

Model Card

Differential Equations - Separable First Order DIFF'L Eqns

Problem:

If $\frac{dy}{dx} = y \cdot \sec^2 x$ and $y=5$ when $x=0$,

Then $y =$

(A) $e^{\tan x} + 4$ (D) $\tan x + 5$

(B) $e^{\tan x} + 5$ (E) $\tan x + 5e^x$

(C) $5e^{\tan x}$

Solution:

$$\frac{dy}{dx} = y \cdot \sec^2 x$$

Separate Variables \rightarrow

$$\frac{1}{y} dy = \sec^2 x dx$$

Integrate Both Sides

$$\int \frac{1}{y} dy = \int \sec^2 x dx$$

$$\ln|y| = \tan x + K$$

$$y = e^{\tan x + K}$$

$$y = C e^{\tan x}$$

$$e^{\tan x + K} = e^{\tan x} \cdot e^K$$

given $y=5$ when $x=0$

$$5 = C e^{\tan(0)}$$

$$5 = C$$

$$\therefore y = 5e^{\tan x}$$

Concepts and Connections

Differential Equation

\hookrightarrow eq'n w/ a derivative

\hookrightarrow e.g. $\frac{dy}{dx}$

How to solve:

① Separate variables

\hookrightarrow x on one side,
 y on the other side

② Integrate

③ Apply initial conditions if given.