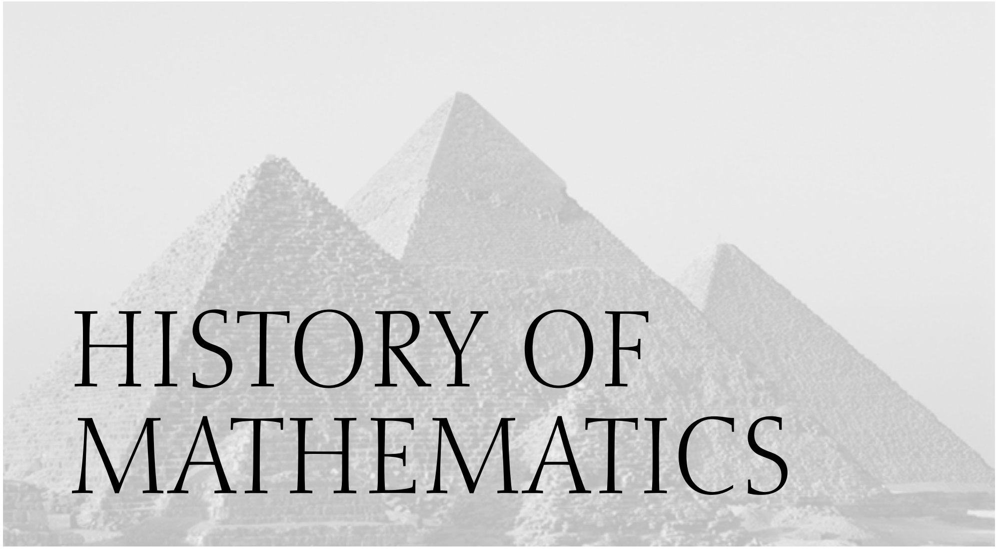


HISTORY OF MATHEMATICS

WHAT IS MATHEMATICS?

What is the **origin** of the **word "mathematics"**?

According to most sources, the word "mathematics" is derived from the Latin *mathmaticus* and from the Greek *mathēmatikos,* meaning "mathematical." (Other forms include *mathēma,* meaning "learning," and *manthanein,* meaning "to learn.")

In simple terms, **what is mathematics**?

Mathematics is often referred to as the science of quantity. The two traditional branches of mathematics have been arithmetic and geometry, using the quantities of numbers and shapes. And although arithmetic and geometry are still of major importance, modern mathematics expands the field into more complex branches by using a greater variety of quantities.

Who were the **first humans** to **use simple forms of mathematics**?

No one really knows who first used simple forms of mathematics. It is thought that the earliest peoples used something resembling mathematics because they would have known the concepts of one, two, or many. Perhaps they even counted using items in nature, such as 1, represented by the Sun or Moon; 2, their eyes or wings of a bird; clover for 3; or legs of a fox for 4.

Archeologists have also found evidence of a crude form of mathematics in the tallying systems of certain ancient populations. These include notches in wooden sticks or bones and piles or lines of shells, sticks, or pebbles. This is an indication that certain prehistoric peoples had at least simple, visual ways of adding and subtracting things, but they did not yet have a numbering system such as we have today.

Early humans used all sorts of images to represent numbers, including the fox, the image of which was used to indicate the number 4. *Stone/Getty Images.*

EARLY COUNTING AND NUMBERS

What are some **examples** of **how early peoples counted**?

There were several different ways that early civilizations recorded the numbers of things. Some of the earliest archeological evidence of counting dates from about 35000 to 20000 BCE, in which several bones bear regularly spaced notches. Most of these marked bones have been found in western Europe, including in the Czech Republic and France. The purpose of the notches is unclear, but most scientists believe they do represent some method of counting. The marks may represent an early hunter's number of kills; a way of keeping track of inventory (such as sheep or weapons); or a way to track the movement of the Sun, Moon, or stars across the sky as a kind of crude calendar.

Not as far back in time, shepherds in certain parts of West Africa counted the animals in their flocks by using shells and various colored straps. As each sheep passed, the shepherd threaded a corresponding shell onto a white strap, until nine shells were reached. As the tenth sheep went by, he would remove the white shells and put one on a blue strap, representing ten. When 10 shells, representing 100 sheep, were on the blue strap, a shell would then be placed on a red strap, a color that represented what we would call the next decimal up. This would continue until the entire flock was counted. This is also a good example of the use of base 10. (For more information about bases, see "Math Basics.")

Certain cultures also used gestures, such as pointing out parts of the body, to represent numbers. For example, in the former British New Guinea, the Bugilai culture used the following gestures to represent numbers: 1, left hand little finger; 2, next finger; 3, middle finger; 4, index finger; 5, thumb; 6, wrist; 7, elbow; 8, shoulder; 9, left breast; 10, right breast.

Another method of counting was accomplished with string or rope. For example, in the early 16th century, the Incas used a complex form of string knots for accounting and sundry other reasons, such as calendars or messages. These recording strings were called *quipus,* with units represented by knots on the strings. Special officers of the king called *quipucamayocs,* or "keepers of the knots," were responsible for making and reading the quipus.

Why did the need for mathematics arise?

The reasons humans developed mathematics are the same reasons we use math in our own modern lives: People needed to count items, keep track of the seasons, and understand when to plant. Math may even have developed for religious reasons, such as in recording or predicting natural or celestial phenomena. For example, in ancient Egypt, flooding of the Nile River would wash away all landmarks and markers. In order to keep track of people's lands after the floods, a way to measure the Earth had to be invented. The Greeks took many of the Egyptian measurement ideas even further, creating mathematical methods such as algebra and trigonometry.

How did certain **ancient cultures** count **large numbers**?

It is not surprising that one of the earliest ways to count was the most obvious: using the hands. And because these "counting machines" were based on five digits on each hand, most cultures invented numbering systems using base 10. Today, we call these *base numbers*—or base of a number system—the numbers that determine place values. (For more information on base numbers, see "Math Basics.")

However, not every group chose 10. Some cultures chose the number 12 (or base 12); the Mayans, Aztecs, Basques, and Celts chose base 20, adding the ten digits of the feet. Still others, such as the Sumerians and Babylonians, used base 60 for reasons not yet well understood.

The numbering systems based on 10 (or 12, 20, or 60) started when people needed to represent large numbers using the smallest set of symbols. In order to do this, one particular set would be given a special role. A regular sequence of numbers would then be related to the chosen set. One can think of this as steps to various floors of a building in which the steps are the various numbers—the steps to the first floor are part of the "first order units"; the steps to the second floor are the "second order units"; and so on. In today's most common units (base 10), the first order units are the numbers 1 through 9, the second order units are 10 through 19, and so on.

What is the connection between **counting and mathematics**?

Although early counting is usually not considered to be mathematics, mathematics began with counting. Ancient peoples apparently used counting to keep track of sundry items, such as animals or lunar and solar movements. But it was only when agriculture, business, and industry began that the true development of mathematics became a necessity.

What are the names of the various base systems?

The base 10 system is often referred to as the *decimal system*. The base 60 system is called the *sexagesimal system*. (This should not be confused with the sexadecimal system—also called the hexadecimal system—or the digital system based on powers of 16.) A *sexagesimal counting table* is used to convert numbers using the 60 system into decimals, such as minutes and seconds.

The following table lists the common bases and corresponding number systems:

Base Number System

Base	Number System	Base	Number System
2	binary	9	nonary
3	ternary	10	decimal
4	quaternary	11	undenary
5	quinary	12	duodecimal
6	senary	16	hexadecimal
7	septenary	20	vigesimal
8	octal	60	sexagesimal

What is a **numeral**?

A numeral is a standard symbol for a number. For example, X is the Roman numeral that corresponds to 10 in the standard Hindu-Arabic system.

What were the **two fundamental ideas** in the development of **numerical symbols**?

There were two basic principles in the development of numerical symbols: First, a certain standard sign for the unit is repeated over and over, with each sign representing the number of units. For example, III is considered 3 in Roman numerals (see the Greek and Roman Mathematics section below for an explanation of Roman numerals). In the other principle, each number has its own distinct symbol. For example, "7" is the symbol that represents seven units in the standard Hindu-Arabic numerals. (See below for an explanation of Hindu-Arabic numbers; for more information, see "Math Basics.")

MESOPOTAMIAN NUMBERS AND MATHEMATICS

What was the **Sumerian oral counting system**?

The Sumerians—whose origins are debated, but who eventually settled in Mesopotamia—used base 60 in their oral counting method. Because it required the

Who were the Mesopotamians?

The explanation of who the Mesopotamians were is not easy because there are many historians who disagree on how to distinguish Mesopotamians from other cultures and ethnic groups. In most texts, the label "Mesopotamian" refers to most of the unrelated peoples who used cuneiform (a way of writing numbers; see below), including the Sumerians, Persians, and so on. They are also often referred to as Babylonians, after the city of Babylon, which was the center of many of the surrounding empires that occupied the fertile plain between the Tigris and Euphrates Rivers. But this area was also called Mesopotamia. Therefore, the more correct label for these people is probably "Mesopotamians."

