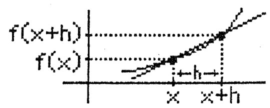
2a (11-97)

Calcu-List



$$\frac{dy}{dx} = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

Vertical Motion in Feet and Seconds:

$a(t) = v'(t) = h''(t)$

$h(t) = -16t^2 + V_0 t + H_0$

$v(t) = -32t + V_0$

$a(t) = -32$

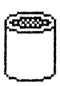
Volumes of Rotation

Discs

$\pi \int r^2 \, dx$

Washers

$\pi \int R^2 - r^2 \, dx$



Shells

$2\pi \int r\,h \, dx$

The Fundamental Theorem of Calculus!

If $f'(x)$ is continuous from a to b then:

$$\int_a^b f'(x)\,dx = f(b) - f(a)$$

If $f(x)$ is continuous from a to b then:

$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$

Chain Rule

$$\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx} \quad \text{or} \quad \frac{d}{dx} f(g(x)) = f'(g(x))\cdot g'(x)$$

Graphing Tips

$\lim_{x\to\pm\infty} f(x) = c$ → Horizontal Asymptote at $y = c$

$\lim_{x\to\pm\infty} f(x) = cx$ → Slant Asymptote with slope c

f(undefined value) $= \frac{c}{0}$ → Vertical Asymptote

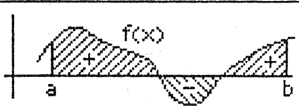
f(undefined value) $= \frac{0}{0}$ → Hole in the graph

y' = slope → + ↗ 0 ↘ −

y'' = concavity → (+) 0 (−)

y' = 0 or Ø → Indicates possible Max or Min

y'' = 0 or Ø → Indicates possible Inflection Point



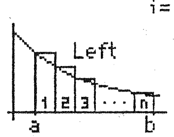
$$\text{Net Area} = \int_a^b f(x)\,dx$$

Trapezoidal Rule (n is the number of trapezoids)

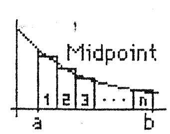
$$\int_a^b f(x)\,dx \approx \frac{b-a}{2n}\left(f(x_0) + 2f(x_1) + 2f(x_2) + \ldots + f(x_n)\right)$$

Approximate Area Using Rectangles of Equal Width

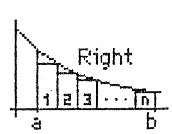
$$\text{Area} = \sum_{i=1}^{n} f(c_i)\cdot\Delta x \qquad \Delta x = \frac{b-a}{n}$$



