SLOPE FIELDS

A slope field is a graphical display which shows the flow of tangent lines to the family of solution curves of a differential equation. Often, this flow diagram provides valuable information about the nature of the solution curve, i.e., whether it is polynomial, exponential, circular, trigonometric, etc. Slope fields can be constructed over a region of the plane by direct substitution into the differential equation, or they can be generated using the graphing calculator and an appropriate program.

There are generally two types of problems that involve slope fields. In the first, we are given a differential equation and asked to produce its slope field diagram. In the second, we are given a slope field and asked to match it to a particular differential equation which has those characteristics.

CONSTRUCTING A SLOPE FIELD.

When constructing slope fields, it is helpful to first create a table of values for a specific interval of x and y. For example, if we know that $dy/dx = xy/4$ for x in $[-2,2]$ and y in $[-2,2]$, we can construct a grid to evaluate the slopes at each integer pair of values. Complete the grid of slope values for this differential equation:

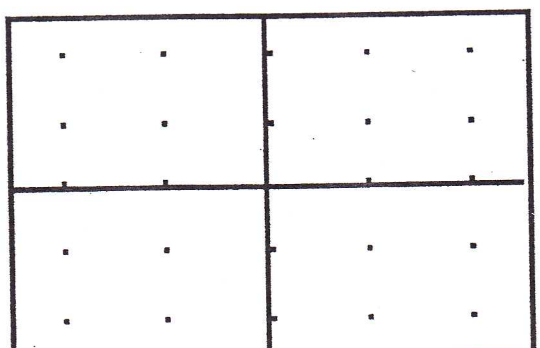
1.

x values

y values	-2	-1	0	1	2
2					
1					
0					
- 1					
- 2					

Once the grid is complete, the slope segments can be drawn through the lattice points of a graph. Draw the slope field for the values obtained above:

2.



3. What kind of function seems to be pictured by the slope field you constructed?

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READING A SLOPE FIELD.

When reading a slope field, it is important to look for clues the slope segments give us about the behavior of the differential equation and, by extension, its family of solutions.
It is possible to read a slope field one segment at a time; however, this can be exhausting for very large fields. It is probably easier in many cases to spot trends in the slope field that tell us something about how x and y are related in the differential equation. Here are some approaches you can use:

- Examine slope field segments along vertical lines. If the segments along each vertical line have the same slope, then the differential equation does not depend on y, because, as y varies, the slope does not.

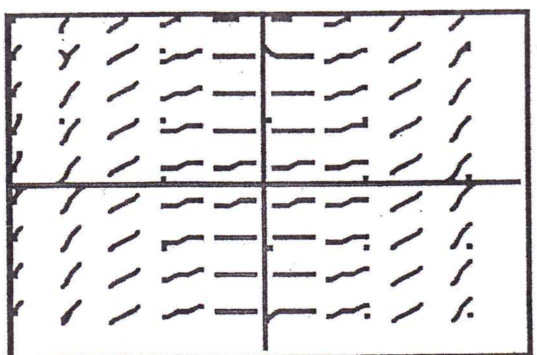
- Examine slope field segments along horizontal lines. If the segments along each horizontal line have the same slope, then the differential equation does not depend on x.

- Examine slope field segments in the first quadrant. If the segments have positive slope, then there are likely no negatives in the expression of the differential equation. If the slopes become larger as x gets larger, then dy/dx varies directly with x; likewise for y. Otherwise, we can determine that the slope is inversely related to one or both variables.

- If the slope field evinces a curve which looks familiar, check by differentiating that curve to see if its slope field fits the graphical data.

NOTE: There are occasional anomalies in the appearance of slope fields, due to the way they are generated on a calculator. Small discrepancies in slope can usually be dismissed.

Consider the following slope field:



[-2.5,2.5] x [-2.5,2.5]

4. What can you deduce from reading the slope segments? ______________________

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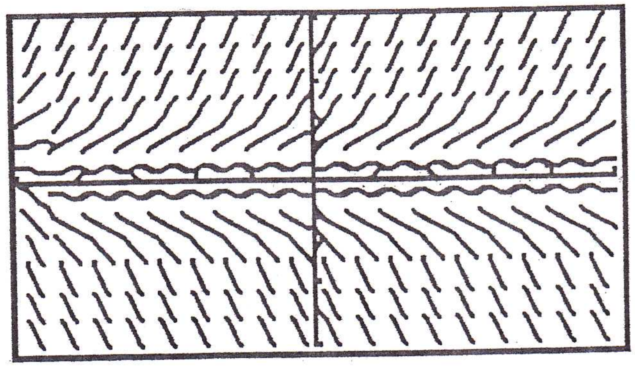
5. Which of these is most likely the differential equation:

(A) $dy/dx = .5xy$ (B) $dy/dx = x^2/y$ (C) $dy/dx = .5x^2$

Here are some more slope fields to practice on. In each, match the slope field with its differential equation.

