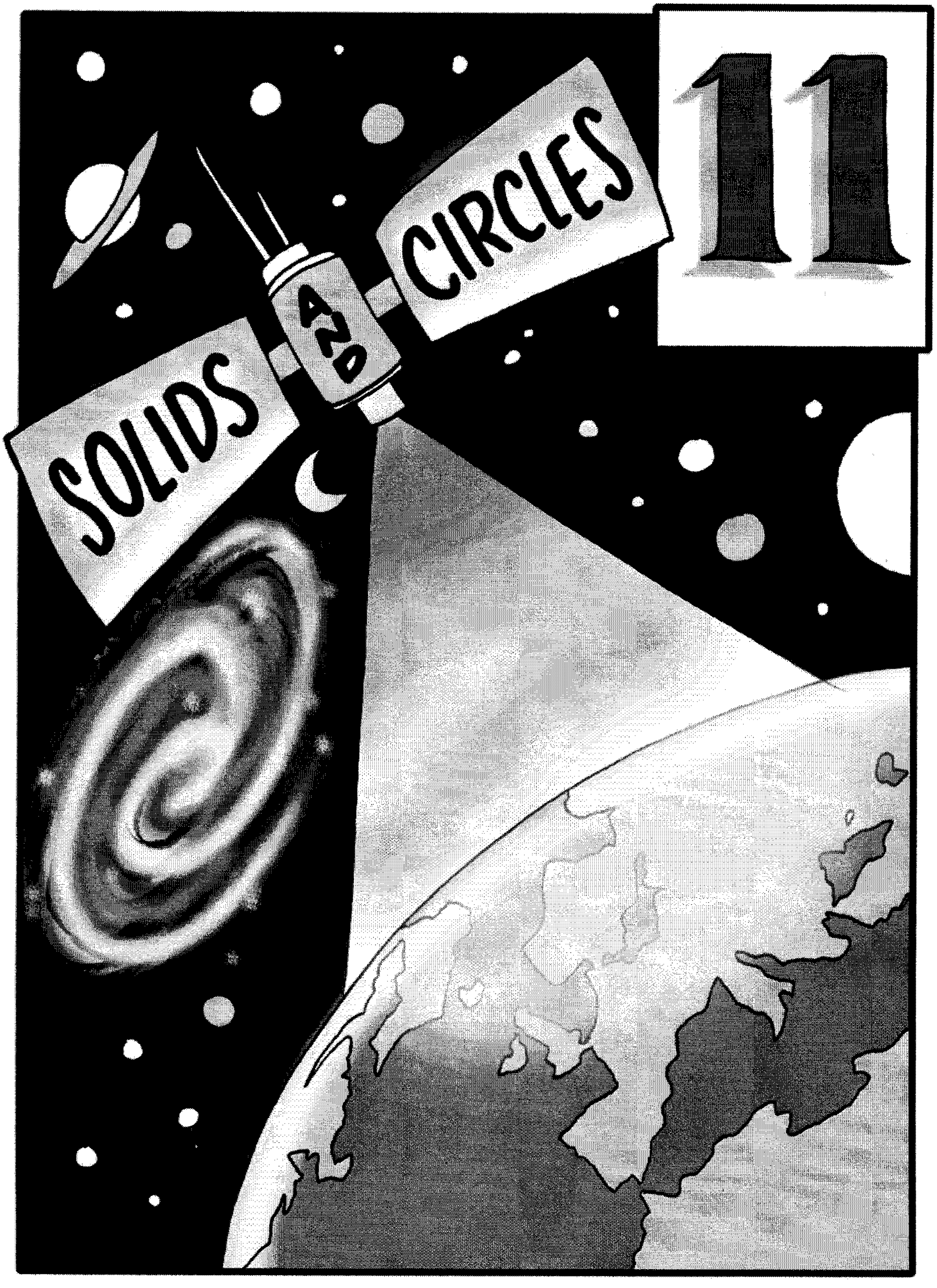

SOLIDS
AND
CIRCLES
11

CHAPTER 11 Solids and Circles

In Chapter 9, you learned how to find the volume and surface area of three-dimensional solids formed with blocks. Then you extended these concepts to include prisms and cylinders. In this chapter, you will complete your study of three-dimensional solids to include pyramids, cones, and spheres. You will learn how to identify the cross-sections of a solid and will **investigate** a special group of solids known as Platonic Solids.

As the word *geometry* literally means the "measurement of the Earth," it is only fitting that Section 11.2 focuses on developing the geometric tools that are used to learn more about the Earth. For example, by studying the height at which satellites orbit the Earth, you will get a chance to develop tools to work with the angle and arc measures that occur when two lines that are tangent to the same circle intersect each other.

Guiding Questions

Think about these questions throughout this chapter:

What's the relationship?

How can I measure it?

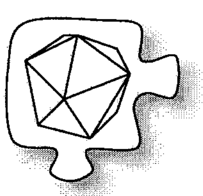
What information do I need?

Is there another way?

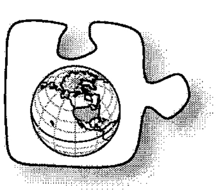
In this chapter, you will learn:

- How to find the volume and surface area of a pyramid, a cone, and a sphere.
- About the properties of special polyhedra, called Platonic Solids.
- How to find the cross-section of a solid.
- How to find the measures of angles and arcs that are formed by tangents and secants.
- About the relationships between the lengths of segments created when tangents or secants intersect outside a circle.

Chapter Outline

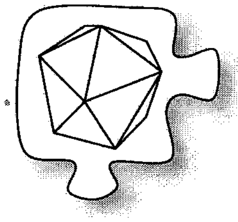


Section 11.1 In this section, you will learn how regular polygons can be used to form three-dimensional solids called "polyhedra." You will extend your knowledge of finding volume and surface area to include other solids, such as pyramids, cones, and spheres.



Section 11.2 By studying the coordinate system of latitude and longitude lines that help use refer to locations on the Earth, you will learn about great circles and how to find the distance between two points on a sphere. You will also **investigate** the geometric relationships created when tangents and secants intersect a circle.

11.1.1 How can I build it?



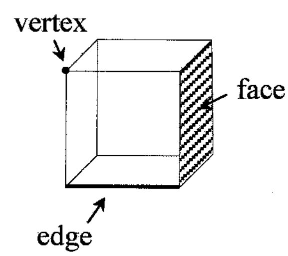
Platonic Solids

In Chapter 9, you explored three-dimensional solids such as prisms and cylinders. You developed methods to measure their sizes using volume and surface area and learned to represent three-dimensional solids using mat plans and two-dimensional (front, right, and top) views.

But what other types of three-dimensional solids can we learn about? During Section 11.1, you and your team will **examine** new types of solids in order to expand your understanding of three-dimensional shapes.

11-1. EXAMINING A CUBE

In Chapter 9, you studied the volume and surface area of three-dimensional solids, such as prisms and cylinders. A **cube** is a special type of rectangular prism because each face is a square.



a. What are some examples of cubes you remember seeing?

b. Find the volume and surface area of a cube with an edge length of 10 units.

c. A "flat side" of a prism is called a **face**, as shown in the diagram above. Notice that the line segment where two faces meet is called an **edge**, while the point where the edges meet is called a **vertex**. How many faces does a cube have? How many edges? How many vertices? ("Vertices" is plural for "vertex.")

d. Confirm with your team that a cube has three square faces that meet at each vertex. Is it possible to have a solid where only two square faces meet at a vertex? Could a solid have four or more square faces at a vertex? Explain.

11-2. OTHER REGULAR POLYHEDRA

A three-dimensional solid made up of flat, polygonal faces is called a **polyhedron** (*poly* is the Greek root for "many," while *hedron* is the Greek root for "faces"). A cube, like the one you studied in problem 11-1, is an example of a **regular polyhedron** because all of the faces are congruent, regular polygons and the same number of faces meet at each vertex. In fact, you found that a cube is the *only* regular polyhedron with square faces.

But what if the faces are equilateral triangles? Or what if the faces are other regular polygons such as pentagons or hexagons?

Your Task: With your team, determine what other regular polyhedra are possible. First, obtain building materials from your teacher. Then work together to build regular polyhedra by testing how the different types of regular polygons can meet at a vertex. For example, what type of solid is formed when three equilateral triangle faces meet at each vertex? Four? Five? Six? Do similar tests for regular pentagons and hexagons. For each regular polyhedron, describe its shape and count its faces. Be ready to discuss your results with the class.

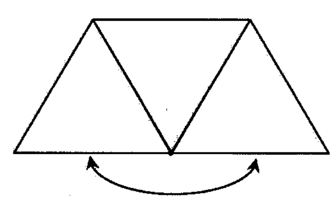
Discussion Points

- What is this task asking you to do?
- How should you start?
- How can your team organize the task among the members to complete the task efficiently?

Further Guidance

11-3. For help in testing the various ways that congruent, regular polygons can build regular polyhedra, follow the directions below.

a. Start by focusing on equilateral triangles. Attach three equilateral triangles so that they are adjacent and share a common vertex, as shown at right. Then fold and attach the three triangles so that they completely surround the common vertex. Complete the solid with as many equilateral triangles as needed so that each vertex is the intersection of three triangles. How would you describe this shape?



Fold so that these edges meet.

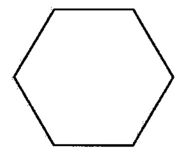
Problem continues on next page →

11-3. *Problem continued from previous page.*

b. Now repeat your test to determine what solids are possible when 4, 5, or more equilateral triangles meet at each vertex. If a regular polyhedron is possible, describe it and state the number of faces it has. If a regular polyhedron is not possible, explain why not.

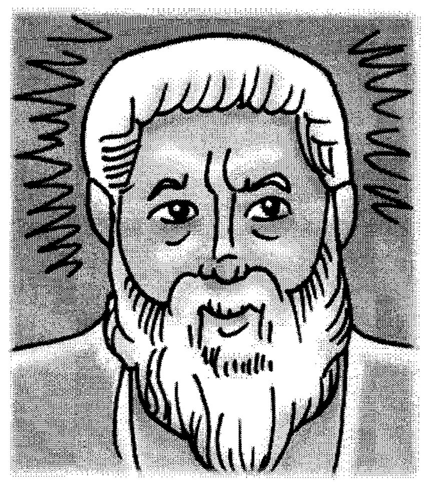
c. What if the faces are regular pentagons? Try building a regular polyhedron so that each vertex is the intersection of three regular pentagons. What if four regular pentagons meet at a vertex? Explain what happens in each case.

d. End your **investigation** by considering regular hexagons. Place three or more regular hexagons at a common vertex and explain what solids are formed. If no solid is possible, explain why.



11-4. POLYHEDRA VOCABULARY

The regular polyhedra you discovered in problem 11-2 (along with the cube from problem 11-1) are sometimes referred to as **Plato's Solids** (or **Platonic Solids)** because the knowledge about them spread about 2300 years ago during the time of Plato, a Greek philosopher and mathematician.



Plato (490 – 430 B.C.)

In addition, polyhedra are classified by the number of faces they have. For example, a cube is a solid with six faces, so it can be called a regular hexahedron (because *hexa* is the Greek root meaning "six" and *hedron* is the Greek root for "face").

Examine the table of names below. Then return to your results from problem 11-2 and determine the name for each regular polyhedron you discovered.

