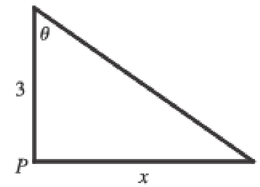
38. We are given that $\dfrac{d\theta}{dt} = 4(2\pi) = 8\pi$ rad/min. $x = 3\tan\theta \quad \Rightarrow$

$\dfrac{dx}{dt} = 3\sec^2\theta\,\dfrac{d\theta}{dt}$. When $x = 1$, $\tan\theta = \frac{1}{3}$, so $\sec^2\theta = 1 + \left(\frac{1}{3}\right)^2 = \frac{10}{9}$

and $\dfrac{dx}{dt} = 3\left(\frac{10}{9}\right)(8\pi) = \frac{80}{3}\pi \approx 83.8$ km/min.

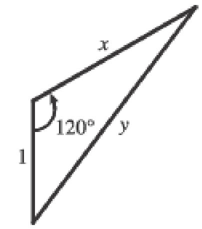


41. We are given that $\dfrac{dx}{dt} = 300$ km/h. By the Law of Cosines,

$y^2 = x^2 + 1^2 - 2(1)(x)\cos 120° = x^2 + 1 - 2x\left(-\frac{1}{2}\right) = x^2 + x + 1$, so

$2y\,\dfrac{dy}{dt} = 2x\,\dfrac{dx}{dt} + \dfrac{dx}{dt} \quad \Rightarrow \quad \dfrac{dy}{dt} = \dfrac{2x+1}{2y}\,\dfrac{dx}{dt}$. After 1 minute, $x = \frac{300}{60} = 5$ km $\quad \Rightarrow$

$y = \sqrt{5^2 + 5 + 1} = \sqrt{31}$ km $\quad \Rightarrow \quad \dfrac{dy}{dt} = \dfrac{2(5)+1}{2\sqrt{31}}(300) = \dfrac{1650}{\sqrt{31}} \approx 296$ km/h.



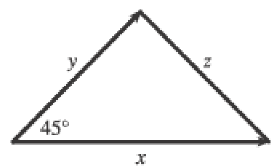
42. We are given that $\dfrac{dx}{dt} = 3$ mi/h and $\dfrac{dy}{dt} = 2$ mi/h. By the Law of Cosines,

$z^2 = x^2 + y^2 - 2xy\cos 45° = x^2 + y^2 - \sqrt{2}\,xy \quad \Rightarrow$

$2z\dfrac{dz}{dt} = 2x\,\dfrac{dx}{dt} + 2y\,\dfrac{dy}{dt} - \sqrt{2}\,x\,\dfrac{dy}{dt} - \sqrt{2}\,y\,\dfrac{dx}{dt}$. After 15 minutes $\left[= \frac{1}{4}\text{ h}\right]$,

we have $x = \frac{3}{4}$ and $y = \frac{2}{4} = \frac{1}{2} \quad \Rightarrow \quad z^2 = \left(\frac{3}{4}\right)^2 + \left(\frac{2}{4}\right)^2 - \sqrt{2}\left(\frac{3}{4}\right)\left(\frac{2}{4}\right) \quad \Rightarrow \quad z = \dfrac{\sqrt{13 - 6\sqrt{2}}}{4}$ and

$$\frac{dz}{dt} = \frac{2}{\sqrt{13 - 6\sqrt{2}}}\left[2\left(\tfrac{3}{4}\right)3 + 2\left(\tfrac{1}{2}\right)2 - \sqrt{2}\left(\tfrac{3}{4}\right)2 - \sqrt{2}\left(\tfrac{1}{2}\right)3\right] = \frac{2}{\sqrt{13 - 6\sqrt{2}}}\,\frac{13 - 6\sqrt{2}}{2} = \sqrt{13 - 6\sqrt{2}} \approx 2.125 \text{ mi/h.}$$

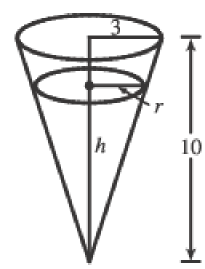


78. Given $dV/dt = 2$, find dh/dt when $h = 5$. $V = \frac{1}{3}\pi r^2 h$ and, from similar

triangles, $\dfrac{r}{h} = \dfrac{3}{10} \quad \Rightarrow \quad V = \dfrac{\pi}{3}\left(\dfrac{3h}{10}\right)^2 h = \dfrac{3\pi}{100}h^3$, so

$2 = \dfrac{dV}{dt} = \dfrac{9\pi}{100}h^2\,\dfrac{dh}{dt} \quad \Rightarrow \quad \dfrac{dh}{dt} = \dfrac{200}{9\pi h^2} = \dfrac{200}{9\pi\,(5)^2} = \dfrac{8}{9\pi}$ cm/s

when $h = 5$.

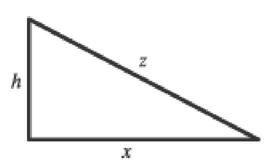


79. Given $dh/dt = 5$ and $dx/dt = 15$, find dz/dt. $z^2 = x^2 + h^2 \quad \Rightarrow$

$2z\,\dfrac{dz}{dt} = 2x\,\dfrac{dx}{dt} + 2h\,\dfrac{dh}{dt} \quad \Rightarrow \quad \dfrac{dz}{dt} = \dfrac{1}{z}(15x + 5h)$. When $t = 3$,

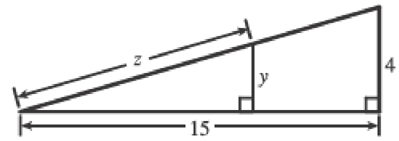
$h = 45 + 3(5) = 60$ and $x = 15(3) = 45 \quad \Rightarrow \quad z = \sqrt{45^2 + 60^2} = 75$,

so $\dfrac{dz}{dt} = \dfrac{1}{75}[15(45) + 5(60)] = 13$ ft/s.



80. We are given $dz/dt = 30$ ft/s. By similar triangles, $\frac{y}{z} = \frac{4}{\sqrt{241}} \Rightarrow$

$y = \frac{4}{\sqrt{241}} z$, so $\frac{dy}{dt} = \frac{4}{\sqrt{241}} \frac{dz}{dt} = \frac{120}{\sqrt{241}} \approx 7.7$ ft/s.



81. We are given $d\theta/dt = -0.25$ rad/h. $\tan\theta = 400/x \Rightarrow$

$x = 400 \cot\theta \Rightarrow \frac{dx}{dt} = -400 \csc^2\theta \, \frac{d\theta}{dt}$. When $\theta = \frac{\pi}{6}$,

$\frac{dx}{dt} = -400(2)^2(-0.25) = 400$ ft/h.