In this text, Mesopotamians will be referred to by their various subdivisions because each brought new ideas to the numbering systems and, eventually, mathematics. These divisions include the Sumerians, Akkadians, and Babylonians.

memorization of so many signs, the Sumerians also used base 10 like steps of a ladder between the various orders of magnitude. For example, the numbers followed the sequence 1, 60, 60^2, 60^3, and so on. Each one of the iterations had a specific name, making the numbering system extremely complex.

No one truly knows why the Sumerians chose such a high base number. Theories range from connections to the number of days in a year, weights and measurements, and even that it was easier to use for their purposes. Today, this numbering system is still visible in the way we tell time (hours, minutes, seconds) and in our definitions of circular measurements (degrees, minutes, seconds).

How did the **Sumerian written counting system change** over time?

Around 3200 BCE, the Sumerians developed a written number system, attaching a special graphical symbol to each of the larger numbers at various intervals (1, 10, 60, 3,600, etc.). Because of the rarity of stone, and the difficulty in preserving leather, parchment, or wood, the Sumerians used a material that would not only last but would be easy to imprint: clay. Each symbol was written on wet clay tablets, then baked in the hot sunlight. This is why many of the tablets are still in existence today.

The Sumerian number system changed over the centuries. By about 3000 BCE, the Sumerians decided to turn their numbering symbols counterclockwise by 90 degrees. And by the 27th century BCE, the Sumerians began to physically write the numbers in a different way, mainly because they changed writing utensils from the old stylus that was cylindrical at one end and pointed at the other to a stylus that was flat. This change in writing utensils, but not the clay, created the need for new symbols. The

Who were the Akkadians?

The region of Mesopotamia was once the center of the Sumerian civilization, a culture that flourished before 3500 BCE. Not only did the Sumerians have a counting and writing system, but they were also a progressive culture, supporting irrigation systems, a legal system, and even a crude postal service. By about 2300 BCE, the Akkadians invaded the area, emerging as the dominant culture. As most conquerors do, they imposed their own language on the area and even used the Sumerians' cuneiform system to spread their language and traditions to the conquered culture.

Although the Akkadians brought a more backward culture into the mix, they were responsible for inventing the abacus, an ancient counting tool. By 2150 BCE, the Sumerians had had enough: They revolted against the Akkadian rule, eventually taking over again.

However, the Sumerians did not maintain their independence for long. By 2000 BCE their empire had collapsed, undermined by attacks from the west by Amorites and from the east by Elamites. As the Sumerians disappeared, they were replaced by the Assyro-Babylonians, who eventually established their capital at Babylon.

new way of writing numbers was called *cuneiform script,* which is from the Latin *cuneus,* meaning "a wedge" and *formis,* meaning "like."

Did any **cultures** use more than one **base number** in **their numbering system**?

Certain cultures may have used a particular base as their dominant numbering system, such as the Sumerians' base 60, but that doesn't mean they didn't use other base numbers. For example, the Sumerians, Assyrians, and Babylonians used base 12, mostly for use in their measurements. In addition, the Mesopotamian day was broken into 12 equal parts; they also divided the circle, ecliptic, and zodiac into 12 sections of 30 degrees each.

What was the **Babylonian numbering system**?

The Babylonians were one of the first to use a positional system within their numbering system—the value of a sign depends on the position it occupies in a string of signs. Neither the Sumerians nor the Akkadians used this system. The Babylonians also divided the day into 24 hours, an hour into 60 minutes, and a minute into 60 seconds, a way of telling time that has existed for the past 4,000 years. For example, the

What is the rule of position?

We are most familiar with the *rule of position,* or place value, as it is applied to the Hindu-Arabic numerals 1, 2, 3, 4, 5, 6, 7, 8, 9, and 0. This is because their values depend on the place or position they occupy in a written numerical expression. For example, the number 5 represents 5 units, 50 is 5 tens, 500 is 5 hundreds, and so on. The values of the 5s depends upon their position in the numerical expression. It is thought that the Chinese, Indian, Mayan, and Mesopotamian (Babylonian) cultures were the first to develop this concept of place value.

way we now write hours, minutes, and seconds is as follows: 6h, 20', 15''; the way the Babylonians would have written this same expression (as sexagesimal fractions) was 6 20/60 15/3600.

Were there any **problems** with the **Babylonian numbering system**?

Yes. One in particular was the use of numbers that looked essentially the same. The Babylonians conquered this problem by making sure the character spacing was different for these numbers. This ended the confusion, but only as long as the scribes writing the characters bothered to leave the spaces.

Another problem with the early Babylonian numbering system was not having a number to represent zero. The concept of zero in a numbering system did not exist at that time. And with their sophistication, it is strange that the early Babylonians never invented a symbol like zero to put into the empty positions in their numbering system. The lack of this important placeholder no doubt hampered early Babylonian astronomers and mathematicians from working out certain calculations.

Did the **Babylonians** finally use a **symbol** to indicate an **empty space** in their numbers?

Yes, but it took centuries. In the meantime, scribes would not use a symbol representing an empty space in a text, but would use phrases such as "the grain is finished" at the end of a computation that indicated a zero. Apparently, the Babylonians did comprehend the concepts of void and nothing, but they did not consider them to be synonymous.

Around 400 BCE, the Babylonians began to record an empty space in their numbers, which were still represented in cuneiform. Interestingly, they did not seem to view this space as a number—what we would call zero today—but merely as a placeholder.

What happened to the Babylonians?

After the Amorites (a Semitic people) founded Babylon, there were several dynasties that ruled the area, including those associated with the famous king and lawmaker, Hammurabi (1792–1750 BCE). It was periodically taken over, including in 1594 BCE by the Kassites and in the 12th century BCE by the Assyrians. Through all these conquests, most of the Babylonian culture retained its own distinctiveness. With the fall of the Assyrian Empire in 612 BCE, the Babylonian culture bloomed, at least until its conquest by Cyris of Persia in 539 BCE. It eventually died out a short time after being conquered by Alexander the Great (356–323 BCE) in 331 BCE (ironically, Alexander died in Babylon, unable to recover from a fever he contracted).

Who **invented** the symbol for **zero**?

Although the Babylonians determined there to be an empty space in their numbers, they did not have a symbol for zero. Archeologists believe that a crude symbol for zero was invented either in Indochina or India around the 7th century and by the Mayans independently about a hundred years earlier. What was the main problem with the invention of zero by the Mayans? Unlike more mobile cultures, they were not able to spread the word around the world. Thus, their claim as the first people to use the symbol for zero took centuries to uncover. (For more information about zero, see "Mathematics throughout History.")

What do we know about **Babylonian mathematical tables**?

Archeologists know that the Babylonians invented tables to represent various mathematical calculations. Evidence comes from two tables found in 1854 at Senkerah on the Euphrates River (dating from 2000 BCE). One listed the squares of numbers up to 59, and the other the cubes of numbers up to 32.

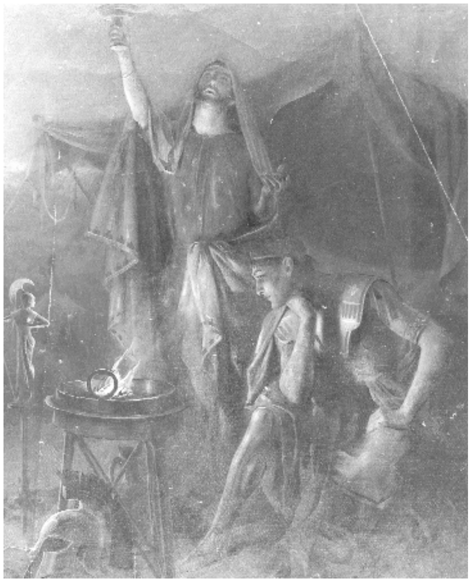
The Babylonians also used a method of division based on tables and the equation $a/b = a \times (1/b)$. With this equation, all that was necessary was a table of reciprocals; thus, the discovery of tables with reciprocals of numbers up to several billion.

They also constructed tables for the equation $\mathrm{n}^3 + \mathrm{n}^2$ in order to solve certain cubic equations. For example, in the equation $ax^3 + bx^2 = c$ (note: this is in our modern algebraic notation; the Babylonians had their own symbols for such an equation), they would multiply the equation by a^2, then divide it by b^3 to get $(ax/b)^3 + (ax/b)^2 = ca^2/b^3$.

If $y = ax/b$, then $y^3 + y^2 = ca^2/b^3$, which could now be solved by looking up the $n^3 + n^2$ table for the value of n that satisfies $n^3 + n^2 = ca^2/b^3$. When a solution was found for y, then x was found by $x = by/a$. And the Babylonians did all this without the knowledge of algebra or the notations we are familiar with today.

What other **significant mathematical contributions** did the **Babylonians** make?

Throughout the centuries, the Babylonians made many mathematical contributions. They were the earliest people to know about the Pythagorean theorem, although it was not known by that name. In fact, Pythagoras, in his travels to the east, may have learned about the theorem that would eventually carry his name from the Babylonians. In addition, the Babylonians possessed all the theorems of plane geometry that the Greeks ascribed to Thales, including the theorem eventually named after him. They also may have been the most skilled algebraists of their time, even though the symbols and methods they used were much different than our modern algebraic notations and procedures.