$c_i = a + (i-1)\cdot\Delta x$



$c_i = a + (i-\frac{1}{2})\cdot\Delta x$

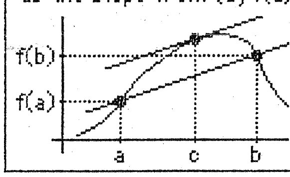


$c_i = a + i\cdot\Delta x$

Derivatives and Integrals

$\frac{d}{dx}(ax^n) = nax^{n-1}$	$\int x^n\,dx = \frac{x^{n+1}}{n+1} + c$
$\frac{d}{dx}(uv) = u'v + uv'$	$\int e^x\,dx = e^x + c$
$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{u'v - uv'}{v^2}$	$\int \frac{1}{x}\,dx = \ln\lvert x\rvert + c$
$\frac{d}{dx}(e^x) = e^x$	$\int \sin x\,dx = -\cos x + c$
$\frac{d}{dx}(\ln x) = \frac{1}{x}$	$\int \cos x\,dx = \sin x + c$
$\frac{d}{dx}(\sin x) = \cos x$	$\int \tan x\,dx = -\ln\lvert\cos x\rvert + c$
$\frac{d}{dx}(\cos x) = -\sin x$	$\int \cot x\,dx = \ln\lvert\sin x\rvert + c$
$\frac{d}{dx}(\tan x) = \sec^2 x$	$\int \sec x\,dx = -\ln\lvert\sec x - \tan x\rvert + c$
$\frac{d}{dx}(\cot x) = -\csc^2 x$	$\int \csc x\,dx = \ln\lvert\csc x - \cot x\rvert + c$
$\frac{d}{dx}(\sec x) = \sec x \tan x$	$\int \sec^2 x\,dx = \tan x + c$
$\frac{d}{dx}(\csc x) = -\csc x \cot x$	$\int \csc^2 x\,dx = -\cot x + c$
$\frac{d}{dx}(\arcsin x) = \frac{1}{\sqrt{1-x^2}}$	$\int \sec x \tan x\,dx = \sec x + c$
$\frac{d}{dx}(\arccos x) = \frac{-1}{\sqrt{1-x^2}}$	$\int \csc x \cot x\,dx = -\csc x + c$
$\frac{d}{dx}(\arctan x) = \frac{1}{1+x^2}$	$\int \frac{1}{\sqrt{1-x^2}}\,dx = \arcsin x + c$
$\frac{d}{dx}(\text{arccot } x) = \frac{-1}{1+x^2}$	$\int \frac{1}{1+x^2}\,dx = \arctan x + c$
$\frac{d}{dx}(\text{arcsec } x) = \frac{1}{\lvert x\rvert\sqrt{x^2-1}}$	$\int \frac{1}{x\sqrt{x^2-1}}\,dx = \text{arcsec}\lvert x\rvert + c$
$\frac{d}{dx}(\text{arccsc } x) = \frac{-1}{\lvert x\rvert\sqrt{x^2-1}}$	

If $f(x)$ is continuous and differentiable from a to b, then there is a x-value c such that the slope at c is the same as the slope from (a, f(a)) to (b, f(b)).



The Mean Value Theorem

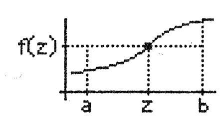
$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

From a to b on a continuous $f(x)$ there is a z such that:

- At z, $f(x)$ takes on the average value.
- $f(z)$ is the average value.

Average Value

$$f(z) = \frac{\int_a^b f(x)\,dx}{b-a}$$



Separable Differential Equations -- Exponential Growth

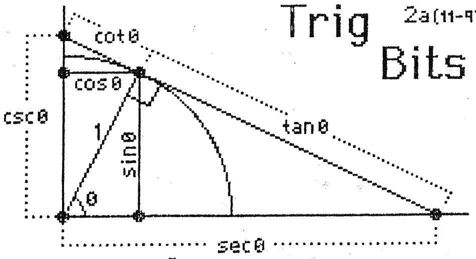
When y is directly proportional to the rate at which y changes: $\Rightarrow \frac{dy}{dt} = ry$

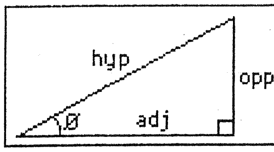
$$\Rightarrow \frac{1}{y}\,dy = r\,dt \Rightarrow \int \frac{1}{y}\,dy = \int r\,dt \Rightarrow \ln y = rt + c$$

$$\Rightarrow e^{\ln y} = e^{rt+c} \Rightarrow y = e^{rt}\cdot e^{c} \Rightarrow \boxed{y = pe^{rt}}$$

Trig Bits

2a(11-97)





$\text{Sin } Ø = \frac{\text{opp}}{\text{hyp}}$ $\text{Tan } Ø = \frac{\text{opp}}{\text{adj}}$

$\text{Cos } Ø = \frac{\text{adj}}{\text{hyp}}$

opp = hyp • sin Ø
adj = hyp • cos Ø

On the Unit Circle

$\text{Sin}\,\theta = y$

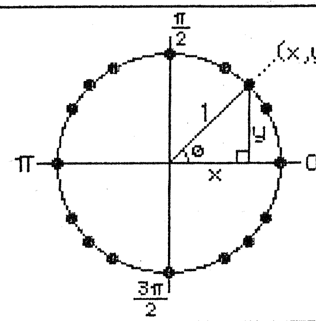
$\text{Cos}\,\theta = x$

$\text{Tan}\,\theta = \frac{y}{x} = \frac{\text{Sin}\,\theta}{\text{Cos}\,\theta}$