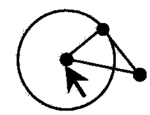
4 faces	Tetrahedron
5 faces	Pentahedron
6 faces	Hexahedron
7 faces	Heptahedron
8 faces	Octahedron

9 faces	Nonahedron
10 faces	Decahedron
11 faces	Undecahedron
12 faces	Dodecahedron
20 faces	Icosahedron

11-5. Find the surface area of each of Plato's Solids you built in problem 11-2 (the regular tetrahedron, octahedron, dodecahedron, and icosahedron) if the length of each edge is 2 inches. Show all work and be prepared to share your method with the class.

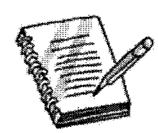
11-6. DUAL POLYHEDRA

Ivan wonders, "What happens when the centers of adjacent faces of a regular polyhedron are connected?" These connections form the edges of a solid, which can be called a **dual polyhedron**.



To **investigate** dual polyhedra, first predict the results for each regular polyhedron with your team using spatial **visualization**. Then use a dynamic geometry tool to test your prediction of what solid is formed when the centers of adjacent faces of a Platonic Solid are connected. Be sure to test all five Platonic Solids (tetrahedron, cube, octahedron, dodecahedron, and icosahedron) and record the results.

11-7. Reflect what you learned about Plato's Solids during this lesson. What connections did you make to previous material? Write an entry in your Learning Log explaining what is special about this group of solids. Name and describe each Platonic Solid. Title this entry "Plato's Solids" and include today's date.



Review & Preview

11-8. Draw a hexagon on your paper.

a. Do all hexagons have an interior angle sum of 720°?

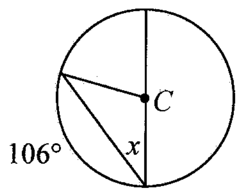
b. Does every hexagon have an interior angle measuring 120°? Explain your **reasoning**.

c. Does every hexagon have 6 sides? Explain your **reasoning**.

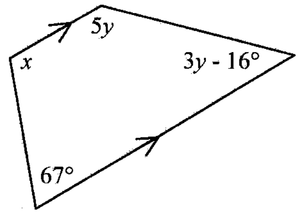
11-9. The **lateral surface** of a cylinder is the surface connecting the bases. For example, the label from a soup can could represent the lateral surface of a cylindrical can. If the radius of a cylinder is 4 cm and the height is 15 cm, find the lateral surface area of the cylinder. Note: It may help you to think of "unrolling" a soup can label and finding the area of the label.

11-10. For each of the relationships represented in the diagrams below, write and solve an equation for x and/or y. **Justify** your method. In part (a), assume that C is the center of the circle.

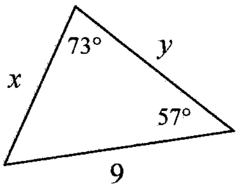
a.



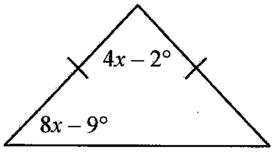
b.



c.



d.



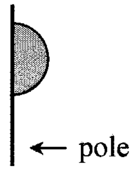
11-11. Garland is having trouble with the copy machine. He's trying to copy a triangle with an area of 36 square units and a perimeter of 42 units.

a. After he pressed the button to copy, Garland noticed the copier's zoom factor (the linear scale factor) was set to 200%. What is the area and perimeter of the resulting triangle?

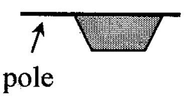
b. Now Garland takes the result from part (a) and accidentally shrinks it by a linear scale factor of $\frac{1}{3}$! What is the area and perimeter of the resulting triangle?

11-12. Three flags are shown below on flagpoles. For each flag, determine what shape appears if the flag is spun very quickly about its pole. If you do not know the name of the shape, describe it.

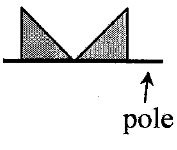
a.



b.



c.



11-13. **Multiple Choice:** $\triangle ABC$ has a right angle at B. If $m\measuredangle A = 42°$ and $BC = 7$ mm, what is the approximate value of AC?

a. 9.4 b. 10.5 c. 7.8 d. 4.7

11-14. Draw a tetrahedron on your paper.

a. How many faces does the tetrahedron have?

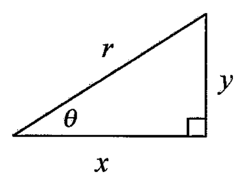
b. How many edges does it have?

c. How many vertices does it have?

11-15. Mia found the volume of a rectangular prism to be 840 mm^3. As she was telling her father about it, she remembered that the base had a length of 10 mm and a width of 12 mm, but she could not remember the height. "Maybe there's a way you can find it by going backwards," her father suggested. Can you help Mia find the height of her prism? Explain your solution.

11-16. On graph paper, graph a circle with center (4, 2) and radius 3 units. Then write its equation.

11-17. **Examine** the diagram of the triangle at right.



a. Write an equation representing the relationship between x, y, and r.

b. Write an expression for $\sin\theta$. What is $\sin\theta$ if $r = 1$?

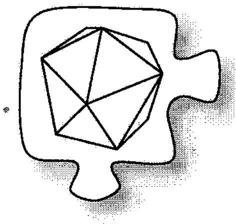
c. Write an expression for $\cos\theta$. What is $\cos\theta$ if $r = 1$?

11-18. In a circle, chord $\overline{AB}$ has length 10 units, while $m\overset{\frown}{AB} = 60°$. What is the area of the circle? Draw a diagram and show all work.

11-19. **Multiple Choice:** Assume that the coordinates of ΔABC are $A(5, 1)$, $B(3, 7)$, and $C(2, 2)$. If ΔABC is rotated 90° clockwise (↻) about the origin, the coordinates of the image of B would be:

a. (–3, 7) b. (–7, 3) c. (7, –3) d. (7, 3)

11.1.2 How can I measure it?



Pyramids

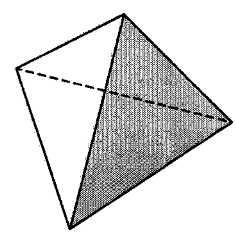
In Lesson 11.1.1, you explored Plato's five special solids: the tetrahedron, the octahedron, the cube (also known as the hexahedron), the dodecahedron, and the icosahedron. You discovered why these are the only regular polyhedra and developed a method to find their surface area.

Today you will **examine** the tetrahedron from a new perspective: as a member of the **pyramid** family. As you work today with your team, you will discover ways to classify pyramids by their shape and will develop new tools of measurement.

11-20. A **pyramid** is a polyhedron with a polygonal base formed by connecting each point of the base to a single given point (the **apex**) that is above or below the flat surface containing the base. Each triangular lateral face of a pyramid is formed by the segments from the apex to the endpoints of a side of the base and the side itself. A tetrahedron is a special pyramid because any face can act as its base.

Obtain the four Lesson 11.1.2 Resource Pages, a pair of scissors, and either tape or glue from your teacher. Have each member of your team build one of the solids. When assembling each solid, be sure to have the printed side of the net on the exterior of the pyramid for reference later. Then answer the questions below.

a. Sketch each pyramid onto your paper. What is the same about each pyramid? What is different? With your team, list as many qualities as you can.

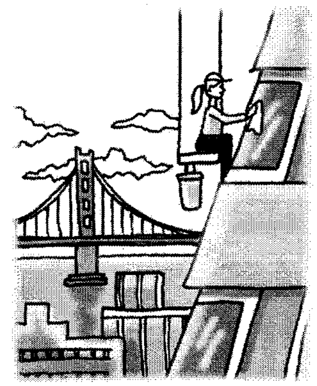


b. A tetrahedron can also be called a **triangular-based pyramid**, because its base is always a triangle. Choose similar, appropriate names for the other pyramids that your team constructed.

c. Find the surface area of pyramids **B** and **D**. Use a ruler to find the dimensions of the edges in centimeters.

d. Compare pyramids **B** and **C**. Which do you think has more volume? **Justify** your **reasoning**.

11-21. THE TRANSAMERICA BUILDING



The TransAmerica building in San Francisco is built of concrete and is shaped like a square-based pyramid. The building is periodically power-washed using one gallon of cleaning solution for every 250 square meters of surface. As the new building manager, you need to order the cleaning supplies for this large task. The problem is that you do not know the height of each triangular face of the building; you only know the vertical height of the building from the base to the top vertex.

Your Task: Determine the amount of cleaning solution needed to wash the TransAmerica building if an edge of the square base is 96 meters and the height of the building is 220 meters. Include a sketch in your solution.

11-22. Read the Math Notes box for this lesson, which introduces new vocabulary terms such as "slant height" and "lateral surface area." Explain the difference between the slant height and the height of a pyramid. How can you use one to find the other?

METHODS AND MEANINGS

MATH NOTES

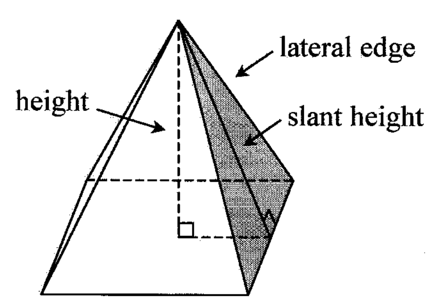
Pyramid Vocabulary

If a face of a pyramid (defined in problem 11-20) or prism is not a base, it is called a **lateral face.**

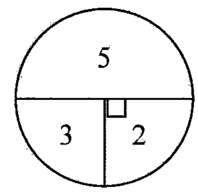
The **lateral surface area** of a pyramid or prism is the sum of the areas of all faces of the pyramid or prism, not including the base(s). The area of the exterior of the TransAmerica building that needs cleaning (from problem 11-21) is an example of lateral surface area, since the exterior of the base of the pyramid cannot be cleaned.

The **total surface area** of a pyramid or prism is the sum of the areas of all faces, including the bases.

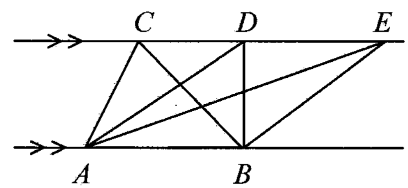
Sometimes saying the word "height" for a pyramid can be confusing, since it could refer to the height of one of the triangular faces or it could refer to the overall height of the pyramid. Therefore, we call the height of each lateral face a **slant height** to distinguish it from the **height** of the pyramid itself. See the diagram at right.



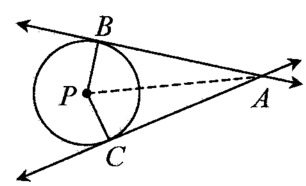
11-23. Marina needs to win 10 tickets to get a giant stuffed panda bear. To win tickets, she throws a dart at the dartboard at right and wins the number of tickets listed in the region where her dart lands. Unfortunately, she only has enough money to play the game three times. If she throws the dart randomly, do you expect that she'll be able to win enough tickets? Assume that each dart will land on the dartboard.



11-24. **Examine** ΔABC, ΔABD, and ΔABE in the diagram at right. If $\overline{CE} \parallel \overline{AB}$, explain what you know about the areas of the three triangles. **Justify** your statements.



11-25. Prove that when two lines that are tangent to the same circle intersect, the lengths between the point of intersection and the points of tangency are equal. That is, in the diagram at right, if $\overleftrightarrow{AB}$ is tangent to $\odot P$ at B, and $\overleftrightarrow{AC}$ is tangent to $\odot P$ at C, prove that $AB = AC$. Use either a flowchart or a two-column proof.