Alexander the Great, depicted here in an 1899 painting of the Battle of Gaugamela, Iraq (331 BCE), by artist Benjamin Ide Wheeler, conquered much of the known world and brought an end to the Babylonian civilization. The rise and fall of civilizations throughout history did much to influence the development of mathematics over the centuries. *Library of Congress.*

EGYPTIAN NUMBERS AND MATHEMATICS

Who were the **Egyptians**?

The Egyptians rose to prominence around 3000 BCE in the area we now call Egypt, but their society was already advanced, urbanized, and expanding rapidly long before that time. Although their civilization arose about the same time that words and numbers were first written down in Mesopotamia, archeologists do not believe there was any sharing between the two cultures. The Egyptians already had writing and written numerals; plus, the Egyptian signs and symbols were taken exclusively from the flora and fauna of the Nile River basin. In addition, the Egyptians developed the utensils for writing signs about a thousand years earlier.

Hieroglyphs can often be found on such Egyptian structures as the Obelisks of Hatshepsut, Karnak Temple, near the ancient city of Thebes. *Robert Harding World Imagery/Getty Images.*

What type of **numerals** did the **Egyptians** use?

By about 3000 BCE, the Egyptians had a writing system based on hieroglyphs, or pictures that represented words. Their numerals were also based on hieroglyphs. They used a base-10 system of numerals: one unit, one ten, one hundred, and so on to one million. The main drawback to this system was the number of symbols needed to define the numbers.

Did the **Egyptians** eventually **develop different numerals**?

Yes, the Egyptians used another number system called hieratic numerals after the invention of writing on papyrus. This allowed larger numbers to be written in a more compact form. For example, there were separate symbols for 1 through 9; 10, 20, 30, and so on; 100, 200, 300, and so on; and 1,000, 2000, 3,000, and so on.

The only drawback was that the system required memorization of more symbols—many more than for hieroglyphic notation. It took four distinct hieratic symbols to represent the number 3,577; it took no less than 22 symbols to represent the same number in hieroglyphs, but most of those symbols were redundant (see illustration on p. 15).

Both hieroglyphic and hieratic numerals existed together for close to two thousand years—from the third to the first millennium BCE. In general, hieroglyph numerals were used when carved on such objects as stone obelisks, palace and temple walls, and tombs. The hieratic symbols were much faster and easier to scribe, and they were written on papyrus for records, inventories, wills, or for mathematical, astronomical, economic, legal—or even magical—works.

Even though it is thought that the hieratic symbols were developed from the corresponding hieroglyphs, the shapes of the signs changed considerably. One reason in particular came from the reed brushes used to write hieratic symbols; writing on papyrus differed greatly from writing using stone carvings, thus the need to change the symbols to fit the writing devices. And as kingdoms and dynasties changed, the hieratic numerals changed, too, with users having to memorize the many distinct signs.

What are some examples of Egyptian multiplication?

Egyptian multiplication methods did not require a great deal of memorization, just a knowledge of the two times tables. For a simple example, to multiply 12 times 16, they would start with 1 and 12. Then they would double each number in each row (1×2 and 12×2; 2×2 and 24×2; and so on) until the number 16, resulting in the answer 192:

1	12
2	24
4	48
8	96
16	192

Another example computes a number that is not a multiple in the row, such as 37 times 19:

1	19
2	38
4	76
8	152
16	304
32	608

First, do the usual procedure by starting with 1 and 19, then doubling the numbers until you get to 32 (if you double 32 [= 64], you've overshot the number 37). Because 37 is higher than 32, go back over the list on the left-hand side, figure out which numbers, with 32, add up to 37 (1, 4, and 32); then add the numbers that correspond to those numbers, to the right (19, 76, and 608), which equals the answer: 703. And you didn't even need a calculator!

Did the **Egyptians** use **fractions**?

Yes, the Egyptian numbering system dealt with fractions, albeit with symbols that do not resemble modern notation. Fractions were written by placing the hieroglyph for "mouth" over the hieroglyph for the numerical expression. For example, 1/5 and 1/10 would be seen as the first two illustrations represented in the box on p. 15. Other fractions, such as the two symbols for 1/2 (see illustration on p. 15), also have special signs.

What were the **problems** with the **Egyptian number system**?

The Egyptian number system had several problems, the most obvious being that it was not written with certain arithmetic calculations in mind. Similar to Roman

The Egyptian civilization did much to contribute to mathematics, including developing a numbering system and using geometry in architecture to create the famous pyramids and other buildings. *Photographer's Choice/Getty Images.*

numerals, Egyptian numbers could be used for addition and subtraction, but not for simple multiplication and division.

All was not lost, however, as the Egyptians devised a way to do multiplication and division that involved addition. Multiplying and dividing by 10 was easy with hieroglyphics—just replace each symbol in the given number by the sign for the next higher order. To multiply and divide by any other factor, Egyptians devised the tabulations based on the two times tables, or a sequence of duplications.

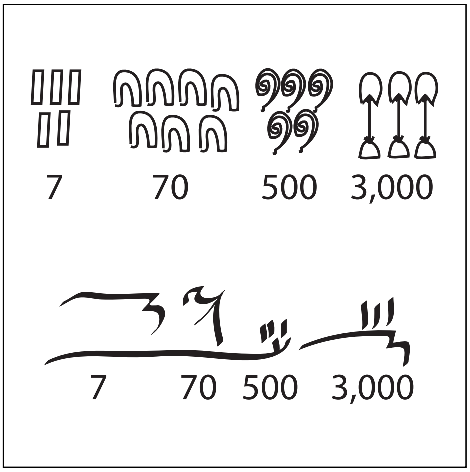
Why did the **Egyptians** need to **develop mathematics**?

Probably the most pressing reason for the development of Egyptian mathematics came from a periodic occurrence in nature: the flooding of the Nile River. With the advent of agriculture in the Nile River valleys, flooding was important, not only to provide fertile soil and water for the irrigated fields, but also to know when the fields would become dry. In addition, along with the growth of the Egyptian society came a need for a more complex way of keeping track of taxes, dividing property, buying and selling goods, and even amassing an army. Thus, the need for counting and mathematics arose, along with the development of a written system of numbers to complete and record the myriad of transactions.

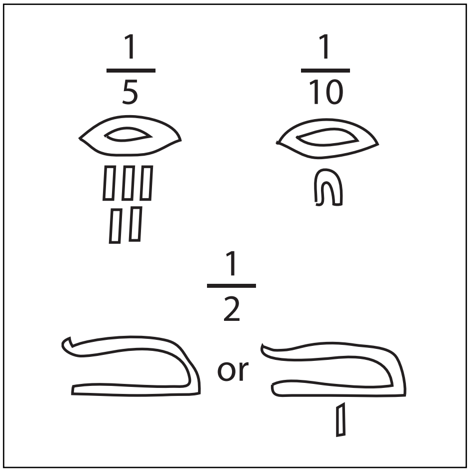
Where does most of our knowledge of **Egyptian mathematics originate**?

Most of our knowledge of Egyptian mathematics comes from writings on papyrus, a type of writing paper made in ancient Egypt from the pith and long stems of the papyrus plant. Most papyri no longer exist, as the material is fragile and disintegrates over time. But two major papyri associated with Egyptian mathematics have survived.

Named after Scottish Egyptologist A. Henry Rhind, the *Rhind papyrus* is about 19 feet (6 meters) long and 1 foot (1/3 meter) wide. It was written around 1650 BCE by Ahmes, an Egyptian scribe who claimed he was copying a 200-year-old document (thus the original information is from about 1850 BCE). This papyrus contains 87 mathematical problems; most of these are practical, but some teach manipulation of the number system (though with no application in mind). For example, the first six problems of the Rhind papyrus ask the following: problem 1. how to divide n loaves between 10 men, in which $n = 1$; in problem 2, $n = 2$; in problem 3, $n = 6$; in prob-



The number 3,577 is represented above using hieroglyphs (top) and hieratic symbols (bottom). Notice these numbers are read from right to left.



The symbols for 1/5, 1/10, and 1/2 are represented above using hieroglyphs.

lem 4, $n = 7$; in problem 5, $n = 8$; and in problem 6, $n = 9$. In addition, 81 out of the 87 problems involve operating with fractions, while other problems involve quantities and even geometry. Rhind purchased the papyrus in 1858 in Luxor; it resides in the British Museum in London.

Written around the 12th Egyptian dynasty, and named after the Russian city, the mathematical information on the *Moscow papyrus* is not ascribed to any one Egyptian, as no name is recorded on the document. The papyrus contains 25 problems similar to those in the Rhind papyrus, and many that show the Egyptians had a good grasp of geometry, including a formula for a truncated pyramid. It resides in the Museum of Fine Arts in Moscow.

GREEK AND ROMAN MATHEMATICS

Why was **mathematics** so **important** to the **Greeks**?

With a numbering system in place and knowledge from the Babylonians, the Greeks became masters of mathematics, with the most progress taking place between the years of 300 BCE and 200 CE, although the Greek culture had been in existence long before that time. The Greeks changed the nature and approach to math, and they considered it one of the—if not the most—important subjects in science. The main reason for their proclivity towards mathematics is easy to understand: The Greeks preferred reasoning over any other activity. Mathematics is based on reasoning, unlike many scientific endeavors that require experimentation and observation.

