1. (a) (i) $\lim_{x \to 2^+} f(x) = 3$

(ii) $\lim_{x \to -3^+} f(x) = 0$

(iii) $\lim_{x \to -3^-} f(x) = -2$ and $\lim_{x \to -3^+} f(x) = 0$

then $\lim_{x \to -3} f(x)$ DNE since $\lim_{x \to -3^-} f(x) \neq \lim_{x \to -3^+} f(x)$

(iv) $\lim_{x \to 4} f(x) = 2$

(v) $\lim_{x \to 0} f(x) = \infty$

(vi) $\lim_{x \to 2^-} f(x) = -\infty$

(b) $x = 0$ is a VA since $\lim_{x \to 0} f(x) = \infty$

$x = 2$ is a VA since $\lim_{x \to 2^-} f(x) = -\infty$

(c) f is discontinuous at $x = -3$ since $\underbrace{\lim_{x \to -3} f(x) \text{ DNE}}_{\text{reasoning in (iii)}}$

f is discontinuous at $x = 0$ since $f(0)$ is undefined

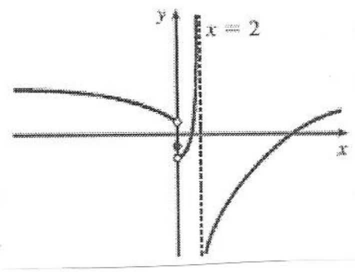
f is discontinuous at $x = 2$ since $\lim_{x \to 2} f(x)$ DNE

$\lim_{x \to 2} f(x)$ DNE since $-\infty = \lim_{x \to 2^-} f(x) \neq \lim_{x \to 2^+} f(x) = 3$

f is discontinuous at $x = 4$ since

$2 = \lim_{x \to 4} f(x) \neq f(4) = 1$

2.



3. $\lim_{x \to 0} (\cos(x + \sin x))$

Let $f(x) = \cos(x + \sin x)$

$g(x) = x$ is a polynomial which is cont on $(-\infty, \infty)$

$h(x) = \sin(x)$ is a trig function which is cont on $(-\infty, \infty)$

$K(x) = \cos(x)$ is a trig function which is cont on $(-\infty, \infty)$

Then $f(x) = K(g(x) + h(x)) = \cos(x + \sin x)$ is cont on $(-\infty, \infty)$

Since f is cont $\lim_{x \to 0} f(x) = f(0) = \cos(0 + \sin(0))$

$= \cos(0)$

$= 1$

4. $\lim_{x \to 3} \frac{x^2 - 9}{x^2 + 2x - 3} = \lim_{x \to 3} \frac{(x+3)(x-3)}{(x+3)(x-1)}$

$= \lim_{x \to 3} \frac{x-3}{x-1}$

$= \frac{0}{2}$

$= 0$

5. $$\lim_{x\to -3} \frac{x^2-9}{x^2+2x-3} = \lim_{x\to -3} \frac{(x+3)(x-3)}{(x+3)(x-1)}$$
$$= \lim_{x\to -3} \frac{x-3}{x-1}$$
$$= \frac{-3-3}{-3-1}$$
$$= \frac{3}{2}$$

6. $$\lim_{x\to 1^+} \frac{x^2-9}{x^2+2x-3} = \lim_{x\to 1^+} \frac{(x+3)(x-3)}{(x+3)(x-1)}$$
$$= \lim_{x\to 1^+} \frac{x-3}{x-1} \qquad \left[\begin{array}{l} \text{as } x\to 1^+,\ (x-3)\to -2 \\ \text{as } x\to 1^+,\ (x-1)\to 0^+ \end{array}\right]$$
$$= -\infty$$

7. $$\lim_{h\to 0} \frac{(h-1)^3+1}{h} = \lim_{h\to 0} \frac{(h^3-3h^2+3h-1)+1}{h}$$
$$= \lim_{h\to 0} \frac{h(h^2-3h+3)}{h}$$
$$= \lim_{h\to 0} (h^2-3h+3)$$
$$= 3$$

8. $$\lim_{t\to 2} \frac{t^2-4}{t^3-8} = \lim_{t\to 2} \frac{(t+2)(t-2)}{(t-2)(t^2+2t+4)}$$
$$= \lim_{t\to 2} \frac{t+2}{t^2+2t+4}$$
$$= \frac{2+2}{2^2+2(2)+4}$$
$$= \frac{1}{3}$$

9. $\lim_{r\to 9} \dfrac{\sqrt{r}}{(r-9)^4} = \infty$

as $r \to 9$, $\sqrt{r} \to 3$

as $r \to 9$, $(r-9)^4 \to 0^+$

10. $\lim_{v\to 4^+} \dfrac{4-v}{|4-v|}$

$$|4-v| = \begin{cases} 4-v & \text{if } 4-v \geq 0 \\ -(4-v) & \text{if } 4-v < 0 \end{cases}$$

$$= \begin{cases} 4-v & \text{if } v \leq 4 \\ -(4-v) & \text{if } v > 4 \end{cases}$$

$$\lim_{v\to 4^+} \frac{4-v}{|4-v|} = \lim_{v\to 4^+} \frac{(4-v)}{-(4-v)} = \lim_{v\to 4^+}(-1) = -1$$