11-26. Solve each equation below, if possible. Show all work.

a. $\frac{3}{5} = \frac{2x}{3} - 8$

b. $\frac{9x}{5000} + \frac{2}{1000} = \frac{28}{5000}$

c. $\frac{2x}{3} + \frac{x}{2} = \frac{2x}{3}$

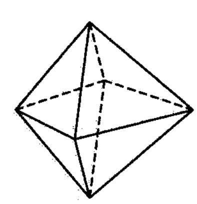
d. $\frac{3}{2}(2x - 5) = \frac{1}{6}$

11-27. On graph paper, plot the points $A(4, 1)$ and $B(10, 9)$.

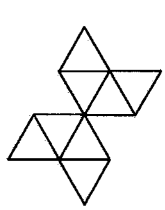
a. Find the distance between points A and B. That is, find AB.

b. If point C is at (10, 1), find $m\measuredangle CAB$. Show all work.

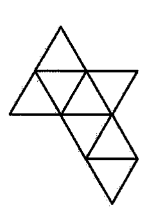
11-28. **Multiple Choice:** Which net below **cannot** create a regular octahedron when folded, like the one at right?



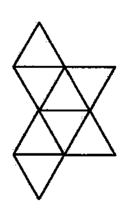
a.



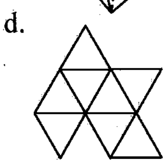
b.



c.

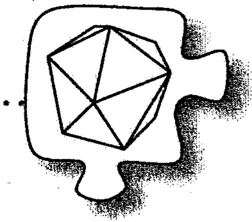


d.



e. None of these

11.1.3 What's the volume?

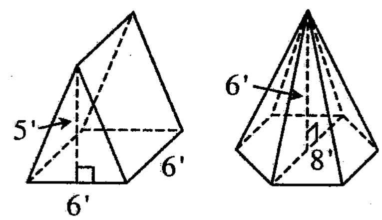


Volume of a Pyramid

Today, as you continue your focus on pyramids, look for and utilize connections to other geometry concepts. The models of pyramids that you constructed in Lesson 11.1.2 will be useful as you develop a method for finding the volume of a pyramid.

11-29. GOING CAMPING

As Soraya shopped for a tent, she came across two models that she liked best, shown at right. However, she does not know which one to pick! They are both made by the same company and appear to have the same quality. She has come to you for help in making her decision.



Tent A *Tent B*

While she says that her drawings are not to scale, below are her notes about the tents:

Tent A is a pup tent with a rectangular base. It has a height of 5 feet, a length of 6 feet, and a width of 6 feet.	*Tent B is a 6-foot-tall teepee. Its base is a regular hexagon, and the greatest diagonal across the floor measures 8 feet.*

With your team, discuss the following questions in any order. Be prepared to share your discussion with the class.

- What are the shapes of the two tents?
- Without doing any calculations, which tent do you think Soraya should buy and why?
- What types of measurement might be useful to determine which tent is better?
- What do you still need to know to answer her question?

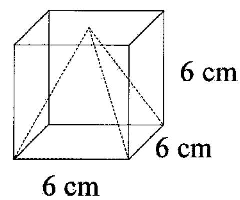
11-30. COMPARING SOLIDS

To analyze Tent B from problem 11-29, you need to know how to find the volume of a pyramid. But how can you find that volume?

To start, consider a simpler pyramid with a square base, such as pyramid **B** that your team built in Lesson 11.1.2. To develop a method to find the volume of a pyramid, first consider what shape(s) we can compare it to. For example, when finding the area of a triangle, you compared it to the area of a rectangle and figured out that the area of a triangle is always half the area of a rectangle with the same base and height. What shape(s) can you compare the volume of pyramid **B** to? Discuss this with your team and be prepared to share your thinking with the class.

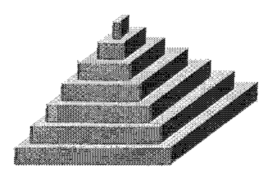
11-31. VOLUME OF A PYRAMID

Soraya thinks that pyramid **B** could be compared to a cube, like the one shown at right, since the base edges and heights of both are 6 cm.

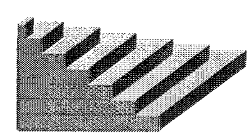


a. What fraction of the cube with edge length 6 cm is pyramid **B**? Discuss this with your team and make an estimate.

b. Soraya remembers comparing pyramids **B** and **C** in Lesson 11.1.2. She decided to compare the volumes by using a model. She constructed a pyramid using foam layers as shown at right. What is the shape of each foam layer? Note: The name for the shape of a layer of a three-dimensional solid is called a **cross-section**.



c. Soraya then slid all of the layers of the pyramid so that the top vertex was directly above one of the corners of the base, like Pyramid C from problem 11-20. Since she did not add or take away any foam layers, how does the volume of this pyramid compare with the pyramid in part (b) above?

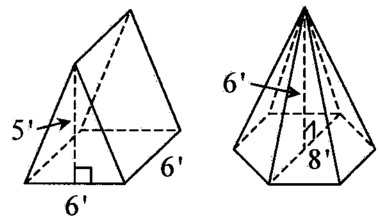


d. Test your estimate from part (a) by using as many pyramid **C**s as you need to assemble a cube. Was your estimate accurate? Now explain how to find the volume of a pyramid.

e. Do you think your method for part (d) works with all pyramids? Why or why not?

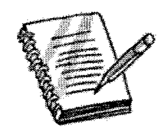
11-32. When the top vertex of a pyramid is directly above (or below) the center of the base, the pyramid is called a **right pyramid**, while all other pyramids are referred to as **oblique pyramids**. **Examine** your models (**A**, **B**, **C**, and **D**) from problem 11-20 and decide which are right pyramids and which are oblique pyramids.

11-33. Now return to problem 11-29 and help Soraya decide which tent to buy for her backpacking trip. To make this decision, compare the volumes, base areas, and surface areas of both tents. Be ready to share your decision with the class.



Tent A *Tent B*

11-34. Write an entry in your Learning Log and explain how to find the volume of a pyramid. Be sure to include an example. Title this entry "Volume of a Pyramid" and include today's date.



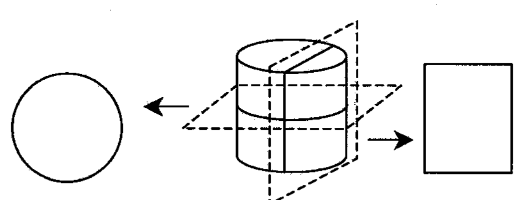
MATH NOTES

METHODS AND MEANINGS

Cross-Sections of Three-Dimensional Solids

The intersection of a three-dimensional solid and a plane is called a **cross-section** of the solid. The result is a two-dimensional diagram that represents the flat surface of a slice of the solid.

One way to **visualize** a cross-section is to imagine the solid sliced into thin slices like a ream of paper. Since a solid can be sliced in any direction and at any angle, you need to know the direction of the slice to find the correct cross-section. For example, the cylinder at right has several different cross-sections depending on the direction of the slice. When this cylinder is sliced vertically, the resulting cross-section is a rectangle, while the cross-section is a circle when the cylinder is sliced horizontally.



A horizontal cross-section is a circle. A cylinder A vertical cross-section is a rectangle.

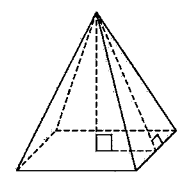
Review & Preview

11-35. Review the information about cross-sections in the Math Notes box for this lesson. Then answer the questions below.

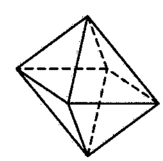
a. Draw a cube on your paper. Is it possible to slice a cube and get a cross-section that is not a quadrilateral? Explain how.

b. Barbara has a solid on her desk. If she slices it horizontally at any level, the cross-section is a triangle. If she slices it vertically in any direction, the cross-section is a triangle. What could her shape be? Draw a possible shape.

11-36. Find the volume and surface area of a square-based pyramid if the base edge has length 6 units and the height of the pyramid is 4 units. Assume the diagram at right is not to scale.



11-37. The solid at right is a regular octahedron.



a. Trace the shape on your paper. How many faces does it have? How many edges? Vertices?

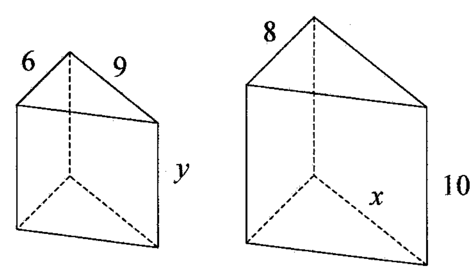
b. If an octahedron is sliced horizontally, what shape is the resulting cross-section?

11-38. Assume that the prisms at right are similar.

a. Solve for x and y.

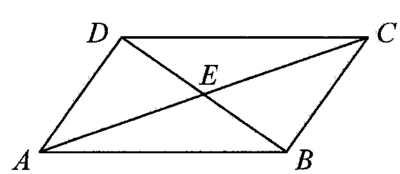
b. What is the ratio of the corresponding sides of Solid B to Solid A?

c. If the base area of Solid A is 27 square units, find the base area of Solid B.



Solid A Solid B

11-39. In the diagram at right, assume that $m\measuredangle ECB = m\measuredangle EAD$ and point E is the midpoint of $\overline{AC}$. Prove that $\overline{AD} \cong \overline{CB}$.



11-40. **Multiple Choice:** The graph of $x^2 + y^2 = 4$ is:

a. a parabola with y-intercept (0, 4)

b. a circle with radius 4 and center (0, 0)

c. a parabola with x-intercepts (–2, 0) and (2, 0)

d. a circle with radius 2 and center (0, 0)

e. None of these

11-41. On graph paper, graph the equation $x^2 + (y - 3)^2 = 25$. Name the x- and y-intercepts.

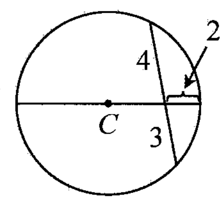
11-42. While volunteering for a food sale, Aimee studied a cylindrical can of soup. She noticed that it had a diameter of 3 inches and a height of 4.5 inches.

a. Find the volume of the soup can.

b. If Aimee needs to fill a cylindrical pot that has a diameter of 14 inches and a height of 10 inches, how many cans of soup will she need?

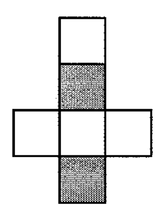
c. What is the area of the soup can label?

11-43. Find the area and circumference of $\odot C$ at right. Show all work.



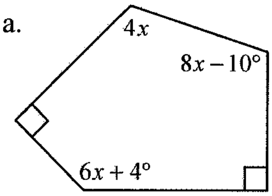
11-44. STEP RIGHT UP!

At a fair, Cyrus was given the following opportunity. He could roll the die formed by the net at right one time. If the die landed so that a shaded die faced up, then Cyrus would win $10. Otherwise, he would lose $5. Is this game fair? Explain how you know.

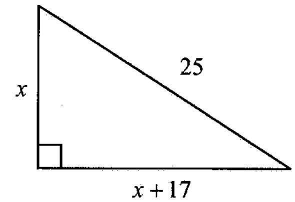


11-45. Write and solve an equation from the geometric relationships provided in the diagrams below.

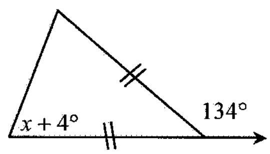
a.



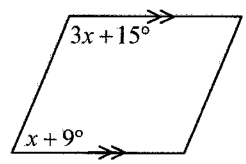
b.