The distance between the Moon and Earth was calculated by Hipparchus of Rhodes using basic trigonometry. *Stone/Getty Images.*

Who were some of the most **influential Ionian, Greek,** and **Hellenic mathematicians**?

The Ionians, Greeks, and Hellenics had some of the most progressive mathematicians of their time, including such mathematicians as Heron of Alexandria, Zeno of Elea, Eudoxus of Cnidus, Hippocrates of Chios, and Pappus. The following are only a few of the more influential mathematicians.

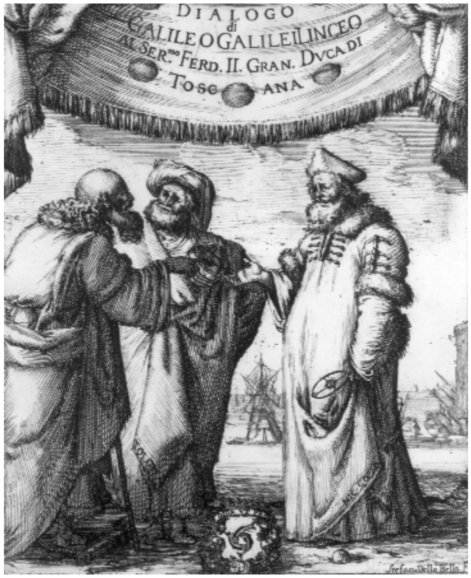
Thales of Miletus (c. 625–c. 550 BCE, Ionian), besides being purportedly the founder of a philosophy school and the first recorded western philosopher known, made great contributions to Greek mathematics, especially by presenting Babylonian mathematics to the Greek culture. His travels as a merchant undoubtedly exposed him to the geometry involved in measurement. Such concepts eventually helped him to introduce geometry to Greece, solving such problems as the height of the pyramids (using shadows), the distance of ships from a shoreline, and reportedly predicting a solar eclipse.

Hipparchus of Rhodes (c. 170–c. 125 BCE, Greek; also seen as Hipparchus of Nicaea) was an astronomer and mathematician who is credited with creating some of the basics of trigonometry. This helped immensely in his astronomical studies, including the determination of the Moon's distance from the Earth. Claudius Ptolemaeus (or Ptolemy) (c. 100–c. 170, Hellenic) was one of the most influential Greeks, not only in the field of astronomy, but also in geometry and cartography. Basing his works on Hipparchus, Ptolemy developed the idea of epicycles in which each planet revolves in a circular orbit, and each goes around an Earth-centered universe. The Ptolomaic way of explaining the solar system—which we now know is incorrect—dominated astronomy for more than a thousand years.

Diophantus (c. 210–c. 290) was considered by some scholars to be the "father of algebra." In his treatise Arithmetica, he solved equations in several variables for integral solutions, or what we call diophantine equations today. (For more about these equations, see "Algebra.") He also calculated negative numbers as solutions to some equations, but he considered such answers absurd.

What were **Archimedes's greatest contributions** to mathematics?

Historians consider Archimedes (c. 287–212 BCE, Hellenic) to be one of the greatest Greek mathematicians of the classic era. Known for his discovery of the hydrostatic principle, he also excelled in the mechanics of simple machines; computed close limits on the value of "pi" by comparing polygons inscribed in and circumscribed about a circle; worked out the formula to calculate the volume of a sphere and cylinder; and expanded on Eudoxus's method of exhaustion that would eventually lead to integral calculus. He also created a way of expressing any natural number, no matter how large; this was something that was not possible with Greek numerals. (For more information about Archimedes, see "Mathematical Analysis" and "Geometry and Trigonometry.")



Ptolemy (center), depicted in this 1632 engraving discussing ideas with Aristotle (left) and Copernicus (right), discovered valuable concepts concerning cartography, geometry, and astronomy. *Library of Congress.*

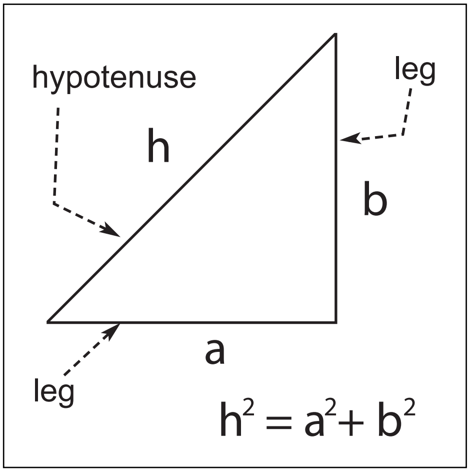
What **Greek mathematician** made major **contributions to geometry**?

The Greek mathematician Euclid (c. 325–c. 270 BCE) contributed to the development of arithmetic and the geometric theory of quadratic equations. Although little is known about his life—except that he taught in Alexandria, Egypt—his contributions to geometry are well understood. The elementary geometry many of us learn in high school is still largely based on Euclid. His 13 books of geometry and other mathematics, titled *Elements* (or *Stoicheion* in Greek), were classics of his day. The first six volumes offer explanations of elementary plane geometry; the other books present the theory of numbers, certain problems in arithmetic (on a geometric basis), and solid geometry. He also defines basic terms such as point and line, certain related axioms and postulates, and a number of statements logically deduced from definitions, axioms, and postulates. (For more information on axioms and postulates, see "Foundations of Mathematics"; for more information about Euclid, see "Geometry and Trigonometry.")

What was **Pythagoras**'s importance to mathematics?

Although the Chinese and Mesopotamians had discovered it a thousand years before, most people credit Greek mathematician and philosopher Pythagoras of Samos (c. 582–c. 507 BCE) with being the first to prove the Pythagorean Theorem. This is a famous geometry theorem relating the length of a right-angled triangle's hypotenuse (h) to the lengths of the other two sides (a and b).

In other words, for any right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the other two sides.



The Pythagorean Theorem is an easy way to determine the length of one side of a right triangle, given one knows the length of the other two sides.

What were **Pythagoras's** other **contributions**?

It is interesting that the Pythagorean Theorem was not Pythagoras's only contribution. He is considered the first pure mathematician. He also founded a school that stressed a fourfold division of knowledge, including number theory (deemed the most important of the pursuits at the school and using only the natural numbers), music, geometry, and astronomy (these subjects were called the *quadrivium* in the Middle Ages). Along with logic, grammar, and rhetoric, these studies collectively formed what was deemed the essential areas of knowledge for any well-rounded person.

Pythagoras not only taught these subjects, but also reincarnation and mysticism, establishing an order similar to, or perhaps influenced by, the earlier Orphic cult. The true lives of Pythagoras and his followers (who worshipped Pythagoras as a demigod) are a bit of a mystery, as they followed a strict code of secrecy and regarded their mathematical studies as something of a black art. The fundamental belief of the Pythagoreans was that "all is number," or that the entire universe—even abstract ethical concepts such as justice—could be explained in terms of numbers. But they also had some interesting non-mathematical beliefs, including an aversion to beans.

Although the Pythagoreans were influential in the fields of mathematics and geometry, they also made important contributions to astronomy and medicine and were the first to teach that the Earth revolved around a fixed point (the Sun). This idea would be popularized centuries later by Polish astronomer Nicolaus Copernicus (1473–1543). By the end of the 5th century BCE, the Pythagoreans had become social outcasts; many of them were killed as people grew angry at the group's interference with traditional religious customs.

Who was the **first** recorded **female mathematician**?

The first known female mathematician was Hypatia of Alexandria (370–415), who was probably taught by her mathematician and philosopher father, Theon of Alexandria. Around 400, she became the head of the Platonist school at Alexandria, lecturing on mathematics and philosophy. Little is known of her writings, and more legend is known of her than any true facts. It is thought that she was eventually killed by a mob.

What is the story behind "Archimedes in the bathtub"?

One of the most famous stories of Archimedes involves royalty: Hiero II of Syracuse, King of Sicily, wanted to determine if a crown (actually, a wreath) he had ordered was truly pure gold or alloyed with silver—in other words, whether or not the Royal Goldsmith had substituted some of the gold with silver. The king called in Archimedes to solve the problem. The Greek mathematician knew that silver was less dense than gold (in other words, silver was not as heavy as gold), but without pounding the crown into an easily weighed cubic shape, he didn't know how to determine the relative density of the irregularly shaped crown.

Perplexed, the mathematician did what many people do to get good ideas: he took a bath. As he entered the tub, he noticed how the water rose, which made him realize that the volume of the water that fell out of the tub was equal to that of the volume of his body. Legend has it that Archimedes ran naked through the streets shouting "Eureka!" ("I have found it!") He knew that a given weight of gold represented a smaller volume than an equal weight of silver because gold is much denser than silver, so not as much is needed to displace the water. In other words, a specific amount of gold would displace less water than an equal weight of silver.

The next day, Archimedes submerged the crown and an amount of gold equal to what was supposed to be in the crown. He found that Hiero's crown displaced less water than an equal weight of gold, thus proving the crown was alloyed with a less dense material (the silver) and not pure gold. This eventually led to the hydrostatic principle, as it is now called, presented in Archimedes's appropriately named treatise, *On Floating Bodies*. As for the goldsmith, he was beheaded for stealing the king's gold.

