7	14	21	28	35	42
$x - \bar{x}$	−17.5	−10.5	−3.5	3.5	10.5	17.5
$(x - \bar{x})^2$	306.25	110.25	12.25	12.25	110.25	306.25

$\sigma^2 = \frac{857.5}{6} = 142.92$ $\sigma = \sqrt{142.92} = 11.95$

12. The mean is $46.\overline{6}$ and the standard deviation is about 8.4.
Three standard deviations is about $3(8.4) = 25.2$.
Values greater than 25.2 units away from the mean are outliers. So 19, the width of the desk, is an outlier.
The mean decreases from ≈ 49.2 to $46.\overline{6}$, and the standard deviation increases from ≈ 0.72 to ≈ 8.4.

PRACTICE AND PROBLEM SOLVING

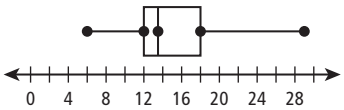
13. mean: $\frac{139}{6} \approx 23.1\overline{6}$
median: $\frac{16+25}{2} = 20.5$
mode: no mode

14. mean: $\frac{169}{8} = 21.125$
median: $\frac{7+7}{2} = 7$
mode: 7

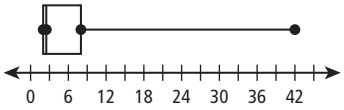
15. mean: $\frac{75}{5} = 15$
median: 15
mode: no mode

16. expected value
$= 0\left(\frac{1}{8}\right) + 1\left(\frac{3}{8}\right) + 2\left(\frac{3}{8}\right) + 3\left(\frac{1}{8}\right)$
$= 1.5$
The expected number of heads is 1.5.

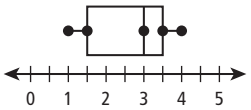
17. 6, 12, 12, 15, 18, 29
minimum: 6
maximum: 29
median: 13.5
first quartile: 12
third quartile: 18
IQR: 6



18. 2, 2, 2, 2, 3, 8, 8, 42
minimum: 2
maximum: 42
median: 2.5
first quartile: 2
third quartile: 8
IQR: 6



19. 1, 2, 3, 3, 4
minimum: 1
maximum: 4
median: 3
first quartile: 1.5
third quartile: 3.5
IQR: 2



20. $\bar{x} = \frac{21}{5} = 4.2$

	4	4	4	4	5
$x - \bar{x}$	−0.2	−0.2	−0.2	−0.2	0.8
$(x - \bar{x})^2$	0.04	0.04	0.04	0.04	0.64

$\sigma^2 = \frac{0.8}{5} = 0.16$ $\sigma = \sqrt{0.16} = 0.4$

21. $\bar{x} = \frac{245}{7} = 35$

	8	12	30	35	48	50	62
$x - \bar{x}$	−27	−23	−5	0	13	15	27
$(x - \bar{x})^2$	729	529	25	0	169	225	729

$\sigma^2 = \frac{2406}{7} \approx 343.71$ $\sigma = \sqrt{343.71} \approx 18.54$

22. $\bar{x} = \frac{132}{4} = 33$

	14	26	40	52
$x - \bar{x}$	−19	−7	7	19
$(x - \bar{x})^2$	361	49	49	361

$\sigma^2 = \frac{820}{4} = 205$ $\sigma = \sqrt{205} \approx 14.32$

23. The mean is about 22.8 and the standard deviation is about 11.5.
Three standard deviations is about 3(11.5) = 34.5. Values greater than 34.5 units away from the mean are outliers. So, 58 is an outlier.
The mean increases from ≈ 19.8 to ≈ 22.8, and the standard deviation increases from ≈ 5.6 to ≈ 11.5.

