6. (a) $\log_{10} 0.1 = -1$ since $10^{-1} = 0.1$.

(b) $\log_8 320 - \log_8 5 = \log_8 \frac{320}{5} = \log_8 64 = 2$ since $8^2 = 64$.

7. (a) $\log_2 6 - \log_2 15 + \log_2 20 = \log_2(\frac{6}{15}) + \log_2 20$ [by Law 2]

$= \log_2(\frac{6}{15} \cdot 20)$ [by Law 1]

$= \log_2 8$, and $\log_2 8 = 3$ since $2^3 = 8$.

(b) $\log_3 100 - \log_3 18 - \log_3 50 = \log_3(\frac{100}{18}) - \log_3 50 = \log_3(\frac{100}{18 \cdot 50})$

$= \log_3(\frac{1}{9})$, and $\log_3(\frac{1}{9}) = -2$ since $3^{-2} = \frac{1}{9}$.

9. $\log_2\left(\frac{x^3 y}{z^2}\right) = \log_2(x^3 y) - \log_2 z^2 = \log_2 x^3 + \log_2 y - \log_2 z^2 = 3\log_2 x + \log_2 y - 2\log_2 z$

[assuming that the variables are positive]

10. $\ln\sqrt{a(b^2+c^2)} = \ln(a(b^2+c^2))^{1/2} = \frac{1}{2}\ln(a(b^2+c^2)) = \frac{1}{2}[\ln a + \ln(b^2+c^2)] = \frac{1}{2}\ln a + \frac{1}{2}\ln(b^2+c^2)$

11. $\ln(uv)^{10} = 10\ln(uv) = 10(\ln u + \ln v) = 10\ln u + 10\ln v$

12. $\ln\frac{3x^2}{(x+1)^5} = \ln 3x^2 - \ln(x+1)^5 = \ln 3 + \ln x^2 - 5\ln(x+1) = \ln 3 + 2\ln x - 5\ln(x+1)$

13. $\log_{10} a - \log_{10} b + \log_{10} c = \log_{10}\frac{a}{b} + \log_{10} c = \log_{10}\left(\frac{a}{b}\cdot c\right) = \log_{10}\frac{ac}{b}$

14. $\ln(x+y) + \ln(x-y) - 2\ln z = \ln((x+y)(x-y)) - \ln z^2 = \ln(x^2-y^2) - \ln z^2 = \ln\frac{x^2-y^2}{z^2}$

15. $\ln 5 + 5\ln 3 = \ln 5 + \ln 3^5$ [by Law 3]

$= \ln(5 \cdot 3^5)$ [by Law 1]

$= \ln 1215$

16. $\ln 3 + \frac{1}{3}\ln 8 = \ln 3 + \ln 8^{1/3} = \ln 3 + \ln 2 = \ln(3 \cdot 2) = \ln 6$

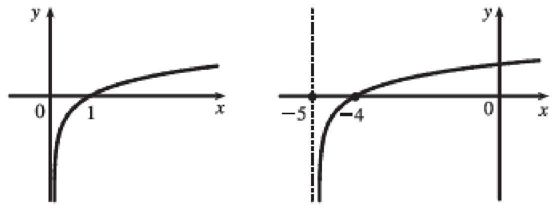
17. $\ln(1+x^2) + \frac{1}{2}\ln x - \ln\sin x = \ln(1+x^2) + \ln x^{1/2} - \ln\sin x = \ln[(1+x^2)\sqrt{x}] - \ln\sin x = \ln\frac{(1+x^2)\sqrt{x}}{\sin x}$

18. $\ln(a+b) + \ln(a-b) - 2\ln c = \ln[(a+b)(a-b)] - \ln c^2$ [by Laws 1, 3]

$= \ln\frac{(a+b)(a-b)}{c^2}$ [by Law 2]

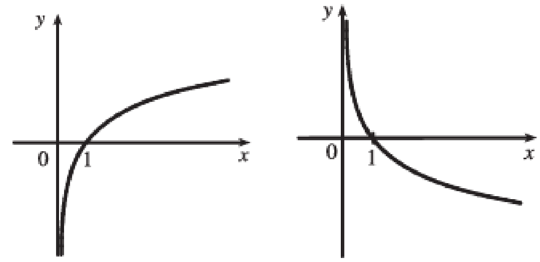
or $\ln\frac{a^2-b^2}{c^2}$

23. (a) Shift the graph of $y = \log_{10} x$ five units to the left to obtain the graph of $y = \log_{10}(x+5)$. Note the vertical asymptote of $x = -5$.



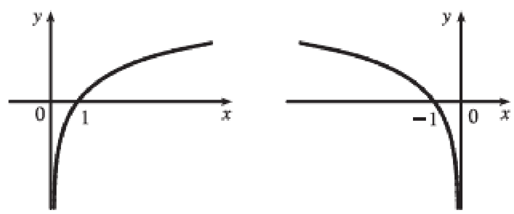
$y = \log_{10} x$ $\qquad$ $y = \log_{10}(x+5)$

(b) Reflect the graph of $y = \ln x$ about the x-axis to obtain the graph of $y = -\ln x$.



