

Projectile Motion Problems. Level 3 and Level 4

Level 3

1. A ball tossed vertically from ground level returns to earth 4 seconds later. Find the initial velocity and maximum height of the ball. *in meters*
2. A bullet is fired in the air vertically from ground level with an initial velocity 200 m/s. Find the bullet's maximum velocity and maximum height.

Level 4

3. Show that for an object falling according to Galileo's Formula, the average velocity over any time interval  $[t_1, t_2]$  is equal to the average of the instantaneous velocities at  $t_1$  and  $t_2$ .

Key

$V_0$  = initial vel or  $V(0)$   
 $S_0$  = initial position or  $S(0)$

Projectile Motion Problems. Level 3 and Level 4

Starts on the ground  $\rightarrow$   $t=0$   
 $S(0)=0$

$S(4)=0$   
 $\downarrow$

Level 3

1. A ball tossed vertically from ground level returns to earth 4 seconds later. Find the initial velocity and maximum height of the ball.

$$S(t) = -\frac{1}{2} (9.8) t^2 + V_0 t + S_0$$

$$s(0) = 0 + 0 + \boxed{S_0 = 0}$$

$$S(4) = -4.9(4)^2 + V_0(4)$$

$$4V_0 = -4.9(4)^2$$

$$\boxed{V_0 = 19.6 \text{ m/s}}$$

max height occurs

when  $V(t) = 0$

$$V(t) = -9.8(t) + 19.6 = 0$$

$t = 2 \leftarrow$  when max velocity occurs

$$\boxed{S(2) = 19.6 \text{ m}}$$

\* not always the same  
 $S(0) = 0$

2. A bullet is fired in the air vertically from ground level with an initial velocity 200 m/s. Find the bullet's maximum velocity and maximum height.

$$S(t) = -4.9t^2 + 200t + 0$$

$$t(-4.9t + 200) = 0$$

$$t = 0, t \approx 40.816 \text{ ground}$$

$V(t) = 0$  max height for parabola 

$$V(t) = -9.8t + 200 = 0$$

$$-9.8t = -200$$

$$t \approx 20.40816327$$

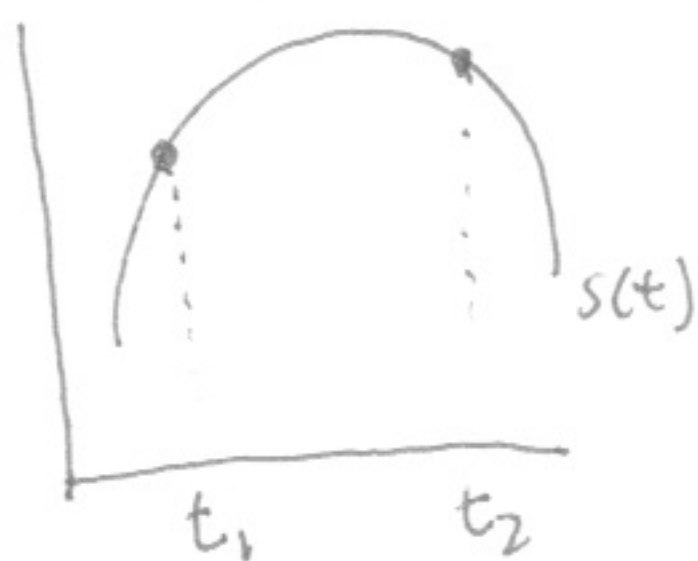
$$S(20.41) \approx \text{max height } 2040.82 \text{ m}$$

max velocity 200 m/s gravity

slows it down to zero and speeds it back to initial velocity of ground by the time it reaches the ground again

Level 4

3. Show that for an object falling according to Galileo's Formula, the average velocity over any time interval  $[t_1, t_2]$  is equal to the average of the instantaneous velocities at  $t_1$  and  $t_2$ .



Show that

$$\frac{v(t_2) + v(t_1)}{2} = \frac{s(t_2) - s(t_1)}{t_2 - t_1}$$

Average velocity

Average of the  
two instantaneous  
velocities  
at  $t_2$  &  $t_1$

$$s(t_2) = s_0 + v_0(t_2) - \frac{1}{2}a(t_2)^2$$
$$- s(t_1) = -s_0 - v_0(t_1) + \frac{1}{2}a(t_1)^2$$

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$$s(t_2) - s(t_1) = v_0(t_2) - v_0(t_1) - \frac{1}{2}a(t_2)^2 + \frac{1}{2}a(t_1)^2$$

$$\text{AVG velocity} = \frac{v_0(t_2 - t_1) - \frac{1}{2}a(t_2^2 - t_1^2)}{t_2 - t_1}$$

$$= \frac{v_0 - \frac{1}{2}a}{1} \frac{v_0(t_2 - t_1) - \frac{1}{2}a(t_2 - t_1)(t_2 + t_1)}{t_2 - t_1}$$

$$= \text{Factor} \frac{[v_0 - \frac{1}{2}a(t_2 + t_1)](t_2 - t_1)}{(t_2 - t_1)}$$

$$= \frac{1}{2}(2v_0) - \frac{1}{2}a(t_2 + t_1)$$

$$\frac{\frac{1}{2}v_0 - \frac{1}{2}a(t_2) + \frac{1}{2}v_0 - \frac{1}{2}a(t_1)}{1}$$

$$v(t) = v_0 - a(t)$$

$$v(t_2) = v_0 - a(t_2)$$

$$v(t_1) = v_0 - a(t_1)$$

or

$$\frac{v_0 - a(t_2) + v_0 - a(t_1)}{2}$$

$$= \boxed{\frac{v(t_2) + v(t_1)}{2}} \quad \text{Q.E.D.}$$