6. Which of the following differential equations has the solution slope field pictured at right?



(A) dy/dx = .5y

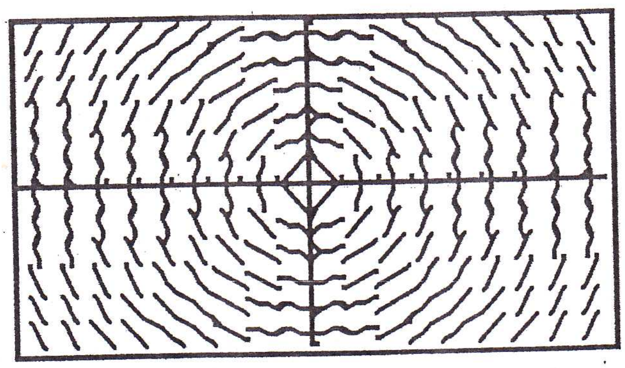
(B) dy/dx = .2x/y

(C) dy/dx = xy

(D) dy/dx = x + y

(E) dy/dx = 1/x

7. Which of the following differential equations has the solution slope field pictured at right?



(A) dy/dx = x^2

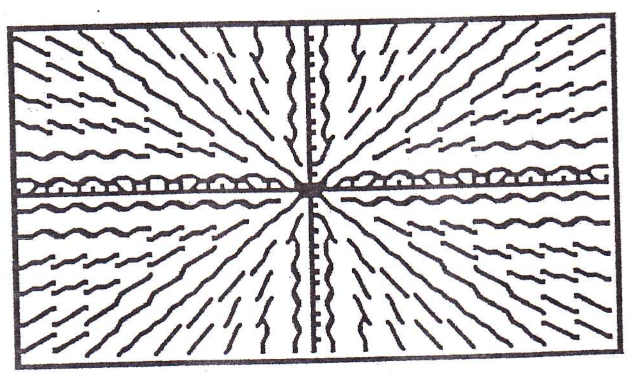
(B) dy/dx = y/x

(C) dy/dx = -y

(D) dy/dx = -x/y

(E) dy/dx = $x^2 + y^2$

8. Which of the following differential equations has the solution slope field pictured at right?



(A) dy/dx = x + y

(B) dy/dx = x - y

(C) dy/dx = x^2

(D) dy/dx = 2y

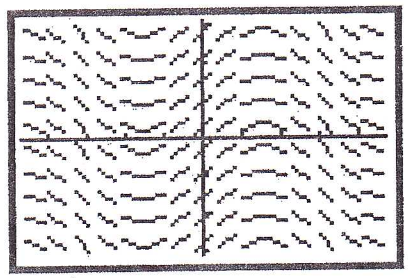
(E) dy/dx = y/x

Reading a Slope Field

A few features are easy to identify and help sort out most problems:

- Look for the places where the slopes are 0; that is, $\frac{dy}{dx} = 0$.
- Look at the slopes along the x-axis.
- Look at the slopes along the y-axis.
- Look to see if the slopes only depend on x.
- Look to see if the slopes only depend on y.
- Look to see where the slopes are positive and where they are negative.

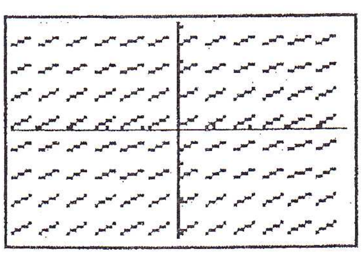
Let's put these ideas into practice.

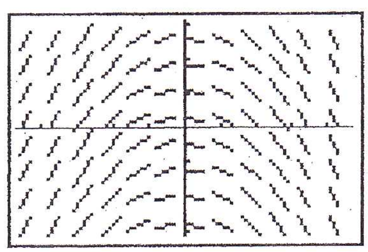


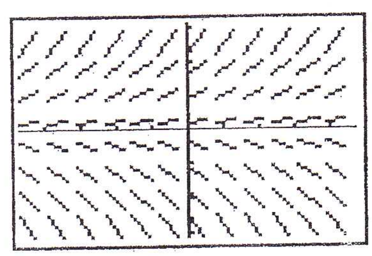
The slope field for the differential equation $\frac{dy}{dx} = \cos(x)$ is shown at the right. As expected, the slopes suggest parallel graphs of functions of the form $y = \sin(x) + C$, giving us a nice visualization of what we know to be the general solution. As useful as this visualization might be, though, slope fields are far more useful than that, as we hope to show in this little exploration.

