2. $f(x)=\ln(x^2+10) \quad\Rightarrow\quad f'(x)=\dfrac{1}{x^2+10}\dfrac{d}{dx}(x^2+10)=\dfrac{2x}{x^2+10}$

4. $f(x)=\ln(\sin^2 x)=\ln(\sin x)^2=2\ln|\sin x| \quad\Rightarrow\quad f'(x)=2\cdot\dfrac{1}{\sin x}\cdot\cos x=2\cot x$

6. $f(x)=\log_5(xe^x) \quad\Rightarrow\quad f'(x)=\dfrac{1}{xe^x\ln 5}\dfrac{d}{dx}(xe^x)=\dfrac{1}{xe^x\ln 5}(xe^x+e^x\cdot 1)=\dfrac{e^x(x+1)}{xe^x\ln 5}=\dfrac{x+1}{x\ln 5}$

Another solution: We can change the form of the function by first using logarithm properties.

$f(x)=\log_5(xe^x)=\log_5 x+\log_5 e^x \quad\Rightarrow\quad f'(x)=\dfrac{1}{x\ln 5}+\dfrac{1}{e^x\ln 5}\cdot e^x=\dfrac{1}{x\ln 5}+\dfrac{1}{\ln 5}$ or $\dfrac{1+x}{x\ln 5}$

8. $f(x)=\ln\sqrt[5]{x}=\ln x^{1/5}=\frac{1}{5}\ln x \quad\Rightarrow\quad f'(x)=\dfrac{1}{5}\cdot\dfrac{1}{x}=\dfrac{1}{5x}$

10. $f(t)=\dfrac{1+\ln t}{1-\ln t} \quad\Rightarrow\quad f'(t)=\dfrac{(1-\ln t)(1/t)-(1+\ln t)(-1/t)}{(1-\ln t)^2}=\dfrac{(1/t)[(1-\ln t)+(1+\ln t)]}{(1-\ln t)^2}=\dfrac{2}{t(1-\ln t)^2}$

12. $h(x)=\ln\left(x+\sqrt{x^2-1}\right) \quad\Rightarrow\quad h'(x)=\dfrac{1}{x+\sqrt{x^2-1}}\left(1+\dfrac{x}{\sqrt{x^2-1}}\right)=\dfrac{1}{x+\sqrt{x^2-1}}\cdot\dfrac{\sqrt{x^2-1}+x}{\sqrt{x^2-1}}=\dfrac{1}{\sqrt{x^2-1}}$

14. $F(y)=y\ln(1+e^y) \quad\Rightarrow\quad F'(y)=y\cdot\dfrac{1}{1+e^y}\cdot e^y+\ln(1+e^y)\cdot 1=\dfrac{ye^y}{1+e^y}+\ln(1+e^y)$

16. $y=\ln(x^4\sin^2 x)=\ln x^4+\ln(\sin x)^2=4\,\ln x+2\,\ln\sin x \quad\Rightarrow\quad y'=4\cdot\dfrac{1}{x}+2\cdot\dfrac{1}{\sin x}\cdot\cos x=\dfrac{4}{x}+2\cot x$

20. $H(z)=\ln\sqrt{\dfrac{a^2-z^2}{a^2+z^2}}=\ln\left(\dfrac{a^2-z^2}{a^2+z^2}\right)^{1/2}=\dfrac{1}{2}\ln\left(\dfrac{a^2-z^2}{a^2+z^2}\right)=\frac{1}{2}\ln(a^2-z^2)-\frac{1}{2}\ln(a^2+z^2) \quad\Rightarrow$

$$H'(z)=\frac{1}{2}\cdot\frac{1}{a^2-z^2}\cdot(-2z)-\frac{1}{2}\cdot\frac{1}{a^2+z^2}\cdot(2z)=\frac{z}{z^2-a^2}-\frac{z}{z^2+a^2}=\frac{z(z^2+a^2)-z(z^2-a^2)}{(z^2-a^2)(z^2+a^2)}$$

$$=\frac{z^3+za^2-z^3+za^2}{(z^2-a^2)(z^2+a^2)}=\frac{2a^2z}{z^4-a^4}$$

22. $y=[\ln(1+e^x)]^2 \quad\Rightarrow\quad y'=2[\ln(1+e^x)]\cdot\dfrac{1}{1+e^x}\cdot e^x=\dfrac{2e^x\ln(1+e^x)}{1+e^x}$

28. $y=\dfrac{\ln x}{x^2} \quad\Rightarrow\quad y'=\dfrac{x^2(1/x)-(\ln x)(2x)}{(x^2)^2}=\dfrac{x(1-2\ln x)}{x^4}=\dfrac{1-2\ln x}{x^3} \quad\Rightarrow$

$$y''=\frac{x^3(-2/x)-(1-2\ln x)(3x^2)}{(x^3)^2}=\frac{x^2(-2-3+6\ln x)}{x^6}=\frac{6\ln x-5}{x^4}$$

30. $y=\ln(\sec x+\tan x) \quad\Rightarrow\quad y'=\dfrac{\sec x\,\tan x+\sec^2 x}{\sec x+\tan x}=\sec x \quad\Rightarrow\quad y''=\sec x\,\tan x$

32. $f(x) = \dfrac{1}{1+\ln x} \quad \Rightarrow \quad f'(x) = -\dfrac{1/x}{(1+\ln x)^2}$ [Reciprocal Rule] $= -\dfrac{1}{x(1+\ln x)^2}$.