11. $$\lim_{u\to 1} \frac{u^4-1}{u^3+5u^2-6u} = \lim_{u\to 1} \frac{(u^2+1)(u^2-1)}{u(u^2+5u-6)}$$

$$= \lim_{u\to 1} \frac{(u^2+1)(u+1)(u-1)}{u(u+6)(u-1)}$$

$$= \lim_{u\to 1} \frac{(u^2+1)(u+1)}{u(u+6)}$$

$$= \frac{(1^2+1)(1+1)}{1(1+6)}$$

$$= \frac{4}{7}$$

12. $$\lim_{x\to 3} \frac{\sqrt{x+6}-x}{x^3-3x^2} = \lim_{x\to 3} \frac{\sqrt{x+6}-x}{x^2(x-3)} \cdot \frac{\sqrt{x+6}+x}{\sqrt{x+6}+x}$$

$$= \lim_{x\to 3} \frac{(x+6)-x^2}{x^2(x-3)(\sqrt{x+6}+x)}$$

$$= \lim_{x\to 3} \frac{-(x^2-x-6)}{x^2(x-3)(\sqrt{x+6}+x)}$$

$$= \lim_{x\to 3} \frac{-(x-3)(x+2)}{x^2(x-3)(\sqrt{x+6}+x)} \quad (\longrightarrow)$$

12 cont'd

$$= \lim_{x \to 3} \frac{-(x+2)}{x^2(\sqrt{x+6} + x)}$$

$$= \frac{-(3+2)}{3^2(\sqrt{3+6} + 3)}$$

$$= \frac{-5}{34}$$

13. $$\lim_{s \to 16} \frac{4-\sqrt{s}}{s-16} \cdot \frac{4+\sqrt{s}}{4+\sqrt{s}} = \lim_{s \to 16} \frac{16-s}{(s-16)(4+\sqrt{s})}$$

$$= \lim_{s \to 16} \frac{-(s-16)}{(s-16)(4+\sqrt{s})}$$

$$= \lim_{s \to 16} \frac{-1}{4+\sqrt{s}}$$

$$= \frac{-1}{4+\sqrt{16}}$$

$$= \frac{-1}{8}$$

14. $$\lim_{v \to 2} \frac{v^2+2v-8}{v^4-16} = \lim_{v \to 2} \frac{(v+4)(v-2)}{(v^2+4)(v^2-4)}$$

$$= \lim_{v \to 2} \frac{(v+4)(v-2)}{(v^2+4)(v+2)(v-2)}$$

$$= \lim_{v \to 2} \frac{v+4}{(v^2+4)(v+2)}$$

$$= \frac{2+4}{(2^2+4)(2+2)}$$

$$= \frac{3}{16}$$

15. $$\lim_{x\to 0} \frac{1-\sqrt{1-x^2}}{x} \cdot \frac{1+\sqrt{1-x^2}}{1+\sqrt{1-x^2}}$$
$$= \lim_{x\to 0} \frac{1-(1-x^2)}{x(1+\sqrt{1-x^2})}$$
$$= \lim_{x\to 0} \frac{x^2}{x(1+\sqrt{1-x^2})}$$
$$= \lim_{x\to 0} \frac{x}{1+\sqrt{1-x^2}}$$
$$= \frac{0}{1+\sqrt{1-0}}$$
$$= 0$$

16. $$\lim_{x\to 1} \left(\frac{1}{x-1} + \frac{1}{x^2-3x+2}\right)$$
$$= \lim_{x\to 1} \left(\frac{1}{(x-1)} + \frac{1}{(x-2)(x-1)}\right)$$
$$= \lim_{x\to 1} \frac{(x-2)+1}{(x-2)(x-1)}$$
$$= \lim_{x\to 1} \frac{x-1}{(x-2)(x-1)}$$
$$= \lim_{x\to 1} \frac{1}{x-2}$$
$$= \frac{1}{1-2}$$
$$= -1$$

17. $2x-1 \le f(x) \le x^2$ for $0<x<3$

$\lim_{x \to 1} (2x-1) = 2(1)-1=1$

$\lim_{x \to 1} (x^2) = 1^2 = 1$

Then by the Squeeze Thm $\lim_{x \to 1} f(x) = 1$

18. Prove that $\lim_{x \to 0} x^2 \cos\left(\frac{1}{x^2}\right) = 0$

$-1 \le \cos\left(\frac{1}{x^2}\right) \le 1$ (Note $x \ne 0$ since $x \to 0$)

$-x^2 \le x^2 \cos\left(\frac{1}{x^2}\right) \le x^2$ (Note $x^2 > 0$ since $x \to 0$)

$\lim_{x \to 0} (-x^2) = 0$

$\lim_{x \to 0} (x^2) = 0$

By the Squeeze Thm $\lim_{x \to 0} x^2 \cos\left(\frac{1}{x^2}\right) = 0$

23.