What is the **origin** of **Roman numerals**?

Because the history of Roman numerals is not well documented, their origin is highly debated. It is thought that the numerals were developed around 500 BCE, partially from primitive Greek alphabet symbols that were not incorporated into Latin. The actual reasons for the seven standard symbols are also argued. Some researchers believe the symbol for 1 (I) was derived from one digit on the hand; the symbol for 5 (V) may have developed because the outstretched hand held vertically forms a "V" from the space between the thumb and first finger; the symbol for 10 (X) may have been two Vs joined at the points, or it may have had to do with the way people or merchants used their hands to count in a way that resembled an "X." All the reasons offered so far have merely been educated guesses.

How ever the symbols were developed, they were used with efficiency and with remarkable aptitude by the Romans. Unlike the ancient Greeks, the Romans weren't

Why were early Greek calendars such a mess?

Unlike the Mesopotamian cultures, the early Greeks paid less attention to astronomy and more to cosmology (they were interested in studying where the Earth and other cosmic bodies stand in relation to the universe). Because of this, their astronomical observations were not accurate, creating confusing calendars. This also led to a major conundrum: Almost every Greek city kept time differently. In fact, during the Greek and Hellenistic times, most dates were given in terms of the Olympiads. This only created another time-keeping problem: If something happened during the 10th Olympiad, it meant the event occurred within a four-year span. Such notation creates headaches for historians, who end up making educated guesses as to the actual dates of Greek events, important people's deaths and births, and other significant historical occurrences.

truly interested in "pure" math, such as abstract geometry. Instead, they concentrated on "applied math," using mathematics and their Roman numerals for more practical purposes, such as building roads, temples, bridges, and aqueducts; for keeping merchant accounts; and for managing supplies for their armies.

Centuries after the Roman Empire fell, various cultures continued to use Roman numerals. Even today, the symbols are still in existence; they are used on certain timepieces, in formal documents, and for listing dates in the form of years. For example, just watch the end credits of your favorite movie or television program and you will often see the movie's copyright date represented with Roman numerals.

What are the **basic Roman numerals** and how are they used?

There are only seven basic Roman numerals, as seen in the following chart:

Number	Roman Numeral
1	I
5	V
10	X
50	L
100	C
500	D
1000	M

There are many rules, of course, to this method of writing numerals. For example, although the way to write a large number like 8,000 would be "MMMMMMMM," this is very cumbersome. In order to work with such large numbers, one rule was to write a

> ### What was the "House of Wisdom"?
>
> Around 786, the fifth Caliph of the Abbasid Dynasty began with Caliph Harun al-Rashid, a leader who encouraged learning, including the translation of many major Greek treatises into Arabic, such as Euclid's *Elements*. Al-Ma'mun (786–833), the next Caliph, was even more interested in scholarship, creating the House of Wisdom in Baghdad, one of several scientific centers in the Islamic Empire. Here, too, Greek works such as Galen's medical writings and Ptolemy's astronomical treatises were translated, not by language experts ignorant of mathematics, but by scientists and mathematicians such as Al-Kindi (801–873), Muhammad ibn Musa al-Khuwarizmi (see below), and the famous translator Hunayn ibn Ishaq (809–873).

bar over a numeral, meaning to multiply by 1,000. Thus, 8,000 would be VIII—equal to our Hindu-Arabic number 8—with a bar over the entire Roman numeral.

OTHER CULTURES AND EARLY MATHEMATICS

What did the **Chinese** add to the **study of mathematics**?

Despite the attention the Greeks have received concerning the development of mathematics, the Chinese were by no means uninterested in it. About the year 200 BCE, the Chinese developed place value notation, and 100 years later they began to use negative numbers. By the turn of the millennium and a few centuries beyond, they were using decimal fractions (even for the value of "pi" [π]) and the first magic squares (for more information about math puzzles, see "Recreational Math"). By the time European cultures had begun to decline—from about 530 to 1000 CE—the Chinese were contributing not only to the field of mathematics, but also to the study of magnetism, mechanical clocks, physical laws, and astronomy.

What is the most **famous Chinese mathematics book**?

The *Jiuzhang suanshu,* or *Nine Chapters on the Mathematical Art,* is the most famous mathematical book to come out of ancient China. This book dominated mathematical development for more than 1,500 years, with contributions by numerous Chinese scholars such as Xu Yue (c. 160–c. 227), though his contributions were lost. It contains 246 problems meant to provide methods to solve everyday questions concerning engineering, trade, taxation, and surveying.

Why is Omar Khayyám so famous?

Omar Khayyám is not as well known for his contributions to math as he is for being immortalized by Edward FitzGerald, the 19th-century English poet who translated Khayyám's own 600 short, four-line poems in the *Rubaiyat*. However, FitzGerald's translations were not exact, and most scholars agree that Khayyám did not write the line "a jug of wine, a loaf of bread, and Thou." Those words were actually conceived by FitzGerald. Interestingly enough, versions of the forms and verses used in the *Rubaiyat* existed in Persian literature long before Khayyám, and only about 120 verses can be attributed to him directly.

Who was **Aryabhata I**?

Aryabhata I (c. 476–550) was an Indian mathematician. Around 499 he wrote a treatise on quadratic equations and other scientific problems called *Aryabhatiya* in which he also determined the value of 3.1416 for pi (π). Although he developed some rules of arithmetic, trigonometry, and algebra, not all of them were correct.

What were some of the **contributions by the Arab world** to mathematics?

From about 700 to 1300, the Islamic culture was one of the most advanced civilizations in the West. The contributions of Arabic scholars to mathematics were helped not only by their contact with so many other cultures (mainly from India and China), but also because of the Islamic Empire's unifying, dominant Arabic language. Using knowledge from the Greeks, Arabian mathematics grew; the introduction of Indian numerals (often called Arabic numerals) also helped with mathematical calculations.

What are some **familiar Arabic terms** used in **mathematics**?

There are numerous Arabic terms we use today in our studies of mathematics. One of the most familiar is the term "algebra," which came from the title of the book *Al jabr w'al muqābalah* by Persian mathematician Muhammad ibn Musa al-Khuwarizmi (783–c. 850; also seen as al-Khowarizmi and al-Khwarizmi); he was the scholar who described the rules needed to do mathematical calculations in the Hindu-Arabic numeration system. The book, whose title is roughly translated as *Transposition and Reduction,* explains all about the basics of algebra. (For more information, see "Algebra.")

Another Arabic derivation is "algorithm," which stems from the Latinized version of Muhammad ibn Musa al-Khuwarizmi's own name. Over time, his name evolved from al-Khuwarizmi to Alchoarismi, then Algorismi, Algorismus, Algorisme, and finally Algorithm.

Who was **Omar Khayyám**?

Omar Khayyám (1048–1131), who was actually known as al-Khayyami, was a Persian mathematician, poet, and astronomer. He wrote the *Treatise on Demonstration of Problems of Algebra,* a book that contains a complete classification of cubic equations with geometric solutions, all of which are found by means of intersecting conic sections. He solved the general cubic equation hundreds of years before Niccoló Tartaglia in the 16th century, but his work only had positive roots, because it was completely geometrical (see elsewhere in this chapter for more about Tartaglia). He also calculated the length of the year to be 365.24219858156 days—a remarkably accurate result for his time—and proved that algebra was definitely related to geometry.

MATHEMATICS AFTER THE MIDDLE AGES

Who first introduced **Arabic notation** and the **concept of zero** to Europe?

Italian mathematician Leonardo of Pisa (c. 1170–c. 1250, who was also known as Fibonacci, or "son of Bonacci," although some historians say there is no evidence that he or his contemporaries ever used that name) brought the idea of Arabic notation and the concept of zero to Europe. His book *Liber abaci* (*The Book of the Abacus*) not only introduced zero but also the arithmetic and algebra he had learned in Arab countries. Another book, *Liber quadratorum* (*The Book of the Square*) was the first major European advance in number theory in a thousand years. He is also responsible for presenting the Fibonacci sequence. (For more information about Fibonacci and the Fibonacci sequence, see "Math Basics.")

What were the **major reasons** for **16th-century advances** in European mathematics?

There are several reasons for advances in mathematics at the end of the Middle Ages. The major reason, of course, was the beginning of the Renaissance, a time when there was a renewed interest in learning. Another important event that pushed mathematics was the invention of printing, which made many mathematics books, along with useful mathematical tables, available to a wide audience. Still another advancement was the replacement of the clumsy Roman numeral system by Hindu-Arabic numerals. (For more information about the Hindu-Arabic numerals, see "Math Basics.")

Who was **Scipione del Ferro**?

There were several mathematicians in the 16th century who worked on algebraic solutions to cubic and quartic equations. (For more information on cubic and quartic equations, see "Algebra.") One of the first was Scipione del Ferro (1465–1526), who in 1515 discovered a formula to solve cubic equations. He kept his work a complete secret until just before his death, when he revealed the method to his student Antonio Maria Fiore.

Who was **Adam Ries**?

