Station 3

Zill

D_x, $\frac{d}{dx}$ all mean derivative w/respect to x variable.

In Problems 45–50 use the information $f(1) = 2, f'(1) = -3$, and $g(1) = g'(1) = 6$ to evaluate the given derivative.

GOAL 1) Do it....
2) Explain the process to a peer

45. $\left.\frac{d}{dx}(f(x)g(x))\right|_{x=1}$

46. $\left.\frac{d}{dx}\left(\frac{g(x)}{f(x)}\right)\right|_{x=1}$

47. $\left.\frac{d}{dx}\left(\frac{1+2f(x)}{x-g(x)}\right)\right|_{x=1}$

48. $\left.\frac{d}{dx}\left(\frac{4}{x}+f(x)\right)g(x)\right|_{x=1}$

49. $\left.D_x(x^2f(x)g(x))\right|_{x=1}$

50. $\left.D_x\left(\frac{xf(x)}{g(x)}\right)\right|_{x=1}$

Zill

51. Find an equation of the line(s) with slope -8 and tangent to the graph of $f(x) = 2/x$.

★ 52. Find the value of k such that the tangent line to the graph of $f(x) = (k + x)/x^2$ has slope 5 at $x = 2$.

53. (a) Graph the function $f(x) = 2/(x^2 + 1)$.

(b) Find all the points on the graph of f such that the normal lines pass through the origin.

54. Show that at $x = 1$ the tangent to the graph of $f(x) = (x^2 + 14)/(x^2 + 9)$ is perpendicular to the tangent to the graph of $g(x) = (1 + x^2)(1 + 2x)$.

In Problems 55–58 find the values of x for which $f'(x) > 0$. Explain what this represents.

55. $f(x) = \frac{5}{x^2}$

56. $f(x) = \frac{x^2+3}{x+1}$

57. $f(x) = (2x + 1)(x + 4)$

58. $f(x) = x + 4/x$

59. The Universal Law of Gravitation states that the force F between two bodies of masses m_1 and m_2 separated by a distance r is $F = km_1m_2/r^2$, where k is constant. What is the instantaneous rate of change of F with respect to r when $r = \frac{1}{2}$ km?

why is this a derivative?

60. The potential energy U between two atoms in a diatomic molecule is given by $U(x) = q_1/x^{12} - q_2/x^6$, where q_1 and q_2

Zill key #44–49

on the graph are $(3, 3/2)$ and $(-5, 1/2)$.

44. $y' = (x+1)\cdot 2 + (2x+5)\cdot 1 = 4x+7$. Since the slope of the tangent line is -3, $4x+7 =$ $x = -5/2$. Given $y = (x+1)(2x+5)$, we see that for $x = -5/2$, $y = 0$. Thus, the point graph is $(-5/2, 0)$.

45. $$\frac{d}{dx} f(x)g(x)\Big|_{x=1} = f(1)g'(1) + g(1)f'(1) = 2\cdot 6 + 6(-3) = -6$$

46. $$\frac{d}{dx}\frac{g(x)}{f(x)}\Big|_{x=1} = \frac{f(1)g'(1) - g(1)f'(1)}{[f(1)]^2} = \frac{2(6) - 6(-3)}{2^2} = \frac{15}{2}$$

47. $$\frac{d}{dx}\frac{1+2f(x)}{x-g(x)}\Big|_{x=1} = \frac{[1-g(1)][2f'(1)] - [1+2f(1)][1-g'(1)]}{[1-g(1)]^2}$$
$$= \frac{(1-6)[2(-3)] - (1+2\cdot 2)(1-6)}{(1-6)^2} = \frac{11}{5}$$

48. $$\frac{d}{dx}\left(4x^{-1} + f(x)\right)g(x)\Big|_{x=1} = \left([4x^{-1} + f(x)]g'(x) + g(x)[-4x^{-2} + f'(x)]\right)\Big|_{x=1}$$
$$= [4+2](6) + 6[-4+(-3)] = -6$$