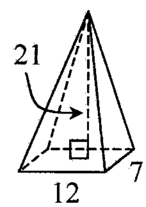
c.



d.



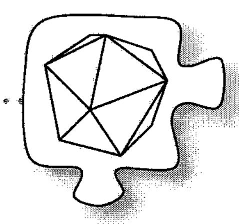
11-46. **Multiple Choice:** Calculate the volume of the rectangle-based pyramid at right.



a. 84 un^3 b. 648 un^3 c. 882 un^3

d. 1764 un^3 e. None of these

11.1.4 What if it's a cone?

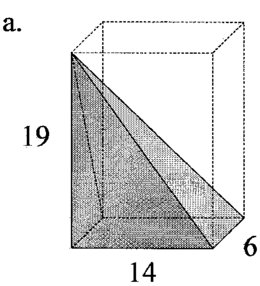


Surface Area and Volume of a Cone

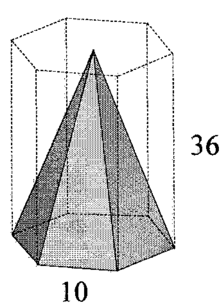
Today you will continue to use what you know about the volume and surface area of prisms and pyramids and will extend your understanding to include a new three-dimensional shape: a cone. As you work with your team, look for connections to previous course material.

11-47. Review what you learned in Lesson 11.1.3 by finding the volume of each pyramid below. Assume that the pyramid in part (a) corresponds to a rectangular-based prism and that the base of the pyramid and prism in part (b) is a regular hexagon.

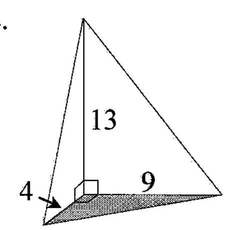
a.



b.



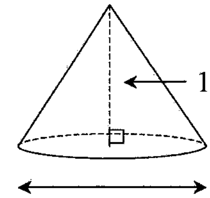
c.



11-48. While finding the volumes of the pyramids in problem 11-47, Jamal asked, "But what if it is a cone? How would you find its volume?" Note that a **cone** is a three-dimensional figure that consists of a circular face, called the **base**, a point called the **apex**, that is not in the flat surface (plane) of the base, and the lateral surface that connects the apex to each point on the circular boundary of the base.

a. Discuss Jamal's question with your team. Then write a response explaining how to find the volume of a cone.

b. Find the volume of the cone at right. Show all work.



11-49. HAPPY BIRTHDAY!

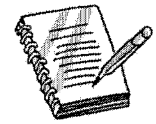
Your class has decided to throw your principal a surprise birthday party tomorrow. The whole class is working together to create party decorations, and your team has been assigned the job of producing party hats. Each party hat will be created out of special decorative paper and will be in the shape of a cone.

Your Task: Use the sample party hat provided by your teacher to determine the size and shape of the paper that forms the hat. Then determine the amount of paper (in square inches) needed to produce one party hat and figure out the total amount of paper you will need for each person in your class to have a party hat.

11-50. The Math Club has decided to sell giant waffle ice-cream cones at the Spring Fair. Lekili bought a cone, but then she got distracted. When she returned to the cone, the ice cream had melted, filling the cone to the very top!

If the diameter of the base of the cone is 4 inches and the slant height is 6 inches, find the volume of the ice cream and the area of the waffle that made the cone.

11-51. Reflect on what you learned during this lesson and write an entry in your Learning Log on the surface area and volume of a cone. What connections did you make to previous material? Be sure to include an example. Title this entry "Surface Area and Volume of a Cone" and include today's date.



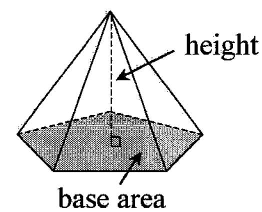
MATH NOTES

METHODS AND MEANINGS

Volume of a Pyramid

In general, the volume of a pyramid is one-third of the volume of the prism with the same base area and height. Thus:

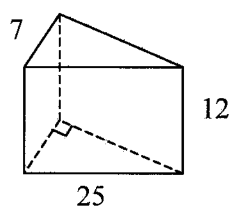
$$V = \frac{1}{3}(\text{base area})(\text{height})$$



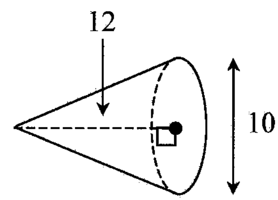
Review & Preview

11-52. Find the volume and total surface area of each solid below. Show all work.

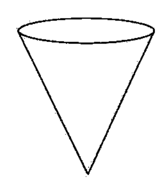
a.



b.



11-53. **Examine** the diagram of the cone at right.

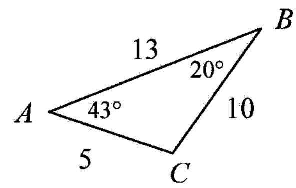


a. How could you slice the cone so that the cross-section is a triangle?

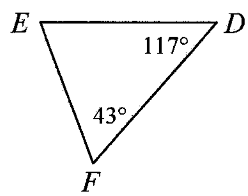
b. What cross-section do you get if you slice the cone horizontally?

c. Lois is thinking of a shape. She says that no matter how you slice it, the cross-section will always be a circle. What shape is she thinking of? Draw and describe this shape on your paper.

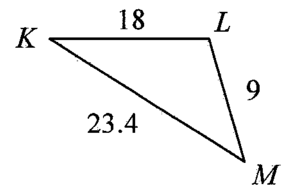
11-54. For each triangle below, decide if it is similar to the triangle at right. If it is similar, **justify** your conclusion and complete the similarity statement $\triangle ABC \sim \triangle$______ . If the triangle is not similar, explain how you know. Assume that the diagrams are not drawn to scale.



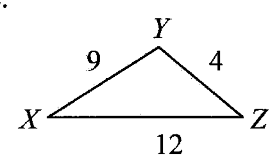
a.



b.



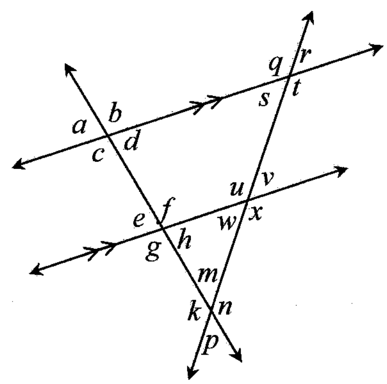
c.



11-55. On graph paper, graph the system of equations at right. Then list all points of intersection in the form (x, y).

$$x^2 + y^2 = 25$$
$$y = x + 1$$

11-56. **Examine** the diagram at right. State the relationship between each pair of angles listed below (such as "vertical angles") and state whether the angles are congruent, supplementary, or neither.



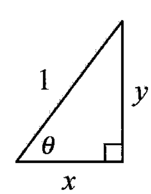
a. $\measuredangle e$ and $\measuredangle a$

b. $\measuredangle t$ and $\measuredangle u$

c. $\measuredangle v$ and $\measuredangle x$

d. $\measuredangle g$ and $\measuredangle v$

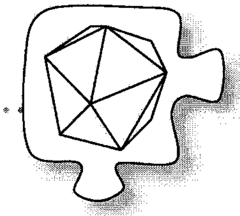
11-57. **Multiple Choice:** In the diagram at right, the value of y is:



a. $\sin\theta$ b. $\cos\theta$ c. $\tan\theta$

d. x e. None of these

11.1.5 What's the relationship?



Surface Area and Volume of a Sphere

This lesson will complete your three-dimensional shape toolkit. You will learn about a new shape that you encounter often in your daily life: a **sphere**. You will also make connections between a cylinder, cone, and sphere of the same radius and height.

As you work with your team, keep the following focus questions in mind:

What's the relationship?

What other tools or information do I need?

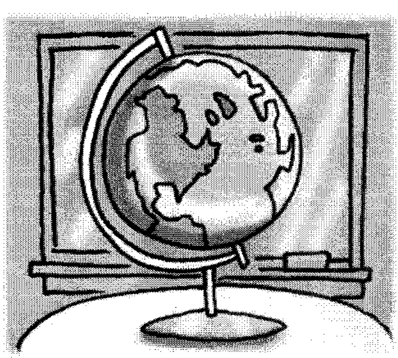
11-58. Alonzo was blowing bubbles to amuse his little sister. He wondered, "Why are bubbles always perfectly round?"

a. Discuss Alonzo's question with the class. Why are free-floating bubbles always shaped like a perfectly round ball?

b. The shape of a bubble is called a **sphere**. What other objects can you remember seeing that are shaped like a sphere?

c. What shapes are related to spheres? How are they related?

11-59. GEOGRAPHY LESSON, Part One



Alonzo learned in his geography class that about 70% of the Earth's surface is covered in water. "That's amazing!" he thought. This information only made him think of new questions, such as "What is the area of land covered in water?", "What percent of the Earth's surface is the United States?", and "What is the volume of the entire Earth?"

Discuss Alonzo's questions with your team. Decide:

- What facts about the Earth would be helpful to know?
- What do you still need to learn to answer Alonzo's questions?

11-60. In order to answer his questions, Alonzo decided to get out his set of plastic geometry models. He has a sphere, cone, and cylinder that each has the same radius and height.

a. Draw a diagram of each shape.

b. If the radius of the sphere is r, what is the height of the cylinder? How do you know?

c. Alonzo's models are hollow and are designed to hold water. Alonzo was pouring water between the shapes, comparing their volumes. He discovered that when he poured the water in the cone and the sphere into the cylinder, the water filled up the cylinder without going over! Determine what the volume of the sphere must be if the radius of the sphere is r units. Show all work.

11-61. Now that Alonzo knows that spheres, cylinders, and cones with the same height and radius are related, he decides to **examine** the surface area of each one. As he paints the exterior of each shape, he notices that the lateral surface area of the cylinder and the surface area of the sphere take exactly the same amount of paint! If the radius of the sphere and cylinder is r, what is the surface area of the sphere?

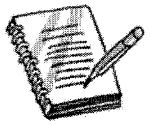
11-62. GEOGRAPHY LESSON, Part Two

Now that you have **strategies** for finding the volume and surface area of a sphere, return to problem 11-59 and help Alonzo answer his questions. That is, determine:

- the area of the Earth's surface that is covered in water.
- the percent of the Earth's surface that lies in the United States.
- the volume of the entire Earth.

Don't forget that in Chapter 10, you determined that the radius of the Earth is about 4,000 miles! Alonzo did some research and discovered that the land area of the United States is approximately 3,537,438 square miles.

11-63. Write an entry in your Learning Log describing the relationships between the volumes of a cube, cylinder, and sphere with the same radius and height. Also be sure to explain how to find the surface area and volume of a sphere and include an example of each. Title this entry "Surface Area and Volume of a Sphere" and include today's date.

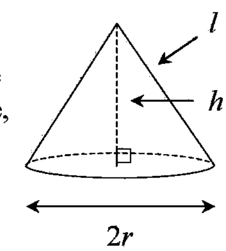


METHODS AND MEANINGS

MATH NOTES

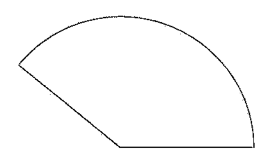
Volume and Lateral Surface of a Cone

Finding the volume of a cone (defined in problem 11-48) is very similar to finding the volume of a pyramid. The volume of a cone is one-third of the volume of the cylinder with the same radius and height. Therefore, the volume of a cone can be found using the formula shown below, where r is the radius of the base and h is the height of the cone.



$$V = \frac{1}{3}(\text{Base Area})(\text{Height}) = \frac{1}{3}\pi r^2 h$$

To find the lateral surface area of a cone, imagine unrolling the lateral surface of the cone to create a sector. The radius of the sector would be the slant height, l, of the cone, and the arc length would be the circumference of the base of the cone, $2\pi r$.