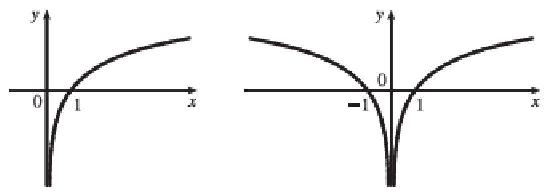
$y = \ln x$ $\qquad$ $y = -\ln x$

24. (a) Reflect the graph of $y = \ln x$ about the y-axis to obtain the graph of $y = \ln(-x)$.



$y = \ln x$ $\qquad$ $y = \ln(-x)$

(b) Reflect the portion of the graph of $y = \ln x$ to the right of the y-axis about the y-axis. The graph of $y = \ln|x|$ is that reflection in addition to the original portion.



$y = \ln x$ $\qquad$ $y = \ln|x|$

25. (a) $2\ln x = 1 \;\Rightarrow\; \ln x = \frac{1}{2} \;\Rightarrow\; x = e^{1/2} = \sqrt{e}$

(b) $e^{-x} = 5 \;\Rightarrow\; -x = \ln 5 \;\Rightarrow\; x = -\ln 5$

27. (a) $2^{x-5} = 3 \;\Leftrightarrow\; \log_2 3 = x - 5 \;\Leftrightarrow\; x = 5 + \log_2 3$.

Or: $2^{x-5} = 3 \;\Leftrightarrow\; \ln(2^{x-5}) = \ln 3 \;\Leftrightarrow\; (x-5)\ln 2 = \ln 3 \;\Leftrightarrow\; x - 5 = \dfrac{\ln 3}{\ln 2} \;\Leftrightarrow\; x = 5 + \dfrac{\ln 3}{\ln 2}$

(b) $\ln x + \ln(x-1) = \ln(x(x-1)) = 1 \;\Leftrightarrow\; x(x-1) = e^1 \;\Leftrightarrow\; x^2 - x - e = 0$. The quadratic formula (with $a = 1$, $b = -1$, and $c = -e$) gives $x = \frac{1}{2}\left(1 \pm \sqrt{1+4e}\right)$, but we reject the negative root since the natural logarithm is not defined for $x < 0$. So $x = \frac{1}{2}\left(1 + \sqrt{1+4e}\right)$.

29. $3xe^x + x^2e^x = 0 \;\Leftrightarrow\; xe^x(3+x) = 0 \;\Leftrightarrow\; x = 0$ or -3

31. $\ln(\ln x) = 1 \;\Leftrightarrow\; e^{\ln(\ln x)} = e^1 \;\Leftrightarrow\; \ln x = e^1 = e \;\Leftrightarrow\; e^{\ln x} = e^e \;\Leftrightarrow\; x = e^e$

33. $e^{2x} - e^x - 6 = 0 \;\Leftrightarrow\; (e^x - 3)(e^x + 2) = 0 \;\Leftrightarrow\; e^x = 3$ or $-2 \;\Rightarrow\; x = \ln 3$ since $e^x > 0$.

35. (a) $e^{2+5x} = 100 \;\Rightarrow\; \ln\left(e^{2+5x}\right) = \ln 100 \;\Rightarrow\; 2 + 5x = \ln 100 \;\Rightarrow\; 5x = \ln 100 - 2 \;\Rightarrow$
$x = \frac{1}{5}(\ln 100 - 2) \approx 0.5210$

(b) $\ln(e^x - 2) = 3 \;\Rightarrow\; e^x - 2 = e^3 \;\Rightarrow\; e^x = e^3 + 2 \;\Rightarrow\; x = \ln(e^3 + 2) \approx 3.0949$

37. (a) $e^x < 10 \quad \Rightarrow \quad \ln e^x < \ln 10 \quad \Rightarrow \quad x < \ln 10 \quad \Rightarrow \quad x \in (-\infty, \ln 10)$

(b) $\ln x > -1 \quad \Rightarrow \quad e^{\ln x} > e^{-1} \quad \Rightarrow \quad x > e^{-1} \quad \Rightarrow \quad x \in (1/e, \infty)$

40. (a) $v(t) = ce^{-kt} \quad \Rightarrow \quad a(t) = v'(t) = -kce^{-kt} = -kv(t)$

(b) $v(0) = ce^0 = c$, so c is the initial velocity.

(c) $v(t) = ce^{-kt} = c/2 \quad \Rightarrow \quad e^{-kt} = \frac{1}{2} \quad \Rightarrow \quad -kt = \ln\frac{1}{2} = -\ln 2 \quad \Rightarrow \quad t = (\ln 2)/k$

45. Let $t = x^2 - 9$. Then as $x \to 3^+$, $t \to 0^+$, and $\lim\limits_{x\to 3^+} \ln(x^2 - 9) = \lim\limits_{t\to 0^+} \ln t = -\infty$ by (8).

46. As $x \to 2^-$, $8x - x^4 = x(8 - x^3) \to 0^+$ since x is positive and $8 - x^3 \to 0^+$. Thus, $\lim\limits_{x\to 2^-} \log_5(8x - x^4) = -\infty$.