A slope field gives us useful information about the solution to a differential equation even when we are unable to "solve" the differential equation itself. By analyzing the *slopes themselves* as they relate to the differential equation, we get an enhanced understanding of how much information a differential equation can convey.

Try this exploration to see how well you can match a differential equation to its slope field purely on the basis of the differential equation itself. Caution: do not attempt to solve the differential equations; they are much harder to *solve* than they look.



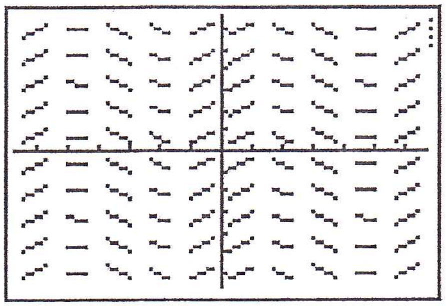




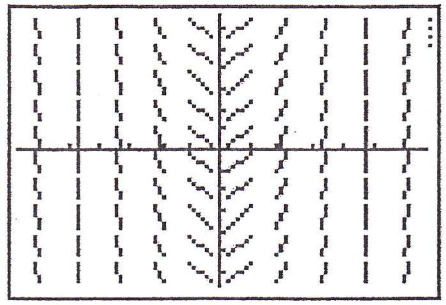
Match these: $\frac{dy}{dx} = y$. $\frac{dy}{dx} = 1$. $\frac{dy}{dx} = -x$.

Below are six examples of **slope fields**. Match them with the correct differential equation. Explain each choice.

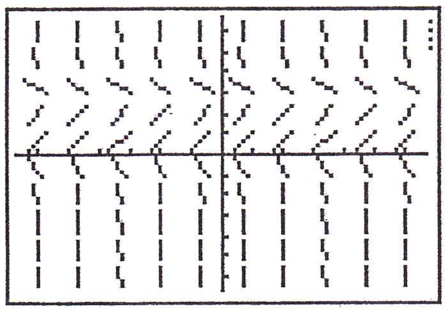
a.



b.



c.



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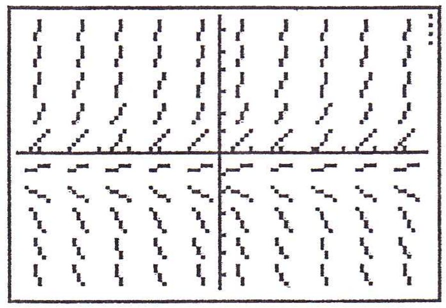
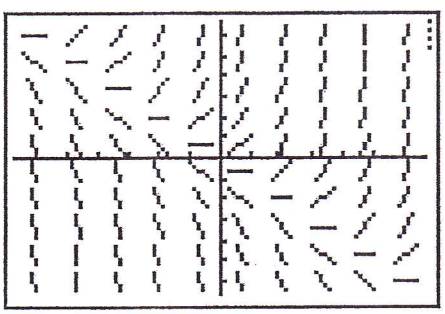
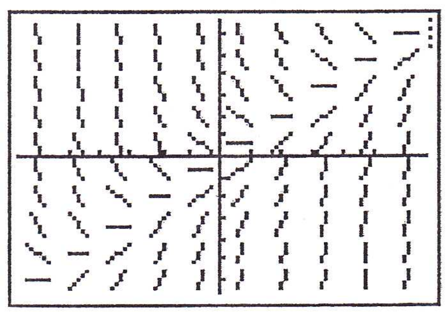


e.



f.



1. $\frac{dy}{dx} = x - y$

2. $\frac{dy}{dx} = 1 + y$

3. $\frac{dy}{dx} = \cos x$

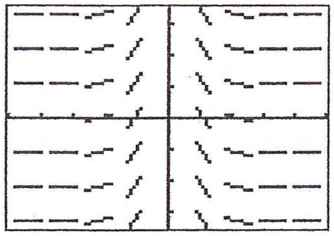
4. $\frac{dy}{dx} = 2x$

5. $\frac{dy}{dx} = x + y$

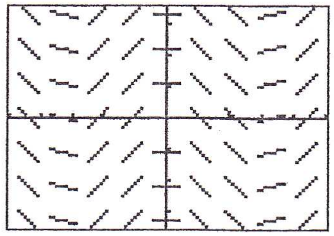
6. $\frac{dy}{dx} = y(3 - y)$

Match each slope field with the equation that the slope field could represent.