$\text{Cot}\,\theta = \frac{x}{y} = \frac{\text{Cos}\,\theta}{\text{Sin}\,\theta}$

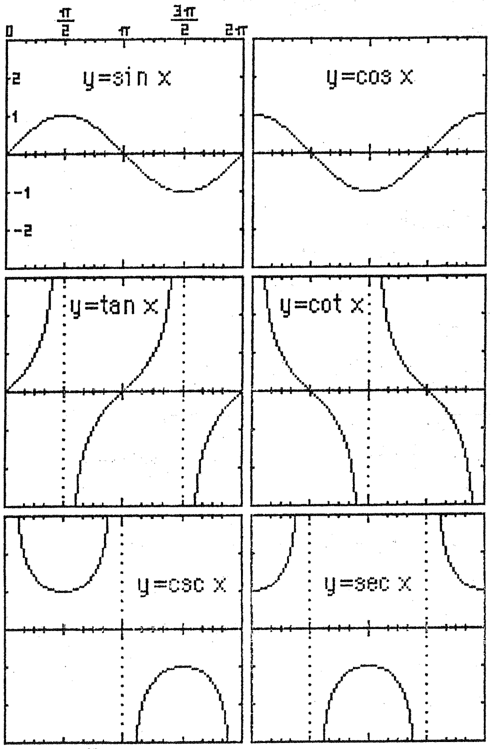
$\text{Sec}\,\theta = \frac{1}{x} = \frac{1}{\text{Cos}\,\theta}$

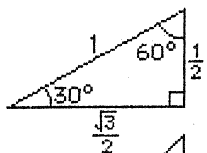
$\text{Csc}\,\theta = \frac{1}{y} = \frac{1}{\text{Sin}\,\theta}$

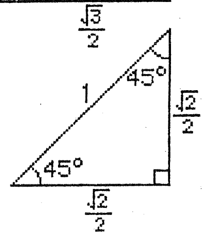


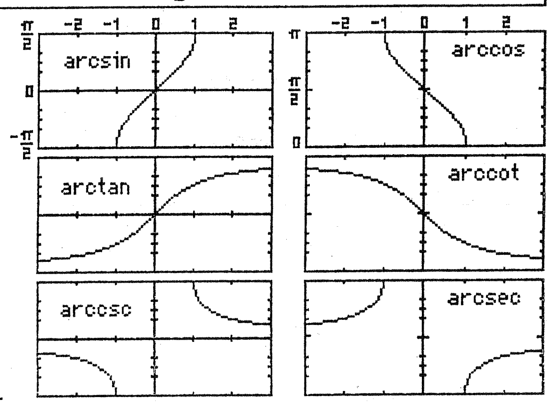
Positive (A,S,T,C)

SinØ CscØ	All
TanØ CotØ	CosØ SecØ









Addition Identities

Sin(a ± b) = Sin a Cos b ± Cos a Sin b

Cos(a ± b) = Cos a Cos b ∓ Sin a Sin b

$\text{Tan}(a \pm b) = \frac{\text{Tan a} \pm \text{Tan b}}{1 \mp \text{Tan a Tan b}}$

Negative Angles

Sin(-a) = -Sin(a)

Cos(-a) = Cos(a)

Tan(-a) = -Tan(a)