47. $\lim\limits_{x\to 0} \ln(\cos x) = \ln 1 = 0$. [$\ln(\cos x)$ is continuous at $x = 0$ since it is the composite of two continuous functions.]

48. $\lim\limits_{x\to 0^+} \ln(\sin x) = -\infty$ since $\sin x \to 0^+$ as $x \to 0^+$.

49. $\lim\limits_{x\to\infty} [\ln(1 + x^2) - \ln(1 + x)] = \lim\limits_{x\to\infty} \ln\dfrac{1 + x^2}{1 + x} = \ln\left(\lim\limits_{x\to\infty} \dfrac{1 + x^2}{1 + x}\right) = \ln\left(\lim\limits_{x\to\infty} \dfrac{\frac{1}{x} + x}{\frac{1}{x} + 1}\right) = \infty$, since the limit in parentheses is ∞.

50. $\lim\limits_{x\to\infty} [\ln(2 + x) - \ln(1 + x)] = \lim\limits_{x\to\infty} \ln\left(\dfrac{2 + x}{1 + x}\right) = \lim\limits_{x\to\infty} \ln\left(\dfrac{2/x + 1}{1/x + 1}\right) = \ln\dfrac{1}{1} = \ln 1 = 0$

51. $f(x) = \log_{10}(x^2 - 9)$. $\quad D_f = \{x \mid x^2 - 9 > 0\} = \left\{x \,\middle|\, |x| > 3\right\} = (-\infty, -3) \cup (3, \infty)$

53. (a) For $f(x) = \sqrt{3 - e^{2x}}$, we must have $3 - e^{2x} \geq 0 \quad \Rightarrow \quad e^{2x} \leq 3 \quad \Rightarrow \quad 2x \leq \ln 3 \quad \Rightarrow \quad x \leq \frac{1}{2}\ln 3$.

Thus, the domain of f is $(-\infty, \frac{1}{2}\ln 3]$.

(b) $y = f(x) = \sqrt{3 - e^{2x}}$ [note that $y \geq 0$] $\quad \Rightarrow \quad y^2 = 3 - e^{2x} \quad \Rightarrow \quad e^{2x} = 3 - y^2 \quad \Rightarrow \quad 2x = \ln(3 - y^2) \quad \Rightarrow$ $x = \frac{1}{2}\ln(3 - y^2)$. Interchange x and y: $y = \frac{1}{2}\ln(3 - x^2)$. So $f^{-1}(x) = \frac{1}{2}\ln(3 - x^2)$. For the domain of f^{-1}, we must have $3 - x^2 > 0 \quad \Rightarrow \quad x^2 < 3 \quad \Rightarrow \quad |x| < \sqrt{3} \quad \Rightarrow \quad -\sqrt{3} < x < \sqrt{3} \quad \Rightarrow \quad 0 \leq x < \sqrt{3}$ since $x \geq 0$. Note that the domain of f^{-1}, $[0, \sqrt{3})$, equals the range of f.

55. $y = \ln(x + 3) \quad \Rightarrow \quad e^y = e^{\ln(x+3)} = x + 3 \quad \Rightarrow \quad x = e^y - 3$.

Interchange x and y: the inverse function is $y = e^x - 3$.

57. $y = f(x) = e^{x^3} \quad \Rightarrow \quad \ln y = x^3 \quad \Rightarrow \quad x = \sqrt[3]{\ln y}$. Interchange x and y: $y = \sqrt[3]{\ln x}$. So $f^{-1}(x) = \sqrt[3]{\ln x}$.

61. $f(x) = e^{3x} - e^x \quad \Rightarrow \quad f'(x) = 3e^{3x} - e^x$. Thus, $f'(x) > 0 \quad \Leftrightarrow \quad 3e^{3x} > e^x \quad \Leftrightarrow \quad \dfrac{3e^{3x}}{e^x} > \dfrac{e^x}{e^x} \quad \Leftrightarrow \quad 3e^{2x} > 1 \quad \Leftrightarrow$ $e^{2x} > \frac{1}{3} \quad \Leftrightarrow \quad 2x > \ln\left(\frac{1}{3}\right) = -\ln 3 \quad \Leftrightarrow \quad x > -\frac{1}{2}\ln 3$, so f is increasing on $\left(-\frac{1}{2}\ln 3, \infty\right)$.

62. $y = 2e^x - e^{-3x} \quad \Rightarrow \quad y' = 2e^x + 3e^{-3x} \quad \Rightarrow \quad y'' = 2e^x - 9e^{-3x}$. Thus, $y'' < 0 \quad \Leftrightarrow \quad 2e^x < 9e^{-3x} \quad \Leftrightarrow$ $e^{4x} < \frac{9}{2} \quad \Leftrightarrow \quad 4x < \ln\frac{9}{2} \quad \Leftrightarrow \quad x < \frac{1}{4}\ln\frac{9}{2}$, so f is concave downward on $\left(-\infty, \frac{1}{4}\ln\frac{9}{2}\right)$.