Sec. 12-7 Normal Distributions

VOCABULARY

A smooth, symmetrical, bell-shaped curve which approximates a binomial distribution is called a **normal curve.** Areas under this curve represent probabilities from **normal distributions.**

Area Under a Normal Curve

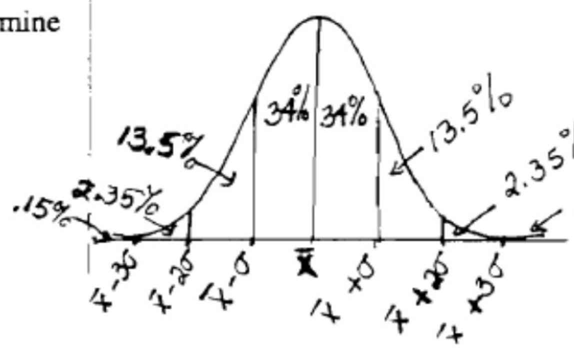
The mean $\bar{x}$ and standard deviation σ of a normal distribution determine the following areas.

The total area under the curve is 1.

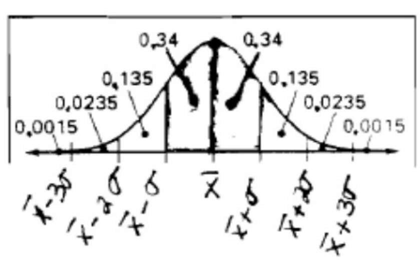
68% of the area lies within 1 standard deviation of the mean.

95% of the area lies within 2 standard deviations of the mean.

99.7% of the area lies within 3 standard deviations of the mean.

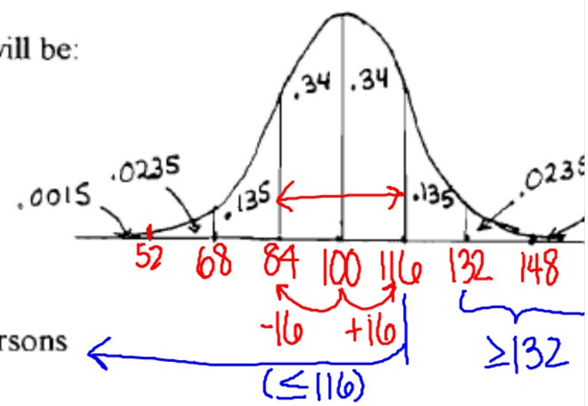


The partial areas can be interpreted as probabilities, as shown.



Example 1: A normal distribution of IQ scores has a mean of 100 and a standard deviation of 16.

Find the probability that a randomly selected person's IQ will be:



a) between 84 and 116 $.34 + .34 = .68$

b) at most 116 $P(\leq 116)$ $.50 + .34 = .84$

Find the probability that the IQ for 3 randomly selected persons will be at least 132.

$$P(\geq 132) = (.025)$$

$$\underset{1st}{.025} \overset{\&}{\times} \underset{2nd}{.025} \overset{\&}{\times} \underset{3rd}{.025}$$

$$\approx .00002$$

If n is large, it becomes tedious to compute binomial probabilities using the formula $P(k \text{ successes}) = {}_nC_k\,(p)^k\,(1-p)^{n-k}$. Under certain conditions, you may use a normal distribution to approximate a binomial distribution.

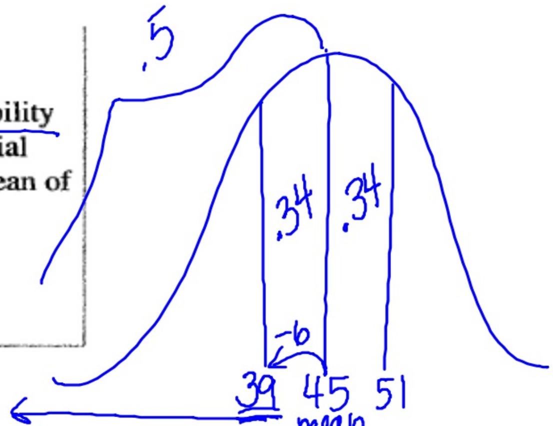
Normal Approximation of a Binomial Distribution

Consider the binomial distribution consisting of n trials with probability p of success on each trial. If $np \geq 5$ and $n(1-p) \geq 5$, then binomial distribution can be approximated by a normal distribution with a mean of

$\bar{x} = np$ mean   mean ≥ 5

and a standard deviation of

$\sigma = \sqrt{np(1-p)}$.



Example 2: A loan officer at a bank may reject a loan application if the borrower does not have enough assets or has too many debts based on total income. At a certain bank, 20% of the loan applications are rejected. Assume there were 225 applications. What is the probability that at most 39 will be rejected?

mean $= \bar{X} = np = 225(.20) = 45$ $\sigma = \sqrt{np(1-p)}$

$P(\leq 39 \text{ rejects}) = .5 - .34 = .16$ $6 = \sigma = \sqrt{225(.2)(.8)}$