24. Possible answer: {3, 3, 9, 9}

25. The mean; 37° is an outlier and affects the mean greatly.

26. 25; $Q_3 + 1.5(\text{IQR})$
$= 5 + 1.5(2)$
$= 8;$
$25 > 8$

27. 15; $Q_1 - 1.5(\text{IQR})$
$= 79 - 1.5(11)$
$= 62.5;$
$15 < 62.5$

28. 92 and 1; $Q_3 + 1.5(\text{IQR})$
$= 36 + 1.5(3)$
$= 40.5$
$92 > 40.5$

$Q_1 - 1.5(\text{IQR})$
$= 33 - 1.5(3)$
$= 28.5$
$1 < 28.5$

29. The time intervals in which the duration time is an outlier are three times the standard deviation away from the mean, < 0.3 min or > 6.9 min. There are no outliers for duration times.

30. The intervals in which the time between eruptions is an outlier are three times the standard deviation away from the mean, < 32.6 min or > 109.4 min. There are no outliers for time between eruptions.

31. Ruth; possible answer: 13

32. Ruth; possible answer: 6

33. Possible answer: Ruth: 36; Aaron: 18

34. Aaron; Ruth's data are more spread out, and the set's IQR is twice the IQR for Aaron's set.

35. expected value $= 500(0.001) - 1(0.999)$
$= -0.499$
The expected gain is −$0.499.

36. expected value $= 100(0.10) - 2(0.30) + 0(0.60)$
$= 9.40$
The expected value is $9.40.

37. B; the student should have squared the differences.

38. Sometimes; possible answer: when 2 coins are tossed, the expected number of heads is 1, a possible outcome. When 3 coins are tossed, the expected number, 1.5, is not a possible outcome.

39a.

Product	1	2	3	4	5	6	8	9	10
Probability	$\frac{1}{36}$	$\frac{1}{18}$	$\frac{1}{18}$	$\frac{1}{12}$	$\frac{1}{18}$	$\frac{1}{9}$	$\frac{1}{18}$	$\frac{1}{36}$	$\frac{1}{18}$
Product	12	15	16	18	20	24	25	30	36
Probability	$\frac{1}{9}$	$\frac{1}{18}$	$\frac{1}{36}$	$\frac{1}{18}$	$\frac{1}{18}$	$\frac{1}{18}$	$\frac{1}{36}$	$\frac{1}{18}$	$\frac{1}{36}$

expected value $= 12\frac{1}{4}$

b. $P\left(\text{product greater than } 12\frac{1}{4}\right)$
$= P(15) + P(16) + P(18) + P(20) + P(24) + P(25) + P(30) + P(36)$
$= \frac{1}{18} + \frac{1}{36} + \frac{1}{18} + \frac{1}{18} + \frac{1}{18} + \frac{1}{36} + \frac{1}{18} + \frac{1}{36}$
$= \frac{13}{36}$

c. $P\left(\text{product less than } 12\frac{1}{4}\right)$
$= 1 - P\left(\text{product greater than } 12\frac{1}{4}\right)$
$= 1 - \frac{13}{36} = \frac{23}{36}$

d. No; possible answer: the expected value is a mean, not a median value.

40a. $\bar{x} = \frac{91}{10} = 9.1$ in.

	9.4	17.0	7.3	7.0	16.1
$x - \bar{x}$	0.3	7.9	−1.8	−2.1	7
$(x - \bar{x})^2$	0.09	62.41	3.24	4.41	49
	5.4	**6.9**	**8.5**	**4.2**	**9.2**
$x - \bar{x}$	−3.7	−2.2	−0.6	−4.9	−0.1
$(x - \bar{x})^2$	13.69	4.84	0.36	24.01	0.01

$\sigma^2 = \frac{162.06}{10} = 16.206$ $\sigma = \sqrt{16.206} \approx 4.0$
The mean annual precipitation is 9.1 in., and the standard deviation is 4.0.

b. One standard deviation from the mean is precipitation less than 5.1 in or greater than 13.1 in.
In the years 1995, 1998, and 2002 the precipitation was more than one standard deviation away from the mean.