Therefore, the area of the sector (the lateral surface area of the cone) is:

$$LA = \frac{2\pi r}{2\pi l}\pi l^2 = \pi r l$$

Review & Preview

11-64. As Shannon peeled her orange for lunch, she realized that it was very close to being a sphere. If her orange has a diameter of 8 centimeters, what is its approximate surface area (the area of the orange peel)? What is the approximate volume of the orange? Show all work.

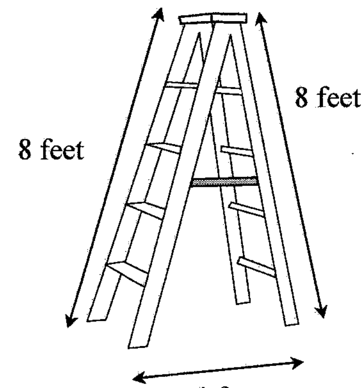
11-65. Review what you know about polyhedra as you answer the questions below. Refer to the table in problem 11-4 if you need help.

a. Find the total surface area of a regular icosahedron if the area of each face is 45 mm^2. Explain your method.

b. The total surface area of a regular dodecahedron is 108 cm^2. What is the area of each face?

c. A regular tetrahedron has an edge length of 6 inches. What is its total surface area? Show all work.

11-66. Hokiri's ladder has two legs that are each 8 feet long. When the ladder is opened safely and locked for use, the legs are 4 feet apart on the ground. What is the angle that is formed at the top of the ladder where the legs meet?



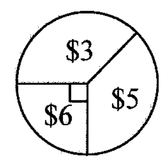
11-67. Find the area of the region that represents the solution of the inequality $x^2 + y^2 \le 72$.

11-68. Solve each system of equations below. Write the solution in the form (x, y). Show all work.

a. $y + 3x = 14$
$y - 3x = 6$

b. $y = 6 - 3x$
$2x + y = 7$

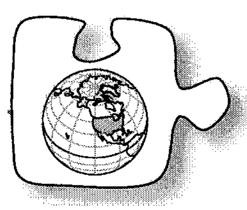
11-69. **Multiple Choice:** The probability of winning \$3 on the spinner at right is equal to the chance of winning \$5. Find the expected value for one spin.



a. \$3.00 b. \$4.50 c. \$4.67

d. \$6.00 e. None of these

11.2.1 Where's this location?



Coordinates on a Sphere

As you learned in Chapter 1, the word *geometry* literally means "measurement of the Earth." In fact, so far in this course, you have used your geometric tools to learn more about Earth. For example, in Lesson 11.1.5, you learned that the United States only makes up 1.8% of the Earth's surface. Also, in Lesson 10.1.1, you learned how Eristothenes used shadows to estimate the Earth's radius.

Today, you will **examine** other earthly questions that can be answered using geometry. Since we can approximate the shape of the earth as a sphere, you will be able to use many of the tools you have used previously. As you work with your team, consider the following focus questions:

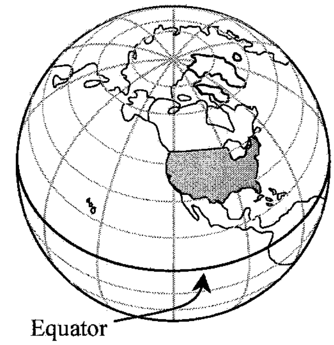
What **strategy** or tool can I use?

Is there another way?

Does this **strategy** always work?

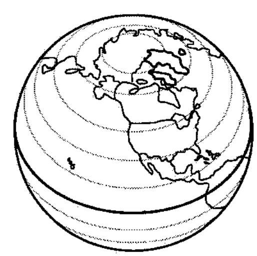
11-70. YOU ARE HERE

In order to help a person describe a location on the Earth, scientists have developed a reference grid on the planet's surface, referred to as **longitude** and **latitude lines**. While these reference markings are referred to as "lines," they are technically circles that wrap around the Earth.

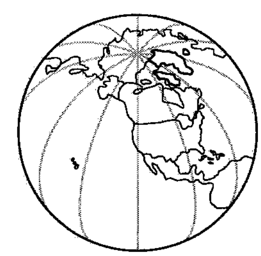


Lines of longitude extend north and south, while lines of latitude extend east and west, as shown in the diagrams below. These lines help to mark off arc measures on the planet's surface. In the diagrams below, the lines of latitude are marked every 15° while the lines of longitude are marked every 30°. The most famous line of latitude is the **equator**, which separates the Earth into two **hemispheres** (half a sphere).

a. The equator is an example of a **great circle**, which means that it is a circle that lies on the sphere and has the same diameter as the sphere. Compare the equator with the other lines of latitude. What do you notice?

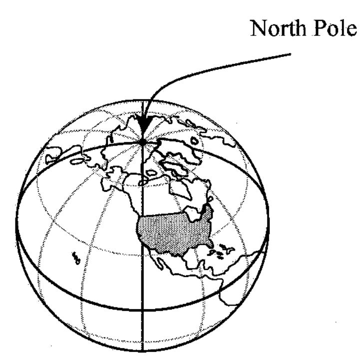


Lines of Latitude



Lines of Longitude

b. Is it possible for two great circles on the same sphere to intersect? If so, draw an example on your paper. If not, explain why not.



c. On the sphere provided by your teacher, carefully draw circles to represent the lines of latitude (every 30°) and longitude (every 30°) on the Earth. Highlight the equator by making it darker or a different color than the other lines of latitude. Also choose one line of longitude to represent 0° (called the **prime meridian**, which passes through Greenwich, England, on the eastern edge of London) and highlight it as well.

d. Norman is exactly 1 mile north of Sula. If they both travel at the same rate due west, will their paths cross? Why or why not? Assume that people can travel over water and all types of terrain.

e. Erin is exactly 1 mile east of Wilber. If they both travel due south at the same rate, will their paths cross? Why or why not?

11-71. DEAR PEN-PAL

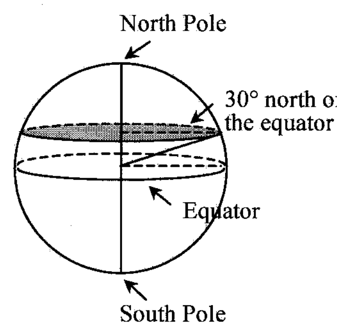
Brianna, who lives in New Orleans, has been writing to her pen-pal in Jacksonville, Florida. "Gosh," she wonders, "How far away is my friend?"

a. On your "globe" from problem 11-70, locate Brianna's home. (New Orleans, LA, is approximately 90° west of the prime meridian and 30° north of the equator.) Mark it with a pushpin.

b. Now, with a second pushpin, mark the location of Brianna's friend, if Jacksonville is 82° west of the prime meridian and 30° north of the equator. Use a rubber band to locate the circle with the smallest radius that passes through these two locations.

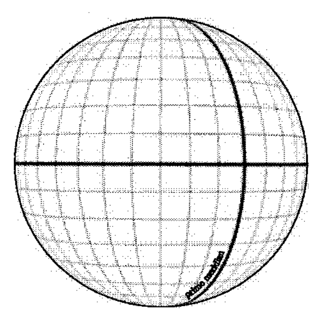
c. What is the measure of the arc connecting these two cities? Show how you know.

d. Brianna thinks that if she knew the circumference of the circle marked with the rubber band, then she could use the arc measure to approximate the distance between the two cities. The shaded circle in the diagram at right represents the cross-section of the earth 30° above the equator. If the radius of the earth is approximately 4000 miles, find the circumference of the shaded circle.



e. Find the distance between New Orleans and Jacksonville.

11-72. Obtain a Lesson 11.2.1 Resource Page from your teacher. On it, mark and label the following locations.



a. London, England, which is on the prime meridian and is approximately 51° **north** of the equator.

b. Narsarssuaq, Greenland, which is approximately 45° **west** of the prime meridian and 61° **north** of the equator.

c. Quito, Equador, which is on the equator and is approximately 79° **west** of the prime meridian.

d. Cairo, Egypt, which is approximately 31° **east** of the prime meridian and 30° **north** of the equator.

e. Buenos Aires, Argentina, which is approximately 58° **west** of the prime meridian and 35° **south** of the equator.

11-73. EXTENSION

a. If a polar bear travels 1 mile south from the North Pole, travels one mile east, and then travels one mile north, where does it end up? Explain what happens and why.

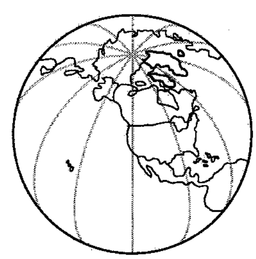
b. Is there another location the polar bear could have started from so that it still ends up where it started after following the same directions? Explain.

MATH NOTES

LOOKING DEEPER

Meridian and Time Zones

The reference lines connecting the north and south poles are called **lines of longitude**, as shown in the diagram at right. These lines help navigators determine how many degrees east or west they have traveled.



Lines of Longitude

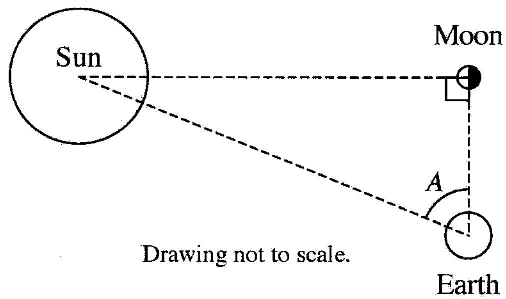
Another name for these lines of longitude is **meridian**, which is Latin from "medius" (which means "middle") and "diem" (which means "day"). Meridian also used to refer to noon, since it was the time the sun was directly overhead. In the morning, it was "ante meridian" or before noon. This is where the abbreviation **a.m.** comes from. Likewise, **p.m.** is short for "post meridian," which means "after noon."

Review & Preview

11-74. The moon is an average distance of 238,900 miles away from the Earth. While that seems very far, how far is it?

a. Compare that distance with the circumference of the Earth's equator. Assume that the Earth's radius is 4000 miles. How many times greater than the Earth's circumference is the distance to the moon?

b. One way to estimate the distance between the Earth and the sun is to consider the triangle formed by the sun, Earth, and moon when the moon appears to be half-full. (See the diagram at right.) When the moon appears from earth to be half-full, it can be assumed that the moon forms a 90° angle with the sun and the Earth.



Using special equipment, Ray found the measure of angle A to be 89.85°. If the moon is 238,900 miles away from the Earth, then how far is the sun from the Earth?

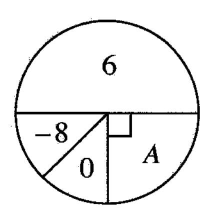
11-75. The length of chord $\overline{AB}$ in $\odot D$ is 9 mm. If the $m\overset{\frown}{AB} = 32°$, find the length of $\overset{\frown}{AB}$. Draw a diagram.

11-76. On your paper, draw a diagram of a square-based pyramid if the side length of the base is 9 cm and the height of the pyramid is 12 cm.

a. Find the volume of the pyramid.

b. If a smaller pyramid is similar to the pyramid in part (a), but has a linear scale factor of $\frac{1}{3}$, find its volume.

