**b.** The probability of a person not having a birthday on February 29, in a four-year span, is $\frac{1460}{1461}$. Since the probability of one person's birthday does not affect the probability of the next person's birthday, the events are independent.

$P(\text{no one born on February 29}) = \left(\frac{1460}{1461}\right)^{150} \approx 0.9$

**c.** Let x represent the size of the smallest group of people.

$\frac{1}{2} \leq P(\text{1 person born on February 29})$

$\frac{1}{2} \leq 1 - P(\text{no one born on February 29})$

$\frac{1}{2} \leq 1 - \left(\frac{1460}{1461}\right)^{x}$

$\frac{1}{2} \leq \left(\frac{1460}{1461}\right)^{x}$

$x \geq 1012.34$

The smallest group is 1013 people.

**37.** $P(\text{lower} \mid \text{woman}) = 1 - P(\text{upper} \mid \text{woman})$

$= 1 - \frac{35}{90} = \frac{11}{18};$

no, $P(\text{lower}) \neq P(\text{lower} \mid \text{woman})$

**38a.**

| Per 10,000 People Tested | | | |
|---|---|---|---|
| | Have Strep | Do Not Have Strep | Total |
| **Test Positive** | 198 | 98 | 296 |
| **Test Negative** | 2 | 9702 | 9704 |
| **Total** | 200 | 9800 | 10,000 |

**b.** $P(\text{have strep} \mid \text{test positive}) = \frac{198}{296} = \frac{99}{148}$

## READY TO GO ON?

**1.** ${}_{10}P_5 = \frac{10!}{(10-5)!} = \frac{10!}{5!}$

$= 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6$

$= 30{,}240$

**2.** ${}_{8}C_4 = \frac{8!}{4!(8-4)!} = \frac{8!}{4!4!}$

$= \frac{8 \cdot 7 \cdot 6 \cdot 5}{4 \cdot 3 \cdot 2 \cdot 1}$

$= 70$

**3.** ${}_{6}P_5 = \frac{6!}{(6-5)!} = \frac{6!}{1!}$

$= 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2$

$= 720$

**4.** $P(\text{iced tea}) = \frac{3}{18} = \frac{1}{6}$

**5.** $P(\text{out of ink and out of ink})$

$= P(\text{out of ink}) \cdot P(\text{out of ink} \mid \text{out of ink})$

$= \frac{2}{9} \cdot \frac{1}{8} = \frac{1}{36}$

**6.** Area of large triangle is $A_t = \frac{1}{2}(15)(4) = 30$

Area of shaded region is $A_s = \frac{1}{2}(4)(4) = 8$

$\frac{A_s}{A_t} = \frac{8}{30} = \frac{4}{15}$

The probability that the point is in the shaded region is $\frac{4}{15}$.

**7.** $P(\text{not rolling a 2}) = 1 - P(\text{rolling a 2})$

$= 1 - \frac{12}{50} = \frac{19}{25}$

**8.** The result of a toss does not affect the probability of the next toss.

$P(\text{tails and tails and tails and tails and heads})$

$= P(\text{tails}) \cdot P(\text{tails}) \cdot P(\text{tails}) \cdot P(\text{tails}) \cdot P(\text{heads})$

$= \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{32}$

**9.** $P(\text{sum} \geq 10)$ changes after a red 6 has occurred.

$P(\text{sum} \geq 10 \text{ and red } 6)$

$P(\text{sum} \geq 10) = \frac{1}{6}$

$P(\text{red } 6 \mid \text{sum} \geq 10) = \frac{1}{2}$

**10.** $P(\text{11th grade} \mid \text{geometry}) = \frac{33}{127}$

**11.** Not replacing the red checker after it is selected affects the probability of the next selection. The events are dependent.

$P(\text{red and black})$

$= P(\text{red}) \cdot P(\text{black} \mid \text{red})$

$= \frac{15}{25} \cdot \frac{10}{24} = \frac{1}{4}$

## 7-4 TWO-WAY TABLES

### CHECK IT OUT!

**1.** Find the total number of books sold: $28 + 52 + 94 + 36 = 210$. Divide each value in the table by 210 to find the joint relative frequencies, and add each row and column to find the marginal relative frequencies.

| | Fiction | Nonfiction | Total |
|---|---|---|---|
| Hardcover | 0.133 | 0.248 | 0.381 |
| Paperback | 0.448 | 0.171 | 0.619 |
| Total | 0.581 | 0.419 | 1 |

**2a.** Find the total enrollment:

$38 + 52 + 86 + 24 = 200$. Divide each value in the table by 200, and add each row and column.

| | | Ballet | | |
|---|---|---|---|---|
| | | Yes | No | Total |
| **Tap** | Yes | 0.19 | 0.26 | 0.45 |
| | No | 0.43 | 0.12 | 0.55 |
| | Total | 0.62 | 0.38 | 1 |