c. median: $\frac{7.3 + 8.5}{2} = 7.9$; IQR: 2.5

TEST PREP

41. D. Data are close together.

42. H. Two different variances for the 2 sets.

43. C. Data cannot have a mean of 50.

CHALLENGE AND EXTEND

44a. The mean, median, and standard deviation grow by a factor of 5.
mean: 20; median: 15; standard deviation: $1.6 \cdot 5 = 8$

b. The mean and median increase by 5, while the standard deviation remains the same.
mean: 9; median: 8; standard deviation: 1.6

45. A deck of 1 card:

Cards in Same Position	1
Probability	1

Expected value = 1

A deck of 2 cards:

Cards in Same Position	2	0
Probability	$\frac{1}{2}$	$\frac{1}{2}$

Expected value = 1

A deck of 3 cards:

Cards in Same Position	3	1	0
Probability	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{3}$

Expected value = 1

A deck of 4 cards:

Cards in Same Position	4	2	1	0
Probability	$\frac{1}{24}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{3}{8}$

Expected value = 1

Following the pattern, the expected number of cards that will be in the same position after a deck of cards is shuffled is 1.

8-2 DATA GATHERING

CHECK IT OUT!

1. There are the same number of perch and walleye in the lake, or a 1:1 ratio. This is not an accurate estimate according to additional research.

2. biased; people visiting a news site online are more likely to be interested in news and subscribe to a daily newspaper. This may not be representative of the entire population.

3. No, this does not include a large part of the population who do not eat out. People in a restaurant on a Tuesday night are much more likely to eat out often.

4. $\frac{\text{emergencies}}{\text{total calls}} = \frac{\text{emergencies}}{\text{total calls}}$

$\frac{11}{25} = \frac{x}{175}$

$25x = 11 \cdot 175$

$x = 77$

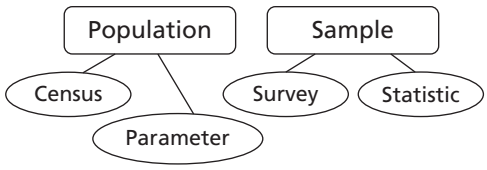
5. No, he is not justified in his evaluation because the sample size was too small. He has a 24% chance of not opening a prize in five boxes.

THINK AND DISCUSS

1. Every individual is surveyed in a census, so no group can be over- or underrepresented.

2. Possible answer: If the parameter is the number of people in the school who will vote for one candidate for school president over another, then it can be estimated by a statistic, or the number of students in a sample that say they will vote for that candidate over the other.

3.



EXERCISES

GUIDED PRACTICE

1. biased

2. parameter

3. Yes; a student that has not gone home from school by 5:00 PM is probably at the school for an extracurricular activity such as a sport.

4. Yes; the sample is a convenience sample that underrepresents people who may not be up so early on a Saturday morning. The baked goods may not be as good later in the day, when they are not as fresh.

5. No; the sample chosen is a convenience sample, which is not likely to be representative of the population. The sample overrepresents students who get to school early in the morning, and who are more likely to have time to eat breakfast at school and make the issue a priority.

6. Yes; the sample chosen is a random sample, so it is likely to be representative of the population.

7. No; the sample is not likely to be representative of the population because it underrepresents students who do not purchase food frequently in the cafeteria, perhaps because of the lack of variety.

8. $\frac{\text{prefer amusement park}}{\text{sophomores}} = \frac{\text{prefer amusement park}}{\text{sophomores}}$

$\frac{8}{20} = \frac{x}{150}$

$20x = 8 \cdot 150$

$x = 60$ sophomores

9. $\frac{\text{satisfied employees}}{\text{total employees}} = \frac{\text{satisfied employees}}{\text{total employees}}$

$\frac{25}{30} = \frac{x}{210}$

$30x = 25 \cdot 210$

$x = 175$ employees