11-77. **Examine** the spinner at right. Assume that the probability of spinning a –8 is equal to that of spinning a 0.

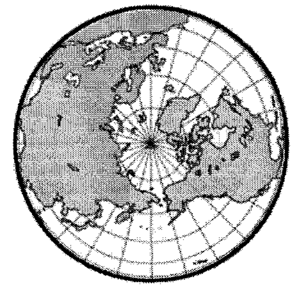


a. Find the spinner's expected value if the value of region A is 8.

b. Find the spinner's expected value if the value of region A is –4.

c. What does the value of region A need to be so that the expected value of the spinner is 0?

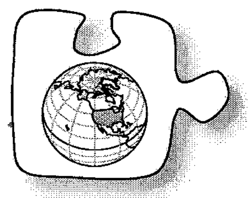
11-78. In the picture of a globe at right, the lines of latitude are concentric circles. Where else might you encounter concentric circles?



11-79. **Multiple Choice:** The volume of a solid is V. If the solid is enlarged proportionally so that the perimeter increases by a factor of 9, what is the volume of the enlarged solid?

a. $9V$ b. $\frac{81}{4}V$ c. $81V$ d. $729V$

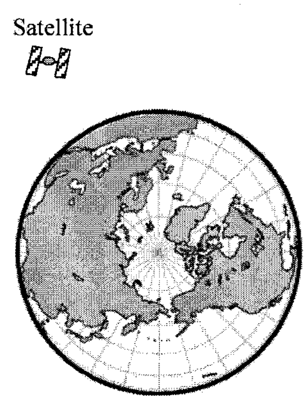
11.2.2 What's the relationship?



Tangents and Arcs

Today, you will develop new geometric tools as you continue to study the Earth and its measure.

11-80. EYE IN THE SKY



Did you know that as of 1997, over 8000 operating satellites orbited the Earth performing various functions such as taking photographs of our planet? One way scientists learn more about the Earth is to carefully **examine** photographs that are taken by an orbiting satellite.

However, how much of the Earth can a satellite see? What does this depend on? In other words, what information would you need to know in order to figure out how much of the planet is in view of a satellite in space? Discuss this with your team and be ready to share your ideas with the rest of the class.

11-81. On the Lesson 11.2.2 Resource Page obtained from your teacher, locate Satellites A, B, and C.

a. On the resource page, draw an angle from Satellite A that shows the portion of the Earth's equator that is visible from the satellite. What is the relationship of the sides of the angle and the circle that represents the equator of the Earth?

Problem continues on next page →

11-81. *Problem continued from previous page.*

b. Draw a quadrilateral *ADEF* that connects Satellite A, the points of tangency, and the center of the Earth (point *E*). If the measure of the angle at Satellite A is 90°, what is the measure of the equator's arc that is in view? Explain how you know.

c. What is the relationship of AD and AF? Prove the relationship using congruent triangles.

d. If $m\measuredangle A = 90°$ and the radius of the Earth is 4000 miles, how far above the surface of the planet is Satellite A?

11-82. What if the satellite is placed higher in orbit? Consider this as you answer the questions below.

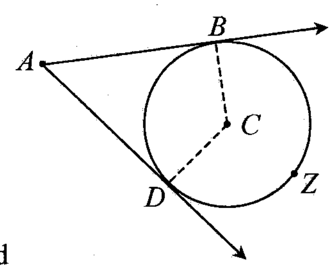
a. Using a different colored pen or pencil, draw the viewing angle from Satellite B on the Lesson 11.2.2 Resource Page. Label the points of tangency *G* and *H*. Will Satellite B see more or less of the Earth's equator than Satellite A?

b. If $m\measuredangle B = 60°$, find the length of the equator in view of Satellite B. Assume that the radius of the Earth is 4000 miles.

c. Use a third color to draw the viewing angle from Satellite C on the resource page. Label the points of tangency *J* and *K*. If $m\measuredangle C = 45°$, find the $m\overset{\frown}{JK}$ and $m\overset{\frown}{JZK}$.

d. Is it possible for a satellite to see 50% of the Earth's equator? Why or why not?

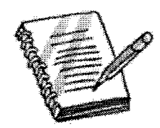
11-83. HOW ARE THEY RELATED?



In problems 11-81 and 11-82, you found the measures of angles and arcs formed by two tangents to a circle that intersect each other.

a. Copy the diagram at right onto your paper. Using intuition, describe how the measure of the angle formed by the tangents ($m\measuredangle A$) seems to be related to the measures of the major and minor arcs formed by the points of tangency ($\overset{\frown}{BD}$ and $\overset{\frown}{BZD}$).

b. If $m\measuredangle A = a$, find $m\overset{\frown}{BD}$ and $m\overset{\frown}{BZD}$ in terms of *a*. Compare the measure of the angle with the measure of the major and minor arcs. What do you notice?



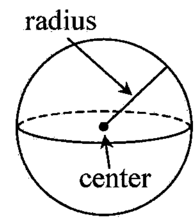
c. Write an entry in your Learning Log describing the relationship between the angles and arcs formed by two intersecting tangents to a circle. Also record what you found out about the lengths of the tangents from the point of tangency to their point of intersection. Title this entry "Tangents and Arcs" and include today's date.

MATH NOTES

METHODS AND MEANINGS

Volume and Lateral Surface of a Sphere

A **sphere** is a three-dimensional solid formed by points that are equidistant from its center.



The **volume of a sphere** is twice the volume of a cone with the same radius and height. Since the volume of a cone with radius r and height $2r$ is $V = \frac{1}{3}\pi r^2(2r) = \frac{2}{3}\pi r^3$, the volume of a sphere with radius r is:

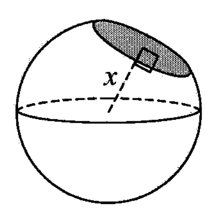
$$V = \frac{4}{3}\pi r^3$$

The **surface area of a sphere** is four times the area of a circle with the same radius. Thus, the surface area of a sphere with radius r is:

$$SA = 4\pi r^2$$

Review & Preview

11-84. While making his lunch, Alexander sliced off a portion of his grapefruit. If the area of the cross-section of the slice (shaded at right) was 3 in^2, and if the diameter of the grapefruit was 5 inches, find the distance between the center of the grapefruit and the slice. Assume the grapefruit is a sphere.

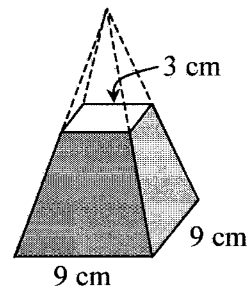


11-85. The approximate surface areas of the seven Earth continents are shown in the table at right. If the radius of the Earth's moon is approximately 1080 miles, how would its surface area compare with the size of the continents?

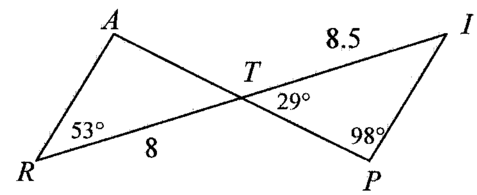
Continent	Area (sq. miles)
Asia	17,212,048.1
Africa	11,608,161.4
North America	9,365,294.0
South America	6,879,954.4
Antarctica	5,100,023.4
Europe	3,837,083.3
Australia/Oceania	2,967,967.3

11-86. Find the area of the regular decagon if the length of each side is 20 units.

11-87. The solid at right is an example of a **truncated pyramid**. It is formed by slicing and removing the top of a pyramid so that the slice is parallel to the base of the pyramid. If the original height of the square-based pyramid at right was 12 cm, find the volume of this truncated pyramid. (Hint: you may find your results from problem 11-76 useful.)



11-88. **Examine** the triangles at right. Are they similar? Are they congruent? Explain how you know. Then write an appropriate similarity or congruence statement.



11-89. **Multiple Choice:** Which shape below has the <u>least</u> area?

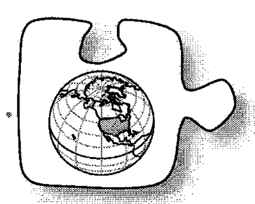
a. A circle with radius 5 units

b. A square with side length 9 units

c. A trapezoid with bases of length 8 and 10 units and height of 9 units

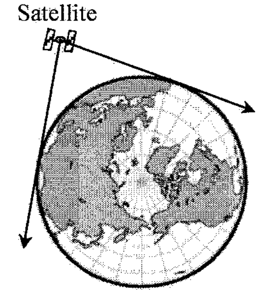
d. A rhombus with side length 9 units and height of 8 units.

11.2.3 What is the measure?



Secant and Tangent Relationships

In Lesson 11.2.2, you studied the angles and arcs formed by tangents when a satellite orbits the Earth, as shown in the diagram at right. Today, you will consider a related question: What if the sides of the angle intersect the circle more than once? What are the relationships between the angles and arcs formed when this happens? And what can you learn about the lengths of the segments created by the points of intersection?



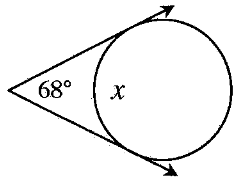
As you work with your team, carefully record your team's conjectures. And while you work, keep the following questions in mind:

What patterns do I see?

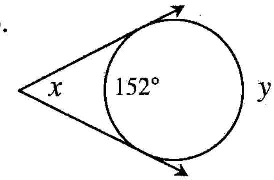
Is this relationship always true?

11-90. Review what you learned in Lesson 11.2.2 by solving for the given variables in the diagrams below. Show all work.

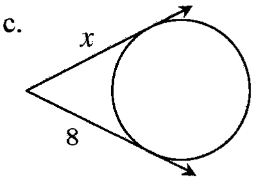
a.



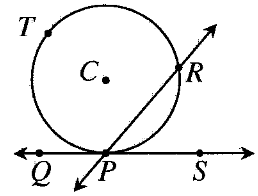
b.



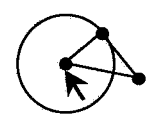
c.



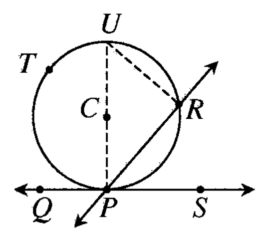
11-91. While a tangent is a line that intersects a circle (like $\odot C$ in the diagram at right) at exactly one point, a **secant** is a line that intersects a circle twice. $\overleftrightarrow{PR}$ is an example of a secant, while $\overleftrightarrow{QS}$ is an example of a tangent.



a. What happens to the measure of the angles and arcs when a secant intersects the circle at the point of tangency? Namely, how are the angles located at P in the diagram above related to $m\overset{\frown}{PR}$ and $m\overset{\frown}{PTR}$? First make an educated guess. Then test your ideas out using a dynamic geometry tool. Write a conjecture and be ready to share it with the class.



b. Uri wants to prove his conjecture from part (a) for a non-special secant (meaning that $\overleftrightarrow{PR}$ is not a diameter). He decided to extend a diameter from point P and to create an inscribed angle that intercepts $\overset{\frown}{PR}$. With your team, **examine** Uri's diagram carefully and consider all the relationships you can identify that could be useful.



To get you started, use the list below.

i. What is $m\measuredangle URP$?

ii. How is $m\measuredangle RUP$ related to $m\overset{\frown}{PR}$?

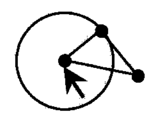
iii. What is the sum of the angles in $\triangle PRU$?

iv. How are $\measuredangle UPR$ and $\measuredangle RPS$ related?

