14.7: Half-Angle Identities

★ identities 15-19

examples: find exact values

1 $\sin 112.5^\circ$ (Q2 +)

$= \sin\left(\frac{225^\circ}{2}\right)^\circ = \pm\sqrt{\frac{1-\cos 225^\circ}{2}}$ ($\theta = 225^\circ$)

$= \sqrt{\frac{\left[1-\left(-\sqrt{2}/2\right)\right]2}{[2]2}}$

$= \sqrt{\frac{2+\sqrt{2}}{4}} = \frac{\sqrt{2+\sqrt{2}}}{2}$

2 $\tan 105^\circ$ (Q2 −)

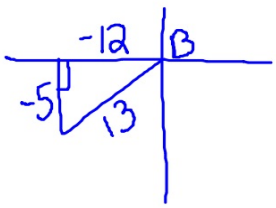
$= \tan\left(\frac{210}{2}\right)^\circ$ ($\theta = 210^\circ$)

$= \frac{1-\cos 210^\circ}{\sin 210^\circ}$

$= \frac{\left[1-\left(-\frac{\sqrt{3}}{2}\right)\right](-2)}{\left[-\frac{1}{2}\right](-2)}$

$= -2-\sqrt{3}$

3 Given: $\tan B = \frac{5}{12}$, $\pi \le B < \frac{3\pi}{2}$



Q3

$\frac{\pi}{2} \le \frac{B}{2} < \frac{3\pi}{4}$ Q2

find $\cos\frac{B}{2}$ Q2 −

$$= -\sqrt{\frac{1+\cos B}{2}} = -\sqrt{\frac{1-12/13}{2}}$$

$$= -\sqrt{1/13 \cdot 1/2} = -\sqrt{\frac{1}{26}}$$

$$= -\frac{\sqrt{26}}{26}$$

Prove: $\cos\frac{B}{2} = \pm\sqrt{\frac{1+\cos B}{2}}$

Proof:

$X = \frac{B}{2}$

$\cos 2x = 2\cos^2 x - 1$

$\cos 2\left(\frac{B}{2}\right) = 2\cos^2\left(\frac{B}{2}\right) - 1$

$\cos B = 2\cos^2\frac{B}{2} - 1$

$\sqrt{\frac{1+\cos B}{2}} = \sqrt{\cos^2\frac{B}{2}}$

$\pm\sqrt{\frac{1+\cos B}{2}} = \cos\frac{B}{2}$