m°	m^r	SinØ	CosØ	TanØ	CotØ	CscØ	SecØ	Arc Ansr
0	0	0	1	0	Undef	Undef	1	0
30	π/6	1/2	√3/2	√3/3	√3	2	2√3/3	π/6
45	π/4	√2/2	√2/2	1	1	√2	√2	π/4
60	π/3	√3/2	1/2	√3	√3/3	2√3/3	2	π/3
90	π/2	1	0	Undef	0	1	Undef	π/2
120	2π/3	√3/2	-1/2	-√3	-√3/3	2√3/3	-2	2π/3
135	3π/4	√2/2	-√2/2	-1	-1	√2	-√2	3π/4
150	5π/6	1/2	-√3/2	-√3/3	-√3	2	-2√3/3	5π/6
180	π	0	-1	0	Undef	Undef	-1	π
210	7π/6	-1/2	-√3/2	√3/3	√3	-2	-2√3/3	
225	5π/4	-√2/2	-√2/2	1	1	-√2	-√2	
240	4π/3	-√3/2	-1/2	√3	√3/3	-2√3/3	-2	
270	3π/2	-1	0	Undef	0	-1	Undef	-π/2
300	5π/3	-√3/2	1/2	-√3	-√3/3	-2√3/3	2	-π/3
315	7π/4	-√2/2	√2/2	-1	-1	-√2	√2	-π/4
330	11π/6	-1/2	√3/2	-√3/3	-√3	-2	2√3/3	-π/6
360	2π	0	1	0	Undef	Undef	1	0

Half Angle Formulas

$\text{Sin}\frac{Ø}{2} = \pm\sqrt{\frac{1 - \text{Cos } Ø}{2}}$

$\text{Cos}\frac{Ø}{2} = \pm\sqrt{\frac{1 + \text{Cos } Ø}{2}}$

$\text{Tan}\frac{Ø}{2} = \frac{1 - \text{Cos } Ø}{\text{Sin } Ø} = \frac{\text{Sin}Ø}{1 + \text{Cos } Ø}$

Double Angle Formulas

$\text{Sin}(2Ø) = 2 \text{ Sin } Ø \text{ Cos } Ø$

$\text{Cos}(2Ø) = \text{Cos}^2 Ø - \text{Sin}^2 Ø$

$\text{Cos}(2Ø) = 2 \text{ Cos}^2 Ø - 1$

$\text{Cos}(2Ø) = 1 - 2 \text{ Sin}^2 Ø$

Triangle Identities

$\text{Sin}^2 Ø + \text{Cos}^2 Ø = 1$

$1 + \text{Tan}^2 Ø = \text{Sec}^2 Ø$

$1 + \text{Cot}^2 Ø = \text{Csc}^2 Ø$

Cofunction Identities

$\text{Sin}\left(\frac{\pi}{2} - Ø\right) = \text{Cos } Ø$

$\text{Cos}\left(\frac{\pi}{2} - Ø\right) = \text{Sin } Ø$

$\text{Tan}\left(\frac{\pi}{2} - Ø\right) = \text{Cot } Ø$

Conversions: $m^\circ \cdot \frac{\pi}{180^\circ} = m^r$ $m^r \cdot \frac{180^\circ}{\pi} = m^\circ$

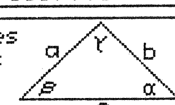
Aproximate π's: 22/7 or 355/113

$y = A \text{ Sin}[B(x - C)] + D$

$y = A \text{ Cos}[B(x - C)] + D$

A = Amplitude
B = Frequency in 2π
(2π/B = Period)
C = x shift
D = y shift

Laws of Sines and Cosines



$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$

$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \text{Cos}\,\alpha$

AP Calculus Workshop - Larry Peterson

Calculus: Words to Live By

1. **Particle Motion:**
Position to velocity (take derivative)
Velocity to acceleration (take derivative)
To go back – integrate – don't forget "C"
If velocity is positive – you are moving in a positive direction (right, up, ...)
If velocity and acceleration have opposite signs – you are slowing down.
Distance means how far you have traveled in any direction...
   - if you have calculator: take the integral of the absolute value of velocity
   - if you don' have a calculator: find the point where velocity changes from positive to negative (a zero) and make two or more integrals

Displacement means the net distance from where you started – a regular integral will do
Speed is the absolute value of velocity (special formula for parametric BC)

2. **Function questions:**
Even functions: $f(x) = f(-x)$ - symmetric with y-axis
Odd functions: $f(-x) = -f(x)$ symmetric with the origin
Domain: Look to exclude x-values – look for negatives under square roots, denominators equal to zero – logs of non-positive numbers. These will give you the values you need to exclude from your domain.
Range: Find the x-coordinate of the minimum. Then find the y-coordinate. Your range goes from $(-\infty$, max] or [min, $\infty)$ or [min,max]