49. $$D_x\left(x^2 f(x)g(x)\right)\Big|_{x=1} = \left(x^2[f(x)g'(x) + g(x)f'(x)]\right)\Big|_{x=1} + (f(x)g(x)2x)\Big|_{x=1}$$
$$= f(1)g'(1) + g(1)f'(1) + 2f(1)g(1) = 2(6) + 6(-3) + 2(2)(6) = 18$$

$$\frac{g(x)[xf'(x) + f(x)] - xf(x)g'(x)}{\ldots}\Big| = \frac{g(1)f'(1) + g(1)f(1) - f(1)g\ldots}{[g(1)]^2}$$

$$= \frac{\cdots (1+2\cdot 2)(1-6)}{(1-6)^2} = \frac{11}{5}$$

Zill #48–53 key

48. $$\frac{d}{dx}\left(4x^{-1}+f(x)\right)g(x)\Big|_{x=1} = \left([4x^{-1}+f(x)]g'(x)+g(x)[-4x^{-2}+f'(x)]\right)\Big|_{x=1}$$
$$= [4+2](6)+6[-4+(-3)] = -6$$

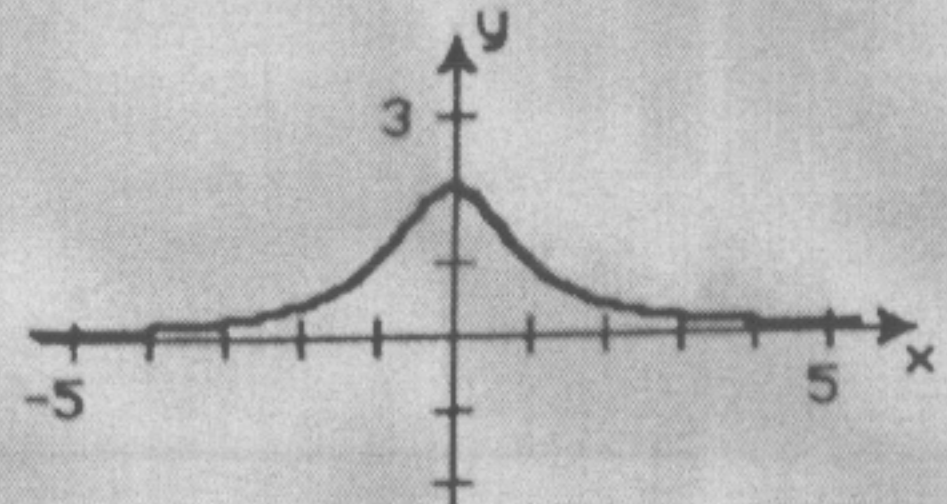
49. $$D_x\left(x^2f(x)g(x)\right)\Big|_{x=1} = \left(x^2[f(x)g'(x)+g(x)f'(x)]\right)\Big|_{x=1} + (f(x)g(x)2x)\Big|_{x=1}$$
$$= f(1)g'(1)+g(1)f'(1)+2f(1)g(1) = 2(6)+6(-3)+2(2)(6) = 18$$

50. $$D_x\left(\frac{xf(x)}{g(x)}\right)\Big|_{x=1} = \frac{g(x)[xf'(x)+f(x)]-xf(x)g'(x)}{[g(x)]^2}\Big|_{x=1} = \frac{g(1)f'(1)+g(1)f(1)-f(1)g'(1)}{[g(1)]^2}$$
$$= \frac{6(-3)+6(2)-2(6)}{6^2} = -\frac{1}{2}$$

51. $f(x)=2x^{-1}$; $f'(x)=-2x^{-2}$. Solving $-2x^{-2}=-8$ we find $x=1/2$. Then the point on the graph where the tangent line has slope -8 is $(1/2,4)$. The equation of the tangent line is $y-4=-8(x-1/2)$ or $y=-8x+8$.

52. $f'(x)=\dfrac{x^2-(k+x)2x}{x^4}=\dfrac{-x-2k}{x^3}$. Solving $\dfrac{-2-2k}{2^3}=5$ we obtain $k=-21$.