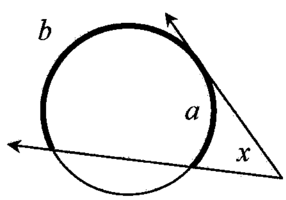
c. Using the relationships you explored in part (b), prove that if $m\measuredangle RPS = x$ in the diagram from part (b), then $m\overset{\frown}{PR} = 2x$. Remember to **justify** each step.

11-92. Uri now has this challenge for you: *What happens when secants and tangents intersect outside a circle?* To consider this, you need to **examine** two separate cases: One is when a secant and tangent intersect outside a circle (case *i* below). The other is when two secants intersect outside a circle (case *ii* below). As with your earlier **investigations**,

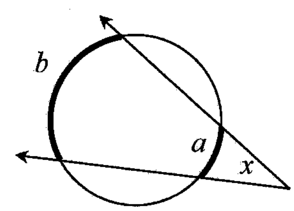
- First make a prediction about the relationship between the measures of x, a, and b for each case.
- Then use your dynamic geometry tool to test your conjectures.
- For each case, write an algebraic statement (equation) that relates x, a, and b. Be ready to share each equation with the rest of the class.



i.

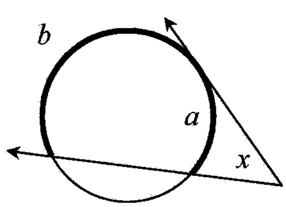


ii.

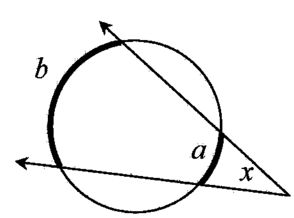


11-93. Now prove your conjectures from problem 11-92. For each diagram, add a line segment that will help to create an inscribed angle. Then use angle relationships (such as the sum of the angles of a triangle must be 180°) to then find the measures of all the angles in terms of x, a, and b. Be sure to show that in each case, $x = \frac{b-a}{2}$. Remember to **justify** each statement.

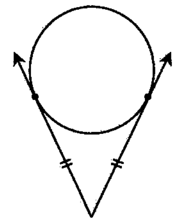
i.



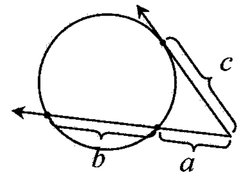
ii.



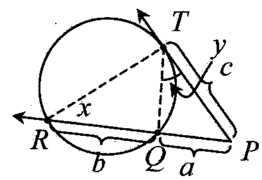
11-94. Camille is interested in the lengths of segments that are created by the points of intersection. She remembers proving in Lesson 11.2.2 that the lengths of the tangents between their intersection and the points of tangency are equal, as shown in the diagram at right. She figures that there must be some relationships in the lengths created by the intersections of secants, too.



a. The first case she wants to consider is when a tangent and secant intersect outside a circle, as shown in the figure at right. Copy this diagram onto your paper.



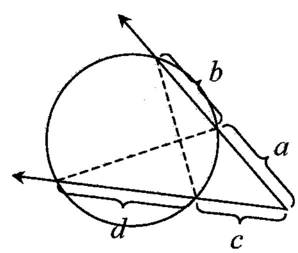
b. "To find a relationship, I think we need to add some line segments to create some inscribed angles and triangles," Camille tells her team. She decides to add the line segments shown at right. Show why the angles marked x and y must be congruent.



c. "If some of these triangles are similar, I can use that to find a relationship between these side lengths," Camille explains. Help her prove that $\Delta PQT \sim \Delta PTR$.

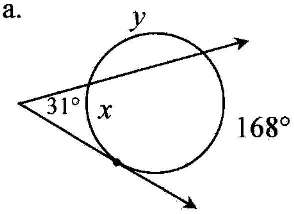
d. Use the fact that $\Delta PQT \sim \Delta PTR$ to write a proportion using a, b, and c. Simplify this equation as much as possible to find an equation that helps you understand the relationship between a, b, and c.

e. Use the same process to find the relationship between the lengths created when two secants intersect outside a circle. Two extra segments have been added to the diagram to help create similar triangles. Be ready to **justify** your relationship.

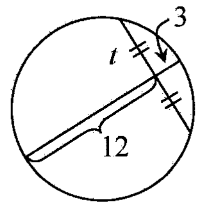


11-95. Use all your circle relationships to solve for the variables in each of the diagrams below.

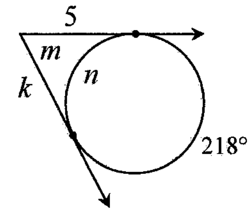
a.



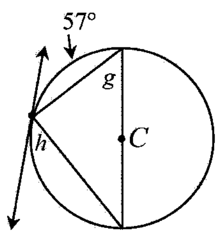
b.



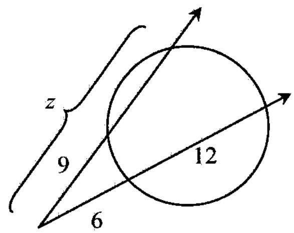
c.



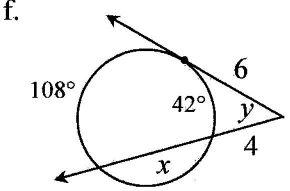
d. C is the center



e.



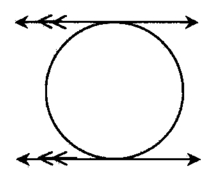
f.



MATH NOTES

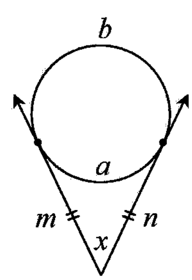
METHODS AND MEANINGS

Intersecting Tangents



Two lines that are tangent to the same circle may not intersect. When this happens, the tangent lines are parallel, as shown in the diagram at right. The arcs formed by the points of tangency are both 180°.

However, when the lines of tangency intersect outside the circle, some interesting relationships are formed. For example, the lengths m and n from the point of intersection to the points of tangency are equal.



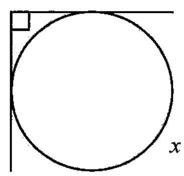
The angle and arcs are related to the angle outside the circle as well. If x is the measure of the angle formed by the intersection of the tangents, a represents the measure of the minor arc, and b represents the measure of the major arc, then:

$$a = 180° - x \quad \text{and} \quad b = 180° + x$$

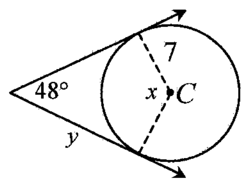
Review & Preview

11-96. Solve for the variables in each of the diagrams below. Assume point C is the center of the circle in part (b).

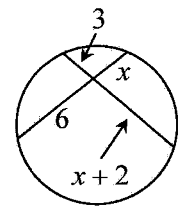
a.



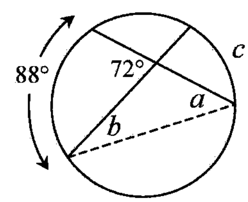
b.



c.



11-97. In part (c) of problem 11-96, you used the relationship between the segment lengths formed by intersecting chords to find a missing length. But how are the arc measures of two random intersecting chords related? **Examine** the diagram at right.



a. Solve for a, b, and c using what you know about inscribed angles and the sum of the angles of a triangle.

b. Compare the result for c with 88° and 72°. Is there a relationship?

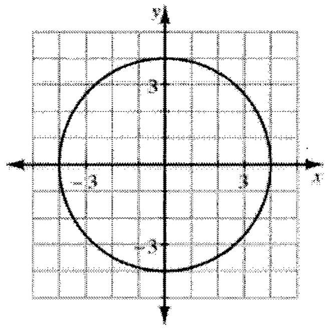
11-98. Perhaps you think the Earth is big? Consider the sun!

a. Assume that the radius of the Earth 4000 miles. The sun is approximately 109 times as wide. Find the sun's radius.

b. The distance between the Earth and the moon is 238,900 miles. Compare this distance with the radius of the sun you found in part (a).

c. If the sun were hollow, how many Earths would fill the inside of it?

11-99. Write the equation for the graph at right.

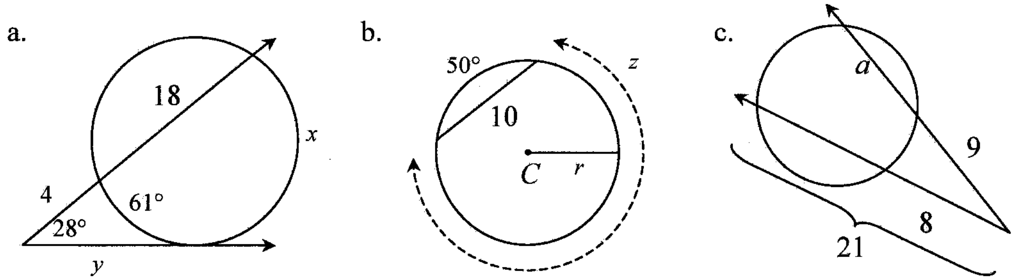


11-100. On your paper, draw a diagram of a square-based pyramid. If the base has side length 6 units and the height of the pyramid is 10 units, find the total surface area. Show all your work.

11-101. **Multiple Choice:** Which of the following cannot be the measure of an exterior angle of a regular polygon?

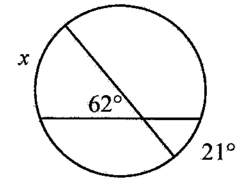
a. 18° b. 24° c. 28° d. 40°

11-102. Solve for the variables in each of the diagrams below. Assume that point C is the center of the circle in part (b).



11-103. Which has greater volume: a cylinder with radius 38 units and height 71 units or a rectangular prism with dimensions 34, 84, and 99 units? Show all work and support your **reasoning**.

11-104. Copy the diagram at right onto your paper. Use the process from problem 11-97 to find the measure of x. Show all work.



11-105. Mr. Kyi has placed 3 red, 7 blue, and 2 yellow beads in a hat. If a person selects a red bead, he or she wins \$3. If that person selects a blue bead, he or she loses \$1. If the person selects a yellow bead, he or she wins \$10. What is the expected value for one draw?

11-106. A solar eclipse occurs when the moon passes between the Earth and the sun and is perfectly aligned so that it blocks the Earth's view of the sun.

Note: This diagram is not to scale.

How do scientists figure out what areas of the Earth will see an eclipse? To find out, copy the diagram above onto your paper. Then use tangents representing the sun's rays to find the portion of the Earth's equator that will see the total eclipse.

11-107. **Multiple Choice:** The Mona Lisa, by Leonardo da Vinci, is arguably the most famous painting in existence. The rectangular artwork, which hangs in the Musée du Louvre, measures 77 cm by 53 cm. When the museum created a billboard with an enlarged version of the portrait for advertisement, they used a linear scale factor of 20. What was the area of the billboard?

a. 4081 cm^2 b. 32,638,000 cm^2

c. 81,620 cm^2 d. 1,632,400 cm^2

e. None of these

Chapter 11 Closure What have I learned?

Reflection and Synthesis

The activities below offer you a chance to reflect on what you have learned during this chapter. As you work, look for concepts that you feel very comfortable with, ideas that you would like to learn more about, and topics you need more help with. Look for **connections** between ideas as well as **connections** with material you learned previously.

① TEAM BRAINSTORM

With your team, brainstorm a list for each of the following three topics. Be as detailed as you can. How long can you make your list? Challenge yourselves. Be prepared to share your team's ideas with the class.