3. **Tangent lines:**
The derivative at one point on a curve = slope of tangent line at that point
You also need a point on the curve! If given x, find y in the original equation.
A normal line is perpendicular to a tangent line at the point of tangency, slope is the opposite and reciprocal of the derivative

4. **Minimums, Maximums and POI's**
The minimum value or maximum value is a **Y-VALUE!!!**
To find max's and min's- first derivative set equal to zero or undefined and sign chart, EXPLAIN the sign chart to the reader!!
A function has a <u>minimum</u> if $f' = 0$ or undefined <u>and</u> $f'' =$ positive
A function has a <u>maximum</u> if $f' = 0$ or undefined <u>and</u> $f'' =$ negative
Poi's – second derivative set equal to zero or undefined and sign chart, EXPLAIN the chart
Concave up and concave down – second derivative (+) or (-) respectively
Cusps: First derivative undefined and function is defined
Critical Points: First derivative is zero or undefined
Asymptotes: First derivative and function are both undefined
Be careful of the word "absolute" vs local
Always check endpoints on closed intervals for the max or min y-values

5. **Related rates**
A rate is a derivative with respect to time.
Write down any given rates with units
Write down any equations or formulas you come up with relating the quantities given in problem.
Look for ways to substitute
If you are taking derivative with respect to time (t) – don't forget to implicitly differentiate both sides.
If you need to integrate – you must separate your variables before doing so!!!
Don't forget +C so you can use the initial conditions.

6. **Differential Equations**

Separate variables through multiplication or division

Keep constants on the "x" side (the independent variable side)

Proportional means $y = kf(x)$ - Don't forget the k

Don't forget +C

Remember every problem is not an *ln*!

7. **Area and volume:**

Draw a picture.

<u>Look carefully at your radius and the height and how they relate to the curve!!!</u>

- Discs: $\pi\int_a^b (f(x))^2\,dx$
- Washers: $\pi\int_a^b \left(R^2 - r^2\right)dx$ (Outside radius squared – Inside radius squared)
- Known cross sections: Set up the *Area of one slice times the thickness*
- Shells: $2\pi\int_a^b rht$ (will never be tested but you can use them, especially if the equation is difficult to solve for the other variable)

8. **Other stuff**

- A Riemann sum means rectangles. You must show that you are adding up rectangles. If the problem says midpoint Riemann sum, make sure to use the y-value of the midpoint of the rectangle as your height. A picture is helpful. This is an estimate for integral. Same kind of deal with trapezoids.
- Use your calculator to integrate unless the problem states you must show your integration. Make sure you show your correct set-up integral.
- Don't simplify beyond the very basic or obvious. Rarely should you square an expression or cube an expression. But don't leave EVERYTHING un-simplified. You will be expected to add like terms, etc.
- If you need to evaluate a tricky function – put it into the calculator in steps if you think you are messing up your parentheses. Use your table to evaluate. (Table-set ASK for independent variable) Or store your value for "x" and then type in the expression in terms of the stored variable.
- Read carefully!!! Many points have been lost by revolving around the wrong axis, forgetting units, putting "=" instead of "$\approx$" or not rounding appropriately.
- Substitute the right value. If your equation is in terms of "x" don't substitute a value for "t".
- Just because the math is messy or the answer "doesn't look nice" doesn't mean it is wrong. If you know you should take a derivative – take a derivative regardless of how the answer looks.
- Try to figure out what you are being asked – look for words like "tangent line", "rate", "minimum" or "maximum". CIRCLE THESE IN THE PROBLEM AS YOU READ IT. It will remind you at the end of problem exactly you need to find and what kind of answer you should give to the reader of the problem!!