Adam Ries (1492–1559) was the first person to write several books teaching the arithmetic method by the old abacus and new Indian methods; his books also presented the basics of addition, subtraction, multiplication, and division. Unlike most books of his time that were written in Latin and only understood by mathematicians, scientists, and engineers, Ries's works were written in his native German and were therefore understood by the general public. The books were also printed, making them more readily available to a wider audience.

Who was **François Viète**?

French mathematician François Viète (or Franciscus Vieta, 1540–1603) is often called the "founder of modern algebra." He introduced the use of letters as algebraic symbols (although Descartes [see below] introduced the convention of letters at the end of the alphabet [x, y, …] for unknowns and letters at the beginning of the alphabet [a, b, …] for knowns), and connected algebra with geometry and trigonometry. He also included trigonometric tables in his *Canon Mathematicus* (1571), along with the theory behind their construction. This book was originally meant to be a mathematical introduction to his unpublished astronomical treatise, *Ad harmonicon coeleste*. (For more about Viète, see "Algebra" and "Geometry and Trigonometry.")

What **century** produced the greatest **revolution** in **mathematics**?

Many mathematicians and historians believe that the 17th century saw not only the unprecedented growth of science but also the greatest revolution in mathematics. This century included the discovery of logarithms, the study of probability, the interactions between mathematics, physics, and astronomy, and the development of one of the most profound mathematical studies of all: calculus.

Who explained the **nature of logarithms**?

Scottish mathematician John Napier (1550–1617) first conceived the idea of logarithms in 1594. It took him 20 years, until 1614, to publish a canon of logarithms called *Mirifici logarithmorum canonis descripto* (*Description of the Wonderful Canon of Logarithms*). The canon explains the nature of logarithms, gives their rules of use, and offers logarithmic tables. (For more about logarithms, see "Algebra.")

What was the scandal between mathematicians working on cubic and quartic equations?

The early work on cubic equations was a tale of telling secrets, all taking place in Italy. No sooner had Antonio Maria Fiore (1526?–?)—considered a mediocre mathematician by scholars—received the secret of solving the cubic equation from Scipione del Ferro than he was spreading the rumor of its solution. A self-taught Italian mathematical genius known as Niccoló Tartaglia (1500–1557?; nicknamed "the stutterer") was already discovering how to solve many kinds of cubic equations. Not to be outdone, Tartaglia pushed himself to solve the equation $x^3 + mx^2 = n$, bragging about it when he had accomplished the task.

Fiore was outraged, which proved to be a fortuitous event for the study of cubic (and eventually quartic) equations. Demanding a public contest between himself and Tartaglia, the mathematicians were to give each other 30 problems with 40 to 50 days in which to solve them. Each problem solved earned a small prize, but the winner would be the one to solve the most problems. In the space of two hours, Tartaglia solved all Fiore's problems, all of which were based on $x^3 + mx^2 = n$. Eight days before the end of the contest, Tartaglia had found the general method for solving all types of cubic equations, while Fiore had solved none of Tartaglia's problems.

But the story does not end there. Around 1539, Italian physician and mathematician Girolamo Cardano (1501–1576; known in English as Jerome Cardan) stepped into the picture. Impressed with Tartaglia's abilities, Cardano asked him to visit. He also convinced Tartaglia to divulge his secret solution of the cubic equation, with Cardano promising not to tell until Tartaglia published his results.

Apparently, keeping secrets was not a common practice in Italy at this time, and Cardano beat Tartaglia to publication. Cardano eventually encouraged his student Luigi (Ludovico) Ferrari (1522–?) to work on solving the quartic equation, or the general polynomial equation of the fourth degree. Ferrari did just that, and in 1545 Cardano published his Latin treatise on algebra, *Ars Magna* (*The Great Art*), which included a combination of Tartaglia's and Ferrari's works in cubic and quartic equations.

Who originated **Cartesian coordinates**?

Cartesian coordinates are a way of finding the location of a point using distances from perpendicular axes. (For more information about coordinates, see "Geometry and Trigonometry.") The first steps toward such a coordinate system were suggested by French philosopher, mathematician, and scientist René Descartes (1596–1650; in

Latin, Renatus Cartesius); he was the first to publish a work explaining how to use coordinates for finding points in space. Around the same time, Pierre de Fermat developed the same idea independently (see below). Both Descartes's and Fermat's ideas would lead to what is now known as Cartesian coordinates.

Descartes is also considered by some to be the founder of analytical geometry. He contributed to the ideas involved in negative roots and exponent notation, explained the phenomenon of rainbows and the formation of clouds, and even dabbled in psychology.

Who was **Pierre de Fermat**?

French mathematician Pierre de Fermat (1601–1665) made many contributions to early methods leading to differential calculus; he was also considered by some to be the founder of modern number theory (see "Math Basics") and did much to establish coordinate geometry, eventually leading to Cartesian coordinates. He supposedly proved a theorem eventually called "Fermat's Last Theorem." It states that the equation $x^n + y^n = z^n$ has no non-zero integer solutions for x, y, and z when n is greater than 2. But there is no proof of Fermat's "proof," making most mathematicians skeptical about his supposed discovery.

Was **Fermat's last theorem** finally **solved**?

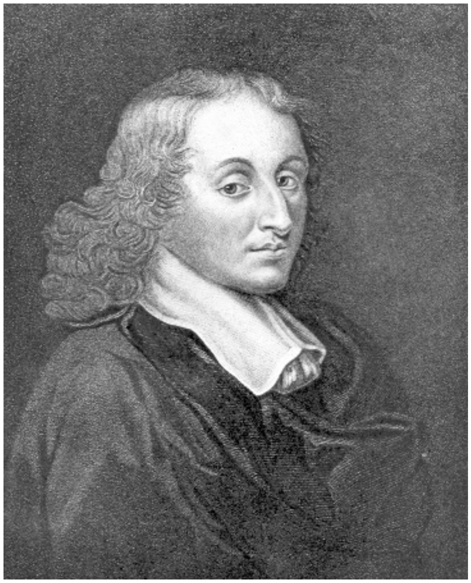
Just before the end of the 19th century, German industrialist and amateur mathematician Paul Wolfskehl, on the brink of suicide, began to explore a book on Fermat's Last Theorem. Enchanted with the numbers, he forgot about dying and instead believed that mathematics had saved him. To repay such a debt, he left 100,000 marks (about $2 million in today's money) to the Göttingen Academy of Science as a prize to anyone who could publish the complete proof of Fermat's Last Theorem. Announced in 1906 after Wolfskehl's death, thousands of incorrect proofs were turned in, but no true proof was offered.

But people kept trying—and failing. Fermat's Last Theorem was finally solved in 1994 by English mathematician Andrew John Wiles (1953–). Wiles was offered the Wolfskehl prize in 1997. By that time, the original $2 million had been affected by not only hyperinflation but also the devaluation of the mark, reducing its value to $50,000. But for Wiles, it didn't matter; his proving the Last Theorem had been a childhood dream.

It is interesting to note that some mathematicians do not believe Wiles uncovered the true proof of Fermat's Last Theorem. Instead, because many of the mathematical techniques used by Wiles were developed within recent decades (some even by Wiles himself), Wiles's proof—although a masterpiece of mathematics—could not possibly be the same as Fermat's. Still other mathematicians wonder about Fermat's words in claiming that he had found a proof. Was it really a proven or flawed proof he was talking about? Or was he such a genius that he took the proof he was able to see, in his time, to his grave? Like so many mysteries of history, we may never know.

Who began the mathematical study of **probability**?

French scientist and religious philosopher Blaise Pascal (1623–1662) is known not only for the study of probability but for many other mathematically oriented advances, such as a calculation machine (invented at age 19 to help his father with tax calculations, but it performed only additions), hydrostatics, and conic sections. He is also credited (along with Fermat) as the founder of modern theory of probability. (For more information about probability, see "Applied Mathematics.")



Seventeenth-century scientist Blaise Pascal was the founder of mathematical probability, as well as other achievements, such as devising one of the first calculating machines.

Who was **Sir Isaac Newton**?

Sir Isaac Newton (1642–1727) was an English mathematician and physicist considered by some to be one of the greatest scientists who ever lived. He was credited with inventing differential calculus in 1665 and integral calculus the following year. (For more information about calculus, see "Mathematical Analysis.") The list of his achievements—mathematical and scientific—does not end there: He is also credited as the discoverer of the general binomial theorem, he worked on infinite series, and he even made advancements in optics and chemistry.

Some of Newton's greatest contributions include the development of the law of universal gravitation, rules of planetary orbits, and sundry other astronomical concepts. By 1687, Newton had written one of his most famous books, *The Principia* or *Philosophiae naturalis principia mathematica* (*The Mathematical Principles of Natural Philosophy*), which is often called the greatest scientific book ever written. In it Newton presents his theories of motion, gravity, and mechanics. Although he had developed calculus earlier, he still used the customary classical geometry to work out physical problems within the book.

Who was Baron **Gottfried Wilhelm Leibniz**?