$$f(x) = \begin{cases} \sqrt{-x} & \text{if } x < 0 \\ 3-x & \text{if } 0 \le x < 3 \\ (x-3)^2 & \text{if } x > 3 \end{cases}$$

(a) (i) $\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} (3-x) = 3$

(ii) $\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} \sqrt{-x} = 0$

(iii) $\lim_{x \to 0} f(x)$ DNE since $0 = \lim_{x \to 0^-} f(x) \neq \lim_{x \to 0^+} f(x) = 3$

(iv) $\lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} (3-x) = 0$

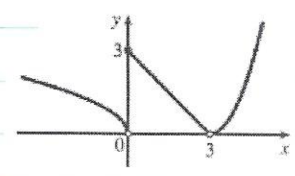
(v) $\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} (x-3)^2 = 0$

(vi) $\lim_{x \to 3} f(x) = 0$ since $\lim_{x \to 3^-} f(x) = \lim_{x \to 3^+} f(x) = 0$

(b) f is discontinuous at $x = 0$ since
$\lim_{x \to 0} f(x)$ DNE (reasoning in (a) part (iii))

f is discontinuous at $x = 3$ since
$f(3)$ is undefined

(c)



24.

$$g(x) = \begin{cases} 2x - x^2 & \text{if } 0 \le x \le 2 \\ 2 - x & \text{if } 2 < x \le 3 \\ x - 4 & \text{if } 3 < x < 4 \\ \pi & \text{if } x \ge 4 \end{cases}$$

(a) $g(2) = 2(2) - 2^2 = 0$

$$\left.\begin{aligned} \lim_{x\to 2^-} g(x) &= \lim_{x\to 2^-} (2x - x^2) = 0 \\ \lim_{x\to 2^+} g(x) &= \lim_{x\to 2^+} (2 - x) = 0 \end{aligned}\right\} \lim_{x\to 2} g(X) = 0$$

g is continuous at $x = 2$ since $\lim_{x\to 2} g(X) = g(2)$

$g(3) = 2 - 3 = -1$

$$\left.\begin{aligned} \lim_{x\to 3^-} g(x) &= \lim_{x\to 3^-} (2 - x) = -1 \\ \lim_{x\to 3^+} g(x) &= \lim_{x\to 3^+} (x - 4) = -1 \end{aligned}\right\} \lim_{x\to 3} g(x) = -1$$

g is continuous at $x = 3$ since $\lim_{x\to 3} g(x) = g(3)$

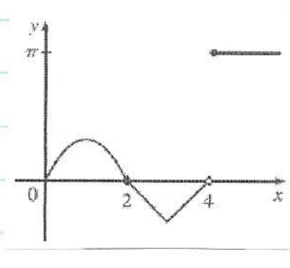
$g(4) = \pi$

$$\left.\begin{aligned} \lim_{x\to 4^-} g(x) &= \lim_{x\to 4^-} (x - 4) = 0 \\ \lim_{x\to 4^+} g(x) &= \lim_{x\to 4^+} \pi = \pi \end{aligned}\right\} \begin{aligned} &\lim_{x\to 4} g(x) \text{ DNE} \\ &\text{since } \lim_{x\to 4^-} g(x) \ne \lim_{x\to 4^+} g(x) \end{aligned}$$

g is discontinuous at $x = 4$ since $\lim_{x\to 4} g(x)$ DNE

g is continuous at $x = 4$ from the right since $\lim_{x\to 4^+} g(x) = g(4)$

(b)



25. $h(x) = \sqrt[4]{x} + x^3 \cos(x)$

$g(x) = \sqrt[4]{x}$ is a root function which is continuous on its domain, $[0, \infty)$

$f(x) = x^3$ is a polynomial which is continuous on $(-\infty, \infty)$

$K(x) = \cos(x)$ is a trig function which is continuous on its domain, $(-\infty, \infty)$

Then $h(x) = g(x) + f(x) \cdot K(x) = \sqrt[4]{x} + x^3 \cos(x)$ is continuous on its domain, $[0, \infty)$

26. $g(x) = \dfrac{\sqrt{x^2-9}}{x^2-2}$

$K(x) = \sqrt{x^2-9}$

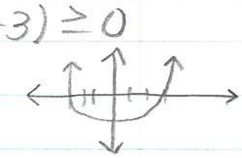
$f(x) = \sqrt{x}$ is a root function which is continuous on its domain, $[0, \infty)$

$h(x) = x^2-9$ is a polynomial which is continuous on $(-\infty, \infty)$

Then $K(x) = f(h(x)) = \sqrt{x^2-9}$ is continuous on its domain, $\mathbb{R}$ such that $x^2-9 \geq 0$

$(x+3)(x-3) \geq 0$

Thus K is cont on $(-\infty, -3] \cup [3, \infty)$



$G(x) = x^2-2$ is a polynomial which is continuous on $(-\infty, \infty)$

Now $g(x) = \dfrac{K(x)}{G(x)} = \dfrac{\sqrt{x^2-9}}{x^2-2}$ is continuous on its domain, $(-\infty, -3] \cup [3, \infty)$

(Note: $x^2-2 \neq 0$)