**b.** Taking ballet: 0.62; of these, also not taking tap: 0.43.

$\frac{0.43}{0.62} = 0.69$

**3.** Create a table of joint and marginal relative frequencies:

| | Pass | Fail | Total |
|---|---|---|---|
| Al's Driving | 0.28 | 0.16 | 0.44 |
| Drive Time | 0.22 | 0.14 | 0.36 |
| Crash Course | 0.1 | 0.1 | 0.2 |
| Total | 0.6 | 0.4 | |

$P$ (passing at Al's Driving) $= \frac{0.28}{0.44} \approx 0.64$

$P$ (passing at Drive Time) $= \frac{0.22}{0.36} \approx 0.61$

$P$ (passing at Crash Course) $= \frac{0.1}{0.2} = 0.5$

Al's Driving School is best.

## THINK AND DISCUSS

**1.** Joint relative frequencies are the values in each category divided by the total number of values, while marginal relative frequencies are found by adding the joint relative frequencies in each column and row.

**2.** To find a conditional relative frequency, divide a joint relative frequency in the two-way table by the marginal relative frequency in that row or column.

**3.**

| Relative Frequencies | | |
|---|---|---|
| Joint | Marginal | Conditional |
| Divide each value by the total number of values. | Add the joint relative frequencies in each row and column. | Divide each joint relative frequency in the two-way table by the marginal relative frequency in that row or column. |

## EXCERCISES

***GUIDED PRACTICE***

**1.** marginal

**2.** conditional

**3.** The joint relative frequencies are determined by dividing the value in each cell in the table by the total number of people surveyed. There were 50 people surveyed, so each value is divided by 50.

| | Under Classmates | Upper Classmates | Total |
|---|---|---|---|
| Morning | 0.16 | 0.28 | 0.44 |
| Afternoon | 0.36 | 0.2 | 0.56 |
| Total | 0.52 | 0.48 | 1 |

The marginal relative frequencies, recorded in the row and column labeled Total, are found by adding the joint relative frequencies in each row and column.

**4.** The joint relative frequencies are determined by dividing the value in each cell in the table by the total number of people surveyed. There were 150 people surveyed, so each value is divided by 150.

| | Arlington | Towson | Parkville | Total |
|---|---|---|---|---|
| Yes | 0.27 | 0.23 | 0.27 | 0.77 |
| No | 0.12 | 0.07 | 0.04 | 0.23 |
| Total | 0.39 | 0.3 | 0.31 | 1 |

The marginal relative frequencies, recorded in the row and column labeled Total, are found by adding the joint relative frequencies in each row and column.

**5a.** The joint relative frequencies are determined by dividing the value in each cell in the table by the total number of sophomores who play an instrument or a sport. Data was collected on 203 people, so each value is divided by 203.

| | | Play Sport | | |
|---|---|---|---|---|
| | | Yes | No | Total |
| Play Instrument | Yes | 0.23 | 0.19 | 0.42 |
| | No | 0.25 | 0.33 | 0.58 |
| | Total | 0.48 | 0.52 | 1 |

**b.** There are $47 + 38 = 85$ students that play an instrument. Of the students, 47 out of 85 also play a sport.
$P$(student that plays an instrument plays a sport) $=$
$47 \div 85 = 0.55$

**c.** There are $47 + 51 = 98$ students that play a sport. Of the students, 47 of 98 also play an instrument.
$P$(student that plays a sport plays an instrument) $=$
$47 \div 98 = 0.48$

**6a.** The joint relative frequencies are determined by dividing the value in each cell in the table by the total number of attempted sales. Data was collected on 36 attempted sales, so each value is divided by 36.

| | Successful | Unsuccessful | Total |
|---|---|---|---|
| Becky | 0.17 | 0.17 | 0.33 |
| Raul | 0.11 | 0.14 | 0.25 |
| Darrell | 0.17 | 0.25 | 0.42 |
| Total | 0.45 | 0.56 | 1 |

**b.** The probability of a salesman being successful is determined by dividing the total number of attempts by the total number of sales
$P$(Becky being successful) $= 6 \div 12 = 0.50$
$P$(Raul being successful) $= 4 \div 9 = 0.44$
$P$(Darrell being successful) $= 6 \div 15 = 0.40$

**c.** Becky has the highest success rate.

***PRACTICE AND PROBLEM SOLVING***

**7.** The joint relative frequencies are determined by dividing the value in each cell in the table by the total number of shirts and sweatshirts sold. There were 60 items of clothing sold, so each value is divided by 60.

|  | Students | Adults | Total |
|---|---|---|---|
| T-Shirts | 0.267 | 0.383 | 0.65 |
| Sweatshirts | 0.117 | 0.233 | 0.35 |
| Total | 0.384 | 0.616 | 1 |

The marginal relative frequencies, recorded in the row and column labeled Total, are found by adding the joint relative frequencies in each row and column.