A contemporary of Isaac Newton, German philosopher and mathematician Baron Gottfried Wilhelm Leibniz (1646–1716) is considered by some to be a largely forgotten mathematician, although his contributions to the field were just as important as Newton's in many ways. He is often called the founder of symbolic logic; he introduced the terms *coordinate, abscissas,* and *ordinate* for the field of coordinate geometry; he invented a machine that could do multiplication and division; he discovered the well-known series for pi divided by 4 ($\pi/4$) that bears his name; and he independently developed infinitesimal calculus and was the first to describe it in print. Because his work on calculus was published three years before Isaac Newton's, Leibniz's system of notation was universally adopted.

Who was considered the **first statistician**?

English statistician and tradesman John Graunt (1620–1674) was the first true statistician and wrote the first book on statistics, although statistics in a simpler form was known long before that. Graunt, a draper by profession, was the first to use a compilation of data, which in this case involved the records of bills of mortality, or the records of how and when people died in London from 1604 to 1661. In his *Natural and Political Observations Made upon the Bills of Mortality,* he determined certain inclinations, such as more boys were born than girls, women tend to live longer than men, etc. He also developed the first mortality table, which showed how long a person might expect to live after a certain age, a concept very familiar to us today, especially in fields such as insurance and health.

Why was the **Bernoulli family** important to mathematics?

The Bernoulli (also seen as Bernouilli) family of the 17th and 18th centuries is synonymous with mathematics and science. One of the developers of ordinary calculus, calculus of variations, and the first to use the word "integral" was Jacob Bernoulli (1654–1705; also known as Jakob, Jacques, or James). He also wrote about the theory of probability, is often credited for developing the field of statistics, and discovered a series of numbers that bear his name: the coefficients of the exponential series expansion of $x/(1 - e^{-x})$.

Not to be outdone, his brother Johann (1667–1748; also known as Jean or John) contributed to the field of integral and exponential calculus, was the founder of calculus of variations, and worked on geodesics, complex numbers, and trigonometry. His son was not far behind: Daniel Bernoulli (1700–1782) was considered the first mathematical physicist, publishing *Hydrodynamica* in 1738, which included his now famous principle named in his honor (Bernoulli's principle); and he brought out two ideas that were ahead of his time by many years: the law of conservation of energy and the kinetic-molecular theory of gases.

The Bernoulli legacy did not end there, with family members continuing to make great mathematical and scientific contributions. There were two Nicolaus Bernoullis: one, the brother of Jacob and Johann (1662–1716), was professor of mathematics at St.

What was in Joseph-Louis Lagrange's letter to Jean le Rond d'Alembert?

Italian-French astronomer and mathematician Comte Joseph-Louis Lagrange (1736–1813) made significant discoveries in mathematical astronomy, including many functions, theories, etc. that bear his name (for example, Lagrange point, Lagrange's equations, Lagrange's theorem, Lagrangian function). His mentor was none other than French scientist Jean le Rond d'Alembert (1717–1783), a physicist who expanded on Newton's laws of motion, contributed to the field of fluid motion, described the regular changes in the Earth's axis, and was the first to use partial differential equations in mathematical physics. He even had time to edit, along with French philosopher Denis Diderot (1713–1784), the *Encyclopedié,* a 17-volume encyclopedia of scientific knowledge published from 1751 to 1772.

Apparently, living in the years of such mathematical enlightenment had its drawbacks. In 1781 Lagrange wrote a letter to d'Alembert about his greatest fear: that the field of mathematics had reached its limit. At that point in time, Lagrange believed everything mathematical had been discovered, uncovered, and calculated. Little did he realize that mathematics was only in its infancy.

Petersburg, Russia's Academy of Sciences; the other, the son of Johann and brother of Daniel (1695–1726), was also a mathematician. Another Johann Bernoulli (1710–1790) was another son of Johann (and brother of Daniel), who succeeded his father in the chair of mathematics at Basel, Switzerland, and also contributed to physics. The younger Johann also had a son named Johann (1746–1807), who was astronomer royal in Berlin and also studied mathematics and geography. Finally, Jacob Bernoulli (1759–1789), yet another son of the younger Johann, succeeded his uncle Daniel in teaching mathematics and physics at St. Petersburg, but he met an untimely death by drowning.

Who was one of the most **prolific mathematicians** who ever lived?

Swiss mathematician Leonhard Euler (1707–1783) is considered to be one of the most prolific mathematicians who ever lived. In fact, his accomplishments are beyond the scope of this text. Suffice it to say that his collected works number more than 70 volumes, with contributions in pure and applied mathematics, including the calculus of variations, analysis, number theory, algebra, geometry, trigonometry, analytical mechanics, hydrodynamics, and the lunar theory (calculation of the motion of the Moon). Euler was one of the first to develop the methods of the calculus on a wide scale. His most famous book, *Elements of Algebra,* rapidly became a classic; and he wrote a geometry textbook (Yale University was the first American college to use the text).

Although half-blind for much of his life—and totally blind for his last 17 years—he had a near-legendary skill at calculation. Among his discoveries are the differential equation named for him (a formula relating the number of faces, edges, and vertices of a polyhedron, although Euler's formula was discovered earlier by René Descartes); and a famous equation connecting five fundamental numbers in mathematics. Like many in the Bernoulli family, Euler eventually worked at the Academy of Sciences in St. Petersburg, Russia, a center of learning founded by Peter the Great.

Who was **Karl Friedrich Gauss**?

German mathematician, physicist, and astronomer Karl Friedrich Gauss (1777–1855; also seen as Johann Carl [or Karl] Friedrich Gauss) was considered one of the greatest mathematicians of his time; some have even compared him to Archimedes and Newton. His greatest mathematical contributions were in the fields of higher arithmetic and number theory. He discovered the law of quadratic reciprocity, determined the method of least squares (independently of French mathematician Adrien-Marie Legendre [1752–1833]), popularized the symbol "i" as the square root of negative 1 (although Euler first used the symbol), did extensive investigations in the theory of space curves and surfaces, made contributions to differential geometry, and much more. In 1801, after the discovery (and subsequent loss) of the first asteroid, Ceres, by Giuseppe Piazzi, he calculated the object's orbit with little data; the asteroid was found again thanks to his calculations. He further calculated the orbits of asteroids found over the next few years.

When was **non-Euclidean geometry** first announced?

Non-Euclidean geometry—or a system of geometry different from that developed by Euclid (see p. 17)—was first announced by Russian mathematician Nikolai Ivanovich Lobachevski (1792–1856; also seen as Lobatchevsky) in 1826. This idea had already been independently developed by the Hungarian János (or Johann) Bolyai (1802–1860) in 1823 and by Karl Friedrich Gauss (1777–1855) in 1816, but Lobachevski was the first to publish on the subject.

In 1854 German mathematician Georg Friedrich Bernhard Riemann (1826–1866) presented several new general geometric principles. His suggestion of another form of non-Euclidean geometry further established this new way of looking at geometry. Riemann was also responsible for presenting the Riemann hypothesis (or zeta function), a complex function that remains an unsolved issue in mathematics today. (For more information about geometry and Riemann, see "Geometry and Trigonometry.")

Who developed the first ideas on **symbolic logic**?

English mathematician George Boole (1815–1864) was the first to develop ideas on symbolic logic, that is, the use of symbols to represent logical principles. He proposed

Why was non-Euclidean geometry important to Albert Einstein?

Non-Euclidian geometry, especially the form suggested by Bernhard Riemann, enabled Albert Einstein (1879–1955) to work on his general relativity theory (1916), showing that the true geometry of space may be non-Euclidean. (For more information about mathematics and Einstein, see "Math in the Physical Sciences.")

this in his treatise, *An Investigation of the Laws of Thought, on Which Are Founded the Mathematical Theories of Logic and Probabilities* (1854). Today, this is called Boolean algebra. (For more information about Boole, see "Algebra"; for more information about symbolic logic, see "Foundations of Mathematics.")

MODERN MATHEMATICS

Who first developed **set theory**?

German mathematician George (Georg) Ferdinand Ludwig Philipp Cantor (1845–1918) was not only known for his work on transfinite numbers, but also for his development of set theory, which is the basis of modern mathematical analysis (for more information on set theory, see "Foundations of Mathematics"). His *Mathematische Annalen* was a basic introduction to set theory. Unlike most long evolutionary histories of mathematical subjects, Cantor's set theory was his creation alone. In the late 19th century, Cantor also developed the Continuum Hypothesis. He realized that there were many different sized infinities, further conjecturing that two particular infinities constructed by different processes were the same size.

What was the ***Principia Mathematica***?

In 1910 the first volume of the *Principia Mathematica* was published by Welsh mathematician and logician Bertrand Arthur William Russell (1872–1970) and English mathematician and philosopher Alfred North Whitehead (1861–1947). This book was an attempt to put mathematics on a logical foundation, developing logic theory as a basis for mathematics. It gave detailed derivations of many major theorems in set theory, examined finite and transfinite arithmetic, and presented elementary measure theory. The two mathematicians published three volumes, but the fourth, on geometry, was never completed.

On their own, both men did a great deal to advance mathematics, too. Russell discovered the Russell paradox (see below), introduced the theory of types, and popularized first-order predicate calculus. Russell's logic consisted of two main ideas: that all

What was Bertrand Russell's "great paradox"?