$\text{Dom}(f) = \{x \mid x > 0 \text{ and } \ln x \neq -1\} = \{x \mid x > 0 \text{ and } x \neq 1/e\} = (0, 1/e) \cup (1/e, \infty)$.

34. $f(x) = \ln \ln \ln x \quad \Rightarrow \quad f'(x) = \dfrac{1}{\ln \ln x} \cdot \dfrac{1}{\ln x} \cdot \dfrac{1}{x}$.

$\text{Dom}(f) = \{x \mid \ln \ln x > 0\} = \{x \mid \ln x > 1\} = \{x \mid x > e\} = (e, \infty)$.

36. $f(x) = \ln(1+e^{2x}) \quad \Rightarrow \quad f'(x) = \dfrac{1}{1+e^{2x}}(2e^{2x}) = \dfrac{2e^{2x}}{1+e^{2x}}$, so $f'(0) = \dfrac{2e^0}{1+e^0} = \dfrac{2(1)}{1+1} = 1$.

38. $y = \ln(x^3 - 7) \quad \Rightarrow \quad y' = \dfrac{1}{x^3-7} \cdot 3x^2 \quad \Rightarrow \quad y'(2) = \dfrac{12}{8-7} = 12$, so an equation of a tangent line at $(2, 0)$ is

$y - 0 = 12(x-2)$ or $y = 12x - 24$.

53. $y = \ln(x^2+y^2) \quad \Rightarrow \quad y' = \dfrac{1}{x^2+y^2}\dfrac{d}{dx}(x^2+y^2) \quad \Rightarrow \quad y' = \dfrac{2x+2yy'}{x^2+y^2} \quad \Rightarrow \quad x^2y' + y^2y' = 2x + 2yy' \quad \Rightarrow$

$x^2y' + y^2y' - 2yy' = 2x \quad \Rightarrow \quad (x^2+y^2-2y)y' = 2x \quad \Rightarrow \quad y' = \dfrac{2x}{x^2+y^2-2y}$

63. $y = f(x) = \ln(1+x^2)$ **A.** $D = \mathbb{R}$ **B.** Both intercepts are 0. **C.** $f(-x) = f(x)$, so the curve is symmetric about the y-axis. **D.** $\lim\limits_{x \to \pm\infty} \ln(1+x^2) = \infty$, no asymptotes. **E.** $f'(x) = \dfrac{2x}{1+x^2} > 0 \quad \Leftrightarrow$

$x > 0$, so f is increasing on $(0, \infty)$ and decreasing on $(-\infty, 0)$.

F. $f(0) = 0$ is a local and absolute minimum.

G. $f''(x) = \dfrac{2(1+x^2) - 2x(2x)}{(1+x^2)^2} = \dfrac{2(1-x^2)}{(1+x^2)^2} > 0 \quad \Leftrightarrow$

$|x| < 1$, so f is CU on $(-1, 1)$, CD on $(-\infty, -1)$ and $(1, \infty)$. IP $(1, \ln 2)$ and $(-1, \ln 2)$.

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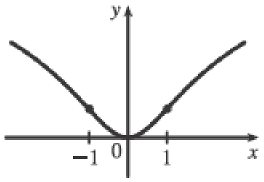

64. $y = f(x) = \ln(x^2 - 3x + 2) = \ln\left[(x-1)(x-2)\right]$ **A.** $D = \{x \text{ in } \mathbb{R} \mid x^2 - 3x + 2 > 0\} = (-\infty, 1) \cup (2, \infty)$.

B. y-intercept: $f(0) = \ln 2$; x-intercepts: $f(x) = 0 \quad \Leftrightarrow \quad x^2 - 3x + 2 = e^0 \quad \Leftrightarrow \quad x^2 - 3x + 1 = 0 \quad \Leftrightarrow$

$x = \frac{3 \pm \sqrt{5}}{2} \quad \Rightarrow \quad x \approx 0.38, 2.62$ **C.** No symmetry **D.** $\lim\limits_{x \to 1^-} f(x) = \lim\limits_{x \to 2^+} f(x) = -\infty$, so $x = 1$ and $x = 2$ are VAs.

No HA **E.** $f'(x) = \dfrac{2x-3}{x^2-3x+2} = \dfrac{2(x-3/2)}{(x-1)(x-2)}$, so $f'(x) < 0$ for $x < 1$ and $f'(x) > 0$ for $x > 2$. Thus, f is decreasing on $(-\infty, 1)$ and increasing on $(2, \infty)$. **F.** No extreme values

G. $$f''(x) = \frac{(x^2-3x+2)\cdot 2 - (2x-3)^2}{(x^2-3x+2)^2}$$

$$= \frac{2x^2 - 6x + 4 - 4x^2 + 12x - 9}{(x^2-3x+2)^2} = \frac{-2x^2 + 6x - 5}{(x^2-3x+2)^2}$$

The numerator is negative for all x and the denominator is positive, so $f''(x) < 0$ for all x in the domain of f. Thus, f is CD on $(-\infty, 1)$ and $(2, \infty)$.

No IP

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