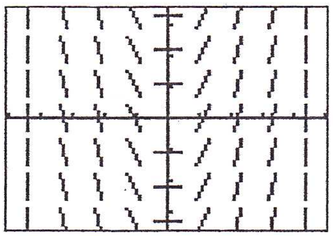
(A)



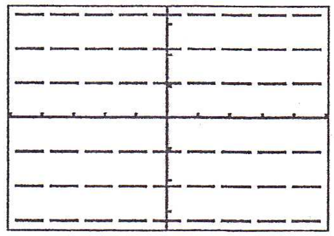
(B)



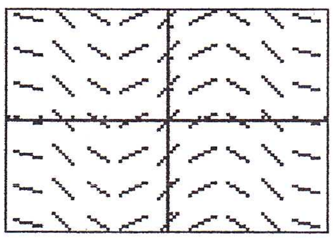
(C)



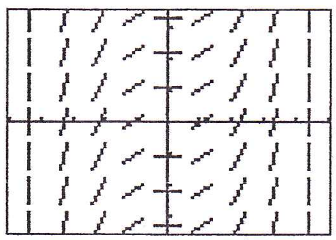
(D)



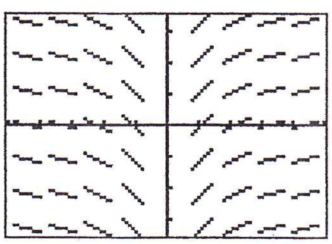
(E)



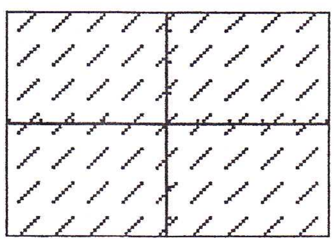
(F)



(G)



(H)



7. $y = 1$

8. $y = x$

9. $y = x^2$

10. $y = \frac{1}{6}x^3$

11. $y = \frac{1}{x^2}$

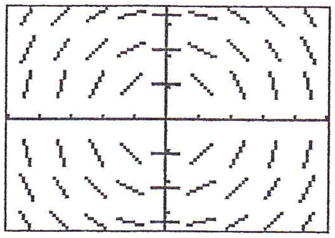
12. $y = \sin x$

13. $y = \cos x$

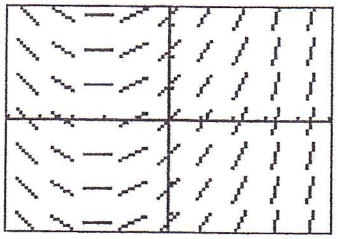
14. $y = \ln|x|$

Match the slope fields with their differential equations.

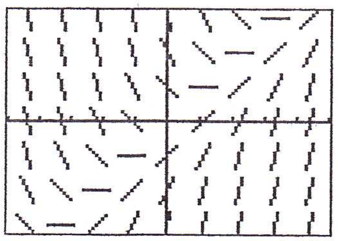
(A)



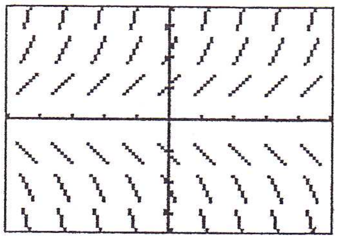
(B)



(C)



(D)



15. $\frac{dy}{dx} = \frac{1}{2}x + 1$

16. $\frac{dy}{dx} = y$

17. $\frac{dy}{dx} = x - y$

18. $\frac{dy}{dx} = -\frac{x}{y}$

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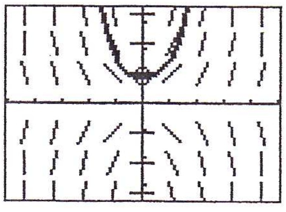
19. The calculator drawn slope field for the differential equation $\frac{dy}{dx} = xy$ is shown in the figure below. The solution curve passing through the point (0, 1) is also shown.
   (a) Sketch the solution curve through the point (0, 2).
   (b) Sketch the solution curve through the point $(0, -1)$.