**8.** Possible answer: First find the total number of individuals in the survey or study by adding all the values in the two-way table. Next find the joint relative frequencies by dividing each cell in the two way table by the total number of individuals in the survey. Record these results in a new table. Finally, add the rows and columns in this new table to find the marginal relative frequencies.

**9a.** The joint relative frequencies are determined by dividing the value in each cell in the table by the total number of customers that were surveyed. There were 118 customers surveyed, so each value is divided by 118.

|  | Satisfied | Dissatisfied | Total |
|---|---|---|---|
| Team 1 | 0.17 | 0.07 | 0.24 |
| Team 2 | 0.29 | 0.1 | 0.39 |
| Team 3 | 0.29 | 0.08 | 0.37 |
| Total | 0.75 | 0.25 | 1 |

**b.** To determine the probability that a customer will be satisfied after working with a particular team, take the number of the customers that were satisfied after working with the team and divide by the total number of customers that worked with the team.
$P$(Being satisfied with Team 1) $= 20 \div 28 = 0.71$
$P$(Being satisfied with Team 2) $= 34 \div 46 = 0.74$
$P$(Being satisfied with Team 3) $= 34 \div 44 = 0.77$

**c.** Team C has the highest rate of customer satisfaction.

**10.** The value is always equal to 1. It represents the portion of the people in the survey who are in the survey, so it must equal 1.

**11.** Maria has made an error. If you add up all the joint relative frequencies in her table the sum is 1.1. The sum should be 1. The sum of the joint relative frequencies in Brennan's table is 1.

**12.** 0.48 is a little less than half. A little less than half of the 107 brownies and muffins is around 50.

**13a.** The joint relative frequencies are determined by dividing the value in each cell in the table by the total number of questions asked. There were 120 questions asked, so each value should be divided by 120.

|  |  | Work less than 5 miles from home? | | |
|---|---|---|---|---|
|  |  | Yes | No | Total |
| Use new system? | Yes | 0.2 | 0.27 | 0.47 |
|  | No | 0.37 | 0.17 | 0.54 |
|  | Total | 0.57 | 0.44 | 1 |

**b.** $P$(Works close to home would use the new system) $= 24 \div 68 = 0.35$

**c.** $P$(Would use the system lives far from home) $= 32 \div 56 = 0.57$

***TEST PREP***

**14.** B; there were $9 + 14 = 23$ teachers polled.

**15.** Use a table to find the joint and marginal relative frequencies:

|  | Yes | No | Total |
|---|---|---|---|
| Junior | 0.223 | 0.313 | 0.536 |
| Senior | 0.167 | 0.298 | 0.464 |
| Total | 0.390 | 0.610 | 1 |

Inspecting the table shows that C is correct.

**16.** $0.25 + x = 0.3$
$x = 0.05$

***CHALLENGE AND EXTEND***

**17.** To find the marginal relative frequencies, add the rows and columns in the joint relative frequency table.

|  | Yes | No | Total |
|---|---|---|---|
| Children | 0.125 | 0.1 | 0.225 |
| Teenagers | 0.725 | 0.05 | 0.775 |
| Total | 0.85 | 0.15 | 1 |

Marginal relative frequencies are in the row and column labeled Total.

**18.** $P$(teenager purchasing a ticket book) $=$ $0.725 \div 0.775 = 0.94 = 94\%$

**19.** According to the table 12.5% of the fair attendees are children who will buy a ticket at the entrance. Because 12.5% of the 80 teenagers and children who attend the fair equals 10, 10 children will buy a ticket at the entrance.

**20.** According to the table, 10% of the fair attendees are children who did not buy a ticket at the entrance. According to the problem, 10% of the total fair goers is equal to 12. 10% of 120 is 12, so there are 120 children and teenagers attending the fair.

**21.** The total of the marginal relative frequencies is always 1, so $1 - 1 = 0$.

**22.** The maximum is 1. It can't be higher because that would represent more than 100% of the data.

## 7-5 COMPOUND EVENTS

### CHECK IT OUT!

**1a.** Each student can only vote once.

**b.** $P(\text{votes for Kline} \cup \text{voted for Vila})$
$= P(\text{votes for Kline}) + P(\text{voted for Vila})$
$= 20\% + 55\% = 75\%$

**2a.** $P(\text{king} \cup \text{heart})$
$= P(\text{king}) + P(\text{heart}) - P(\text{king} \cap \text{heart})$
$= \frac{4}{52} - \frac{13}{52} - \frac{1}{52} = \frac{4}{13}$