AP Calculus
Stuff You Mus' 'now - AB

Trig Stuff
Identities

$\sin 2x = 2\sin x \cos x$

$\cos 2x = \cos^2 x - \sin^2 x$

$\cos 2x = 2\cos^2 x - 1$

$\cos 2x = 1 - 2\sin^2 x$

$\sin^2 x = \dfrac{1-\cos 2x}{2}$

$\cos^2 x = \dfrac{1+\cos 2x}{2}$

$\sin^2 x + \cos^2 x = 1$

$1 + \tan^2 x = \sec^2 x$

$1 + \cot^2 x = \csc^2 x$

$\sec x = \dfrac{1}{\cos x}$

$\csc x = \dfrac{1}{\sin x}$

$\sin(-x) = -\sin x$

$\cos(-x) = \cos x$

$\tan(-x) = -\tan x$

$\cot(-x) = -\cot x$

$\sec(-x) = \sec x$

$\csc(-x) = -\csc x$

Trig Values

θ	sin θ	cos θ	tan θ
0	0	1	0
$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
$\frac{\pi}{2}$	1	0	∞

Algebra Stuff

Slope: $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

Point-slope form: $y - y_0 = m(x - x_0)$

Standard form: $Ax + By = C$

Distance Formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Differential Calculus
Derivative Fc ulas and Rules

$$\frac{d}{dx}\left(x^n\right)=nx^{n-1}$$
$$\frac{d}{dx}(\sin x)=\cos x$$
$$\frac{d}{dx}(\cos x)=-\sin x$$
$$\frac{d}{dx}(\tan x)=\sec^2 x$$
$$\frac{d}{dx}(\cot x)=-\csc^2 x$$

$$\frac{d}{dx}(\sec x)=\sec x\tan x$$
$$\frac{d}{dx}(\csc x)=-\csc x\cot x$$
$$\frac{d}{dx}\left(e^x\right)=e^x$$
$$\frac{d}{dx}\left(a^x\right)=a^x\ln a$$
$$\frac{d}{dx}(\ln x)=\frac{1}{x}$$

$$\frac{d}{dx}(\log_b x)=\frac{1}{x\ln b}$$
$$\frac{d}{dx}\left(\sin^{-1}x\right)=\frac{1}{\sqrt{1-x^2}}$$
$$\frac{d}{dx}\left(\cos^{-1}x\right)=\frac{-1}{\sqrt{1-x^2}}$$
$$\frac{d}{dx}\left(\tan^{-1}x\right)=\frac{1}{1+x^2}$$
$$\frac{d}{dx}\left(\cot^{-1}x\right)=\frac{-1}{1+x^2}$$

$$\frac{d}{dx}\left(\sec^{-1}x\right)=\frac{1}{|x|\sqrt{x^2-1}}$$
$$\frac{d}{dx}\left(\csc^{-1}x\right)=\frac{-1}{|x|\sqrt{x^2-1}}$$
$$\frac{d}{dx}(uv)=uv'+vu'$$
$$\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{vu'-uv'}{v^2}$$
$$\frac{d}{dx}\left(f(g(x))\right)=f'(g(x))g'(x)$$

Uses of the first and second derivative

Curve Sketching

- $y=f(x)$ must be a continuous function on the given interval.
- To find a critical value, set $f'(x)=0$ or undefined.
- Use a labeled first-derivative chart to determine if the function has a relative max or min. Make sure you write sentences explaining why.
- Alternately, you can use the Second Derivative Test if $f'(x_0)=0$. If $f''(x_0)=+$, then x_0 is the x-coordinate of the relative minimum. If $f''(x_0)=-$, then x_0 is the x-coordinate of the relative maximum.
- To find points of inflection, set $f''(x)=0$ or undefined. Use a labeled second-derivative sign chart to show that the sign of $f''(x)$ changes at that point.

Three Important Theorems

Intermediate Value Theorem

If a function, $f(x)$ is continuous on a closed interval [a, b], and *y* is some value between *f(a)* and *f(b)*, then there exists at least one number *x* = *c* in the open interval (a, b) where $f(c) = y$.

In simple words, the function must pass through every y-value between *f(a)* and *f(b)*.