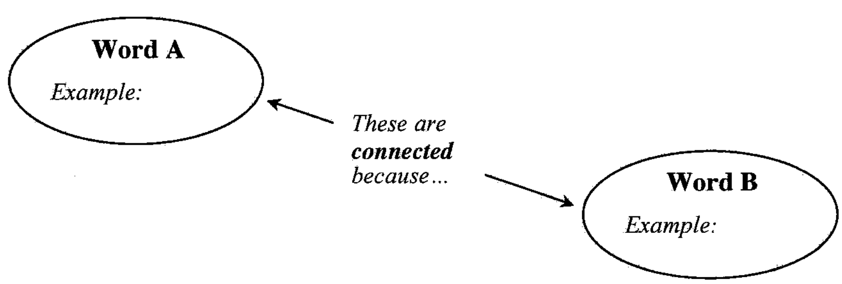
Topics:	What have you studied in this chapter? What ideas and words were important in what you learned? Remember to be as detailed as you can.
Problem Solving:	What did you do to solve problems? What different **strategies** did you use?
Connections:	How are the topics, ideas, and words that you learned in previous courses are **connected** to the new ideas in this chapter? Again, make your list as long as you can.

② MAKING CONNECTIONS

The following is a list of the vocabulary used in this chapter. The words that appear in bold are new to this chapter. Make sure that you are familiar with all of these words and know what they mean. Refer to the glossary or index for any words that you do not yet understand.

arc	base	circle
cone	**cross-section**	**cube**
cylinder	diameter	**edge**
equator	**face**	**great circle**
height	**hemisphere**	lateral face
latitude	**longitude**	oblique
octahedron	**platonic solid**	**polyhedron**
pyramid	radius	secant
slant height	**sphere**	surface area
tangent	**tetrahedron**	volume

Make a concept map showing all of the **connections** you can find among the key words and ideas listed above. To show a **connection** between two words, draw a line between them and explain the **connection**, as shown in the example below. A word can be **connected** to any other word as long as there is a **justified connection**. For each key word or idea, provide a sketch of an example.



Your teacher may provide you with vocabulary cards to help you get started. If you use the cards to plan your concept map, be sure either to re-draw your concept map on your paper or to glue the vocabulary cards to a poster with all of the **connections** explained for others to see and understand.

While you are making your map, your team may think of related words or ideas that are not listed above. Be sure to include these ideas on your concept map.

③ SUMMARIZING MY UNDERSTANDING

This section gives you an opportunity to show what you know about certain math topics or ideas. Your teacher will give you directions for exactly how to do this.

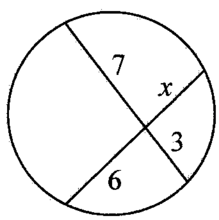
④ WHAT HAVE I LEARNED?

This section will help you evaluate which types of problems you have seen with which you feel comfortable and those with which you need more help. This section will appear at the end of every chapter to help you check your understanding. Even if your teacher does not assign this section, it is a good idea to try these problems and find out for yourself what you know and what you need to work on.

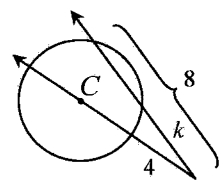
Solve each problem as completely as you can. The table at the end of this closure section has answers to these problems. It also tells you where you can find additional help and practice on problems like these.

CL 11-108. Use all your circle relationships to solve for the variables in each of the diagrams below. Assume that C is the center of the circle for parts (b) and (c).

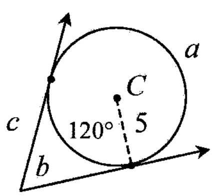
a.



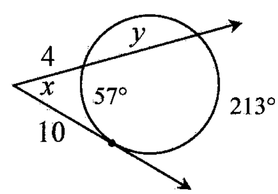
b. The area of $\odot C$ is 25π un^2



c.



d.



CL 11-109. The radius of the moon is approximately 1738 km. Draw a diagram of the moon on your paper.

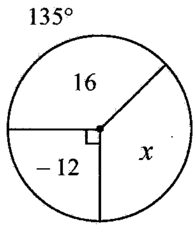
a. If all the Earth's water were distributed on the surface of the moon, it would be about 33.6 km deep! How much water is on the Earth?

b. If all of this water were to be collected and reshaped into a gigantic spherical drop out in space, what would its radius be?

CL 11-110. On the same set of axes, graph the equations below. Name all points of intersection.

$$x^2 + y^2 = 100$$
$$y = \frac{1}{2}x + 5$$

CL 11-111. **Examine** the spinner at right.



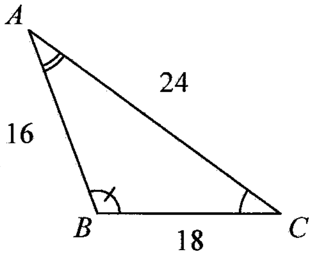
a. Find the expected value of the spinner if $x = 4$.

b. Find the expected value of the spinner if $x = -8$.

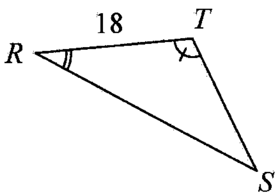
c. Find x so that the expected value of the spinner is 6.

CL 11-112. Find the volume of a pyramid if its base is a regular pentagon with perimeter 20 units and if its height is 7 units.

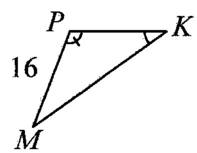
CL 11-113. **Examine** the triangles below. Based on the markings and measurements provided in the diagrams, which are similar to $\triangle ABC$ at right? Which are congruent? Are there any that you cannot determine? **Justify** your conclusion and, if appropriate, write a similarity or congruence statement. **Note:** The diagrams are not drawn to scale.



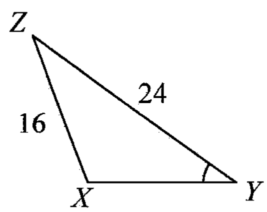
a.



b.



c.

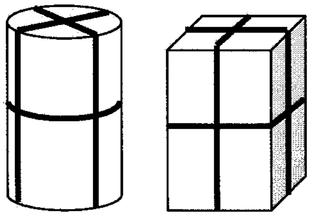


CL 11-114. Talila is planning on giving her geometry teacher a gift. She has two containers to choose from:

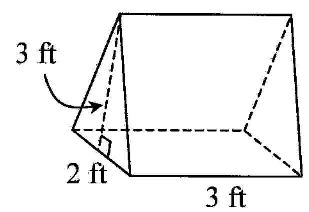
- A cylinder tube with diameter 6 inches and height 10 inches
- A rectangular box with dimensions 5 inches by 6 inches by 9 inches

a. Assuming that her gift can fit in either box, which will require the least amount of wrapping paper?

b. She plans to tie two loops of ribbon about the package as shown at right. Which package will require the least amount of ribbon? Ignore any ties or bows.



CL 11-115. A big warehouse carrying tents has a miniature model that is similar to the full-sized tent. The tent is a triangular-based prism and the miniature model has dimensions shown in the diagram at right.



a. How much fabric does the small tent use? That is, what is its surface area?

b. What is the volume of the small model?

c. If the volume of the full-sized model is 72 ft^3, how tall is the full-sized tent?

d. How much fabric does the full-sized tent use?

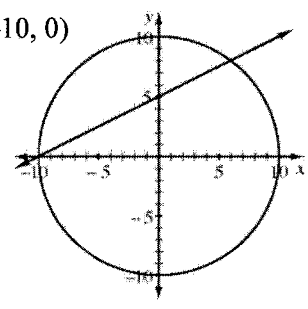
CL 11-116. Check your answers using the table at the end of the closure section. Which problems do you feel confident about? Which problems were hard? Use the table to make a list of topics you need help on and a list of topics you need to practice more.

⑤ HOW AM I THINKING?

This course focuses on five different **Ways of Thinking**: investigating, examining, visualizing, choosing a strategy/tool, and reasoning and justifying. These are some of the ways in which you think while trying to make sense of a concept or to solve a problem (even outside of math class). During this chapter, you have probably used each Way of Thinking multiple times without even realizing it!

Choose three of these Ways of Thinking that you remember using while working in this chapter. For each Way of Thinking that you choose, show and explain where you used it and how you used it. Describe why thinking in this way helped you solve a particular problem or understand something new. Be sure to include examples to demonstrate your thinking.

Answers and Support for Closure Activity #4
What Have I Learned?

Problem	Solution	Need Help?	More Practice
CL 11-108.	a. $x = 3.5$ b. $k = 7$ c. $a = 240°$, $b = 60°$, $c = 5\sqrt{3}$ d. $x = 78°$, $y = 21$	Lessons 10.1.2, 10.1.3, 10.1.4, and 11.2.3 Math Notes box, problem 10-35	Problems 10-7, 10-15, 10-26, 10-37, 10-52, 10-59, 10-93, 11-43, 11-95, 11-96, 11-97, 11-102, 11-104
CL 11-109.	a. 1,300,222,453 km b. 677.1 km	Lesson 11.2.2 Math Notes box	Problems 11-62, 11-64, 11-98
CL 11-110.	(6, 8) and (–10, 0) 	Lesson 10.3.1	Problems 10-88, 10-89, 10-90, 11-16, 11-40, 11-41, 11-55, 11-67, 11-99
CL 11-111.	a. 4.5 b. 0 c. $x = -8$	Lesson 10.3.1 Math Notes box	Problems 10-55, 10-64, 10-66, 10-67, 10-69, 10-75, 10-76, 10-77, 10-81, 10-92, 11-23, 11-69, 11-77, 11-105
CL 11-112.	$V \approx 64.23$ un^3	Lessons 8.1.4, 8.1.5, 8.3.1, 11.1.2, and 11.1.4 Math Notes boxes	Problems 8-45, 8-47, 8-48, 8-67, 8-85, 9-9, 10-41, 11-31, 11-36, 11-46, 11-76

Problem	Solution	Need Help?	More Practice
CL 11-113.	a. $\Delta ABC \sim \Delta RTS$ (AA ~) b. $\Delta ABC \cong \Delta MPK$ (AAS ≅) c. Cannot be determined because there are two possible triangles when SSA is given.	Lessons 3.2.1, 3.2.2, and 3.2.5 Math Notes boxes	Problems 7-6, 7-14, 7-28, 7-53, 7-77, 7-87, 7-104, 8-32, 8-54, 8-80, 9-12, 9-61, 10-32, 10-96, 11-54
CL 11-114.	a. The cylinder needs less paper ($SA = 78\pi$ in^2) b. The prism requires less ribbon (80 inches)	Lessons 8.3.2, 9.1.2, and 9.1.3 Math Notes boxes	Problems 9-15, 9-17, 9-25, 9-26, 9-27, 9-28, 9-33, 9-40, 9-59, 9-89, 10-6, 10-41, 10-95, 11-9, 11-15, 11-42, 11-103
CL 11-115.	a. SA ≈ 31.0 ft^2 b. V ≈ 9 ft^3 c. linear scale factor = 2, height = 6 ft. d. $SA \approx 124$ ft^2	Lessons 9.1.2, 9.1.3, 9.1.5 Math Notes boxes, problems 9-35 and 9-36	Problems 9-37, 9-39, 9-40, 9-45, 9-46, 9-47, 9-48, 9-73, 10-51, 10-60, 11-11, 11-38, 11-76, 11-107