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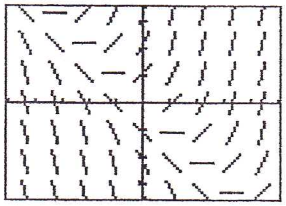
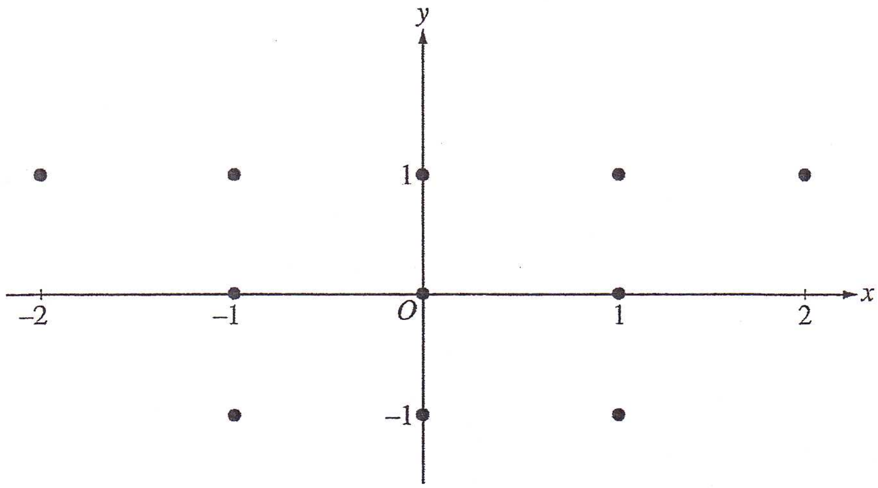
20. The calculator drawn slope field for the differential equation $\frac{dy}{dx} = x + y$ is shown in the figure below.
   (a) Sketch the solution curve through the point (0, 1).
   (b) Sketch the solution curve through the point $(-3, 0)$.



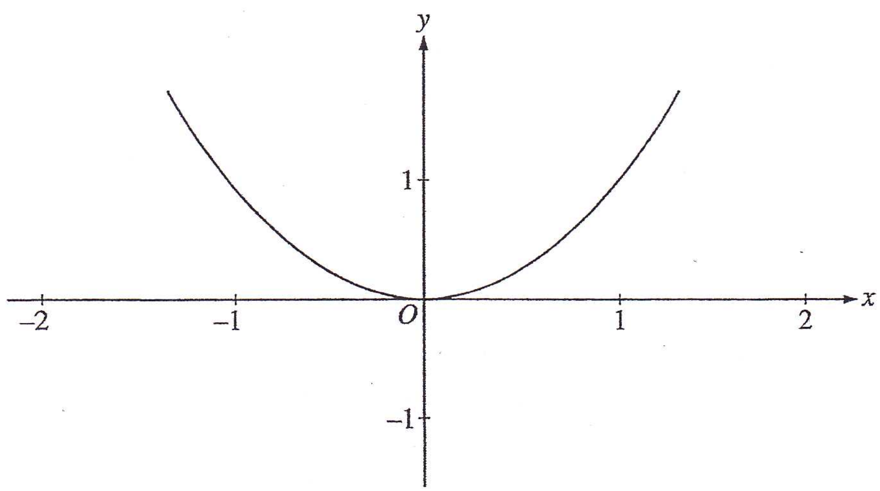
6. Consider the differential equation given by $\frac{dy}{dx} = x(y-1)^2$.

   (a) On the axes provided, sketch a slope field for the given differential equation at the eleven points indicated.

   (Note: Use the axes provided in the pink test booklet.)



   (b) Use the slope field for the given differential equation to explain why a solution could not have the graph shown below.



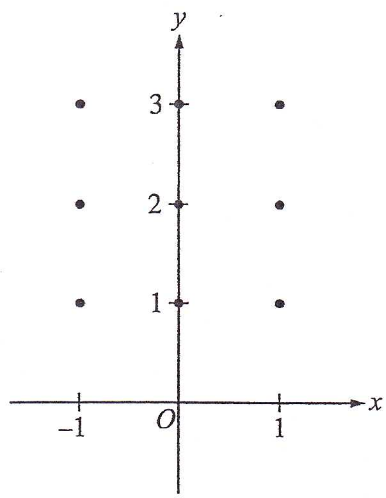
   (c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = -1$.

   (d) Find the range of the solution found in part (c).

END OF EXAMINATION

4. Consider the differential equation given by $\frac{dy}{dx} = \frac{xy}{2}$.

(a) On the axes provided below, sketch a slope field for the given differential equation at the nine points indicated.



(b) Let $y = f(x)$ be the particular solution to the given differential equation with the initial condition $f(0) = 3$. Use Euler's method starting at $x = 0$, with a step size of 0.1, to approximate $f(0.2)$. Show the work that leads to your answer.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = 3$. Use your solution to find $f(0.2)$.

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