Mean Value Theorem for Derivatives

If the function, $f(x)$ is continuous on the closed interval [a, b] AND $f(x)$ is differentiable on the open interval (a, b), then there exists at least one number *x* = *c* in the open interval (a, b) such that $f'(c) = \frac{f(b) - f(a)}{b - a}$.

In simple words, on a smooth, continuous function there is at least one point in the interval (a, b) where the slope of the tangent line is parallel to the secant line drawn through the endpoints of the interval.

Rolle's Theorem

If the function, $f(x)$ is continuous on the closed interval [a, b] AND $f(x)$ is differentiable on the open interval (a, b), AND *f(a)* = *f(b)*, then there exists at least one number *x* = *c* in the open interval (a, b) such that $f'(c) = 0$.

In simple words, if the endpoints of the interval of a differentiable function have the same y-coordinates, then there is at least one point in the interval (a, b) where the slope of the tangent line is equal to zero.
This is really a special case of the Mean Value Theorem.

Integral Formulas

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c;\ n \neq -1$$

$$\int \frac{1}{x} dx = \ln|x| + c$$

$$\int e^x dx = e^x + c$$

$$\int a^x dx = \frac{a^x}{\ln a} + c$$

$$\int \sin x\, dx = -\cos x + c$$

$$\int \cos x\, dx = \sin x + c$$

$$\int \tan x\, dx = -\ln|\cos x| + c$$

$$\int \cot x\, dx = \ln|\sin x| + c$$

$$\int \sec x\, dx = \ln|\sec x + \tan x| + c$$

$$\int \csc x\, dx = -\ln|\csc x + \cot x| + c$$

$$\int \sec^2 x\, dx = \tan x + c$$

$$\int \csc^2 x\, dx = -\cot x + c$$

$$\int \sec x \tan x\, dx = \sec x + c$$

$$\int \csc x \cot x\, dx - \csc x + c$$

$$\int \frac{1}{\sqrt{1-x^2}} dx = \arcsin x + c$$

$$\int \frac{1}{1+x^2} dx = \arctan x + c$$

$$\int \frac{1}{x\sqrt{x^2-1}} dx = arc\sec x + c$$

$$\int \ln x\, dx = x \ln x - x + c$$

Fundamental Theorem of Calculus-- Part 1

$$\int_a^b f(x)\, dx = F(b) - F(a)$$

Where $F'(x) = f(x)$

Fundamental Theorem of Calculus-- Part 2

$$\frac{d}{dx}\left(\int_a^x f(t)\, dt\right) = f(x)$$

Average Value Theorem

If the function $f(x)$ is continuous on the closed interval [a, b], there exists some number *c* such that

$$AvgValue = f(c) = \frac{1}{b-a}\int_a^b f(x)\, dx$$

Trapezoidal Rule

$$\int_a^b f(x)\, dx \approx \frac{1}{2}\left(\frac{b-a}{n}\right) \cdot$$

$$\left(f(x_0)\right) + 2f(x_1) + \ldots + 2(f(x_{n-1}) + f(x_0))$$

Volume of a Solid of Revolution Disk Method

$$V = \pi\int_a^b \left((OR)^2 - (IR)^2\right) dx \ or \ dy$$

Volume of a Solid of Known Cross-Section

$$V = \int_a^b Area(x)\, dx$$

Particle Motion Formulas

velocity $=\frac{d}{dt}(\text{position})$	accleration $=\frac{d}{dt}(\text{velocity})$	displacement $=\int_{t_1}^{t_2} v(t)\,dt$
total distance $=\int_{t_1}^{t_2} \lvert v(t)\rvert\,dt$	average velocity $=\frac{\text{final position - initial position}}{\text{total time}}$	final position $=$ $x(t_2)=x(t_1)+\int_{t_1}^{t_2} v(t)\,dt$

L'Hopital's Rule

If $\lim\limits_{x\to a}\frac{f(x)}{g(x)}=\frac{0}{0}\,or\,\frac{\infty}{\infty}$, then $\lim\limits_{x\to a}\frac{f(x)}{g(x)}=\lim\limits_{x\to a}\frac{f'(x)}{g'(x)}$