53. (a)



(b) $f'(x)=\dfrac{-2(2x)}{(x^2+1)^2}=-\dfrac{4x}{(x^2+1)^2}$. At $x=a$ the slope of the normal line is $\dfrac{(a^2+1)^2}{4a}$ and the

Zill Key
54 - 55

Exercises 2.4

equation of the normal line through $\left(a, \dfrac{2}{a^2+1}\right)$ is $y - \dfrac{2}{a^2+1} = \dfrac{(a^2+1)^2}{4a}(x-a)$. When this line passes through the origin,

$$-\frac{2}{a^2+1} = \frac{(a^2+1)^2}{4a}(-a)$$

$$8 = (a^2+1)^3$$

$$2 = a^2+1$$

and $a = \pm 1$. Thus, the points on the graph where the normal line passes through the origin are $(-1,1)$ and $(1,1)$. From the graph we see that the y-axis is also a normal line, so that another point is $(0,2)$.

54. $f'(x) = \dfrac{(x^2+9)2x - (x^2+14)2x}{(x^2+9)^2} = \dfrac{-10x}{(x^2+9)^2}$; $f'(1) = -\dfrac{1}{10}$

$g'(x) = (1+x^2)2 + (1+2x)2x = 6x^2+2x+2$; $g'(1) = 10$

Since $f'(1)g'(1) = 1$, the tangent lines are perpendicular.

55. $f(x) = 5x^{-2}$; $f'(x) = -10x^{-3} = -\dfrac{10}{x^3}$. For $f'(x)$ to be positive, x^3 must be negative. Therefore, $x < 0$.

Zill Key
54 - 59

54. $f'(x) = \dfrac{(x^2+9)2x - (x^2+14)2x}{(x^2+9)^2} = \dfrac{-10x}{(x^2+9)^2}$; $f'(1) = -\dfrac{1}{10}$

$g'(x) = (1+x^2)2 + (1+2x)2x = 6x^2 + 2x + 2$; $g'(1) = 10$

Since $f'(1)g'(1) = 1$, the tangent lines are perpendicular. 63.

55. $f(x) = 5x^{-2}$; $f'(x) = -10x^{-3} = -\dfrac{10}{x^3}$. For $f'(x)$ to be positive, x^3 must be negative. Therefore, $x < 0$. 64.

56. $f'(x) = \dfrac{(x+1)\cdot 2x - (x^2+3)\cdot 1}{(x+1)^2} = \dfrac{x^2+2x-3}{(x+1)^2} = \dfrac{(x+3)(x-1)}{(x+1)^2}$

For $f'(x)$ to be positive, $(x+3)$ and $(x-1)$ must both be positive or both be negative and $x \neq -1$.

CASE I	$x+3>0$	and	$x-1>0$;	$x>-3$	and	$x>1$
CASE II	$x+3<0$	and	$x-1<0$;	$x<-3$	and	$x<1$

65

Thus, $x > 1$ or $x < -3$.

57. $f'(x) = (2x+1)\cdot 1 + (x+4)\cdot 2 = 4x + 9$. For $f'(x)$ to be positive, $x > -9/4$. 6

58. $f(x) = x + 4x^{-1}$; $f'(x) = 1 - 4x^{-2} = 1 - \dfrac{4}{x^2}$. For $f'(x)$ to be positive, $x^2 > 4$. Therefore, $x > 2$ or $x < -2$.

59. $F(r) = km_1m_2r^{-2}$; $F'(r) = -2km_1m_2r^{-3}$; $F'(1/2) = -2km_1m_2(1/2)^{-3} = -16km_1m_2$

60. $U(x) = q_1x^{-12} - q_2x^{-6}$; $U'(x) = -12q_1x^{-13} + 6q_2x^{-7} = \dfrac{-12q_1 + 6q_2x^6}{x^{13}}$

$F(x) = -U'(x) = \dfrac{12q_1 - 6q_2x^6}{x^{13}}$

$F\left(\sqrt[6]{2q_1/q_2}\right) = F[(2q_1/q_2)^{1/6}] = \dfrac{12q_1 - 6q_2[(2q_1/q_2)^{1/6}]^6}{[(2q_1/q_2)^{1/6}]^{13}} = \dfrac{12q_1 - 12q_1}{\dots}$