In the early 1900s, Bertrand Russell discovered what is known as the "great paradox" as it applies to the set of all sets: The set either contains itself or it does not, but if it does, then it does not, and vice versa. The reason that this paradox became so important was its affect on mathematics. It created problems for those people who tried to base mathematics on logic, and it also indicated that something was wrong with Georg Cantor's intuitive set theory, which at that time was one of the backbones of set theory. (For more about Russell and set theory, see "Foundations of Mathematics.")

mathematical truths can be translated into logical truths (or that the vocabulary of mathematics constitutes a proper subset of the vocabulary of logic) and that all mathematical proofs can be recast as logical proofs (or that the theorems of mathematics constitute a proper subset logical theorems).

Whitehead excelled not only in mathematics and logic but also in the philosophy of science and study of metaphysics. In mathematics, he extended the known range of algebraic procedures, and he was a prolific writer. In philosophy, he criticized the traditional theories for their lack of integrating the direct relationship between matter, space, and time; thus, he created a vocabulary of his own design, which he called the "philosophy of organism."

Who was **Kurt Gödel**?

For about a hundred years, mathematicians such as Bertrand Russell were trying to present axioms that would define the entire field of mathematics on an axiomatic basis. Austrian-American mathematician and logician Kurt Gödel (1906–1978) was the first to suggest that any formal system strong enough to include the laws of mathematics is either incomplete or inconsistent; this was called "Gödel's Incompleteness Theorem." Thus, axioms could not define all of mathematics.

Gödel also stated that the various branches of mathematics are based in part on propositions that are not provable within the system itself, although they may be proved by means of logical (metamathematical) systems external to mathematics. In other words, nothing is as simple as it seems; and, interestingly enough, Gödel's idea also implies that a computer can never be programmed to answer all mathematical questions.

What did **David Hilbert propose** in 1900?

In 1900 German mathematician David Hilbert (1862–1943) proposed 23 unsolved mathematical problems for the new century, most of which only proved to bring up

What was the "Golden Age of Logic"?

Kurt Gödel's work led to what is often described as the Golden Age of Logic. Spanning the years from about 1930 to the late 1970s, it was a time when there was a great deal of work done in mathematical logic. From the beginning, mathematicians broke into many camps that worked on various phases of logic (for more information about logic, see "Foundations of Mathematics"), including:

Proof theory—In which the mathematical proofs started by Aristotle and continued by Boole (see p. 30) were extensively studied, resulting in branches of this mathematics being applied to computing (including artificial intelligence).

Model theory—In which mathematicians investigated the connection between the truth in a mathematical structure and propositions about that structure.

Set theory—In which a breakthrough in 1963 showed that certain mathematical statements were undeterminable, a direct challenge to the major set theories of the time. This showed that Cantor's Continuum Hypothesis (see p. 31) is independent of the axioms of set theory, or that there are two mathematical possibilities: one that says the continuum hypothesis is true, one that says it is false.

Computability theory—In which mathematicians worked out the abstract theorems that would eventually help lead to computer technology. For example, English mathematician Alan Turing proved an abstract theorem that established the theoretical possibility of a single computing machine programmed to complete any computation. (For more information about Turing and computers, see p. 34 and "Math in Computing.")

other problems. By the 1920s Hilbert gathered many mathematicians—called the formalists—to prove that mathematics was consistent. But all did not go well as mathematical complications set in. By 1931 Kurt Gödel's Incompleteness Theorem dashed any more efforts by the formalists by proving that mathematics is either inconsistent or incomplete. (For more about Hilbert, see "Foundations of Mathematics.")

When was **quantum mechanics** developed?

There was not one major year in which quantum mechanics was developed, or even one major scientist who proposed the idea. This modern theory of physics evolved over about 30 years, with many scientists contributing to it. Beginning about 1900 Max Planck proposed that energies of any harmonic oscillator (such as the atoms of a black body radiator) are restricted to certain values. Mathematics came into play here, too, with each value an integral multiple of a basic, minimum value. Planck developed the equation $E = h\nu$ (or h times "nu"), in which E (the energy of the basic quantum) is directly proportional to the ν (the frequency of the oscillator) multiplied by h, or Planck's constant (6.63×10^{-34} joule-second).

From there, mainly with the use of rigorous mathematics, others expanded or added to Planck's idea, including German scientist Albert Einstein (1879–1955), who explained the photoelectric effect; New Zealand-born British physicist Ernest Rutherford (1871–1937) and Danish physicist Neils Bohr (1885–1962), who explained both atomic structure and spectra; Austrian physicist Erwin Schrödinger (1887–1961), who developed wave mechanics; and German physicist Werner Karl Heisenberg (1901–1976), who discovered the uncertainty principle. Out of these studies came quantum mechanics (in the 1920s), quantum statistics, and quantum field theory. Today, quantum mechanics and Einstein's theory of relativity form the foundation of modern physics. These theories continually change or are modified as we get closer to understanding more about the physics—and mathematics—of our universe.

Who was **Alan Turing**?

British mathematician Alan Mathison Turing (1912–1954) was the first person to propose the idea of a simple computer. Called the Turing machine, its operation was limited to reading and writing symbols on tape, moving the tape to the left or right to read the symbols one at a time. This invention is often considered the start of the computer age. In fact, the definition of the word "computable" is a problem that can be solved by a Turing machine. Turing was also instrumental in interpreting and deciphering encrypted German messages using the Enigma cipher machine. (For more information on computers, see "Math in Computing.")

What is **chaos theory**?

Chaos theory is one of the "newest" ideas in mathematics. Developed in the last half of the 20th century, it affects not only math, but also physics, geology, biology, meteorology, and many other fields. Modern ideas about chaos began when theorists in various scientific disciplines started to question the linear analysis used in classical applied mathematics, most of which presumes an orderly periodicity that rarely occurs in nature. In the search to discover regularities, the idea of disorder had been ignored. To overcome this problem, chaos theorists developed deterministic, nonlinear dynamic models that explain irregular, unpredictable behavior. By 1961, American meteorologist Edward Norton Lorenz (1917–) noticed that small variations in the initial values of variables in his primitive computer weather model resulted in major divergent weather patterns. His discovery of a simple mathematical system with chaotic behavior led to the new mathematics of chaos theory.

Chaos theory recognizes the unpredictability of life, including the world's highly complex weather system, which can be influenced by a myriad of factors ranging from changes in temperature and humidity to alterations in geology and agricultural development. *Taxi/Getty Images.*

The use of chaos theory has enabled scientists and mathematicians to reveal the structure in aperiodic, unpredictable dynamic systems. For example, it has been used to examine crystal growth, the expansion of pollution plumes in water and in the air, and even to determine the formation of storm clouds. One of the reasons chaos theory has come to the forefront of science and mathematics is because of advancements in computers; high-end computers allow for a plethora of variables to enter into the complex chaos equations.

Who invented **catastrophe theory**?

Catastrophe theory—or the study of gradually changing forces that lead to so-called catastrophes (or abrupt changes)—was popularized by French mathematician René Thom (1923–2002) in 1972. Unable to use differential calculus in certain situations, Thom used other mathematical treatments of continuous action to produce a discontinuous result. Although it is not as popular as it once was, it is often used in biological and optical applications.

Who is **Benoit Mandelbrot**?

Benoit B. Mandelbrot (1924–) is the Polish-born, French mathematician who invented a branch of mathematics called fractal geometry, which is designed to find order in

Why is there no Nobel Prize for mathematics?

The Nobel Prizes were established at the bequest of Swedish chemical engineer Alfred Bernhard Nobel (1833–1896), the discoverer of dynamite. First awarded in 1901, the Nobel Prizes honor innovators in the fields of chemistry, physics, physiology or medicine, literature, and peace; a prize in economics was added in 1969, but there is no award for mathematics.

The lack of a mathematics prize has many stories attached, including one that states that Nobel's wife jilted him for Norwegian mathematician Magnus Gösta Mittag-Leffler, a notion made less plausible by the fact that Nobel never married. Most historians agree, however, that the reason has to do with Nobel's attitude toward mathematics: He simply did not consider mathematics sufficiently practical. To fill the gap, the Fields Medal of the International Mathematical Congress was established in 1932; it has the equivalent prestige of the Nobel with the limitation that it is only awarded for work done by mathematicians younger than 40 years old, and the monetary value is a mere $15,000 in Canadian dollars (or about $12,000 in U.S. dollars at press time).

But mathematics has not been left out of award-winning ceremonies. In 2003 Norway created the Abel Prize for mathematic achievement. Named after Norwegian mathematician Niels Henrik Abel (1802–1829), who proved that solving fifth-degree algebraic equations (quintics) is impossible, the award gives the winner a prize of six million Norwegian kroners (about $935,000 in American currency).

apparently erratic shapes and processes. A largely self-taught mathematician who did not like pure logical analysis, he was a pioneer of chaos theory, developing and finding applications for fractal geometry. Unlike traditional geometry with its regular shapes and whole-number dimensions, fractal geometry uses shapes found in nature with non-integer (or fractal—thus the name) dimensions. For example, twigs, tree branches, river systems, and shorelines can be examined using fractals. Today, fractals are often applied not only to the natural world but also to the chemical industry, computer graphics, and even the stock market.