Mathematical Induction is a method of proof used to show that an algebraic statement is true for all positive integers.

To prove a statement is true for all positive integers n:

1. Show that the statement is true for $n = 1$. (Basis Step)
2. Assume that the statement is true for $n = k - 1$, where $k - 1$ is any positive integer
   Show that the statement is true for the next positive integer, $n = k$.
   } Inductive Step

Example 1. Prove by induction: $1 + 4 + 7 + \ldots + (3n-2) = \frac{n(3n-1)}{2}$

1. For $n=1$, $3\cdot1-2 = \frac{1(3\cdot1-1)}{2}$ } (replace n with 1 in prove statement)
$$3-2 = 1\cdot\frac{2}{2}$$
$$1 = 1 \checkmark$$

2.

(Substitute K-1 for n in prove statement)

Assume that $1+4+7+\cdots+(3(K-1)-2) = \frac{(K-1)(3(K-1)-1)}{2}$

Show that $1+4+7+\cdots+(3K-2) = \frac{K(3K-1)}{2}$
(substitute K for n in prove st.)

Proof: $1+4+7+\cdots+(3(K-1)-2) = \frac{(K-1)(3(K-1)-1)}{2}$ (Begin with what you assumed)

(simplify)

$1+4+7+\cdots+(3K-5)+(3K-2) = \frac{(K-1)(3K-4)}{2} + 3K-2$

(Add next term (3K-2) to both sides)

$$= \frac{(3K^2-7K+4)+2(3K-2)}{2}$$

(Do all necessary algebra until you reach what you said you would show)

$$= \frac{3K^2-K}{2}$$

$$= \frac{K(3K-1)}{2}$$

Therefore, $1+4+7+\cdots+(3n-2) = \frac{n(3n-1)}{2}$. ("wrap up" statement)

* words in ( ) are directions, & should not be part of your proof. *

Example 2. **Prove by induction:** $\sum_{i=1}^{n} 5^i = \dfrac{5^{n+1}-5}{4}$

1\. For $n=1$, $5^1 = \dfrac{5^{1+1}-5}{4}$

$$5 = \frac{5^2-5}{4}$$

$$5 = 5 \checkmark$$

2\. Assume that $5^1+5^2+5^3+\cdots+5^{k-1} = \dfrac{5^{(k-1)+1}-5}{4}$

Show that $5^1+5^2+5^3+\cdots+5^k = \dfrac{5^{k+1}-5}{4}$.

Proof: $5^1+5^2+5^3+\cdots+5^{k-1} = \dfrac{5^{(k-1)+1}-5}{4}$

$$5^1+5^2+5^3+\cdots+5^{k-1}+5^k = \frac{5^k-5}{4} + 5^k$$

$$= \frac{5^k-5+4\cdot 5^k}{4}$$

$$= \frac{5\cdot 5^k-5}{4} \quad \left(\text{combine like terms } 5^k+4\cdot 5^k\right)$$

$$= \frac{5^{k+1}-5}{4}. \quad \left(\text{combine exponents}\right)$$

Therefore, $\sum_{i=1}^{n} 5^i = \dfrac{5^{n+1}-5}{4}$.