Lesson 2 - 5

Algebraic Proof

Going Deeper

Essential question: *What kinds of justifications can you use in writing algebraic and geometric proofs?*

PREP FOR CC.9–12.G.CO.9

1 ENGAGE Introducing Proofs

In mathematics, a **proof** is a logical argument that uses a sequence of statements to prove a conjecture. Once the conjecture is proved, it is called a **theorem**.

Each statement in a proof must follow logically from what has come before and must have a reason to support it. The reason may be a piece of given information, a definition, a previously proven theorem, or a mathematical property.

The table states some properties of equality that you have seen in earlier courses. You have used these properties to solve algebraic equations and you will often use these properties as reasons in a proof.

Properties of Equality	
Addition Property of Equality add POE	If $a = b$, then $a + c = b + c$.
Subtraction Property of Equality subtr. POE	If $a = b$, then $a - c = b - c$.
Multiplication Property of Equality	If $a = b$, then $ac = bc$.
Division Property of Equality	If $a = b$ and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$.
Reflexive Property of Equality	$a = a$
Symmetric Property of Equality	If $a = b$, then $b = a$.
Transitive Property of Equality	If $a = b$ and $b = c$, then $a = c$.
Substitution Property of Equality	If $a = b$, then b can be substituted for a in any expression.

Remember!

The Distributive Property states that

$a(b + c) = ab + ac$.

Example 4: Identifying Property of Equality and Congruence

Identify the property that justifies each statement.

A. $\angle QRS \cong \angle QRS$ Reflex. Prop. of ≅.

B. $m\angle 1 = m\angle 2$ **so** $m\angle 2 = m\angle 1$ Symm. Prop. of =

C. $\overline{AB} \cong \overline{CD}$ **and** $\overline{CD} \cong \overline{EF}$, **so** $\overline{AB} \cong \overline{EF}$. Trans. Prop of ≅

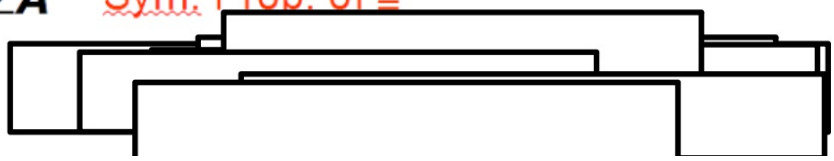
D. $32° = 32°$ Reflex. Prop. of =

4a. $DE = GH$, **so** $GH = DE$. Sym. Prop. of =

4b. $94° = 94°$ Reflex. Prop. of =

4c. $0 = a$, **and** $a = x$. **So** $0 = x$. Trans. Prop. of =

4d. $\angle A \cong \angle Y$, **so** $\angle Y \cong \angle A$ Sym. Prop. of ≅



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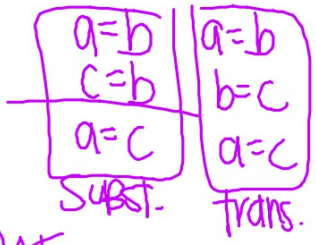


Name the Properties of Equality (POE)

1) If $2x = 3$, then $x = 3/2$ mult/div POE

2) If $XY - AB = WZ - AB$, then $XY = WZ$ +/− POE

3) If $m\angle 1 + m\angle 2 = 90$ and $m\angle 2 = m\angle 3$, then $m\angle 1 + m\angle 3 = 90$ Substitution

4) If $MK = NJ$ and $BG = NJ$, then $MK = BG$ ($a = b$, $c = b$, $a = c$) Substitution

5) If $ST = UV$, then $UV = ST$ Symmetric

REFLECT

1a. Given the equation $3 = x - 2$, you quickly write the solution as $x = 5$. Which property or properties of equality are you using? Explain.

$3 = x - 2$

$5 = x$

add/sub POE

2 (-2)

from both sides

1b. Give an example of an equation that you can solve using the Division Property of Equality. Explain how you would use this property to solve the equation.

$5x = 10$

$x = 2$

Solve the equation $4m - 8 = -12$. Write a justification for each step.

$4m - 8 = -12$	Given equation
$+8 = +8$ Reflex	Addition Property of Equality
$4m = -4$	Simplify.
$\frac{4m}{4} = \frac{-4}{4}$	Division Property of Equality
$m = -1$	Simplify.

Solve the equation $\frac{1}{2}t = -7$. Write a justification for each step.

$\frac{1}{2}t = -7$	Given equation
$2\left(\frac{1}{2}\right)t = 2(-7)$	Multiplication Property of Equality.
$t = -14$	Simplify.

Solve each equation. Write a justification for each step.

1. $\frac{z-5}{6} = -2$

$(6)\frac{z-5}{6} = -2(6)$	Given
$z - 5 = -12$	Mult. Prop. of =
$z = -7$	Add. Prop. of =

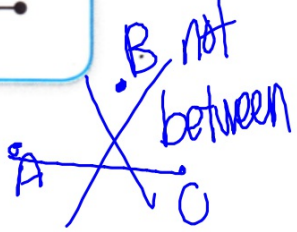
2. $6r - 3 = -2(r + 1)$

$6r - 3 = -2(r + 1)$	Given
$6r - 3 = -2r - 2$	Distrib. Prop.
$8r - 3 = -2$	Add. Prop. of =
$8r = 1$	Add. Prop. of =
$r = \frac{1}{8}$	Div. Prop. of =

A **postulate** (or *axiom*) is a statement that is accepted as true without proof. Like undefined terms, postulates are basic building blocks of geometry. The following postulate states that the lengths of segments "add up" in a natural way.

Segment Addition Postulate

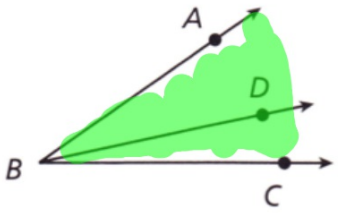
If B is between A and C, then $AB + BC = AC$.



The Angle Addition Postulate is similar to the Segment Addition Postulate.

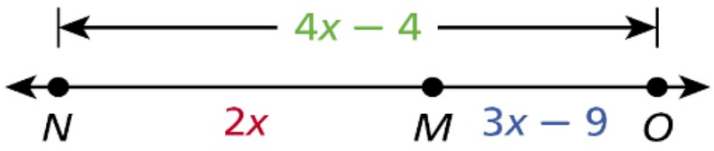
Angle Addition Postulate

If D is in the interior of $\angle ABC$, then $m\angle ABD + m\angle DBC = m\angle ABC$.



Example 3: Solving an Equation in Geometry

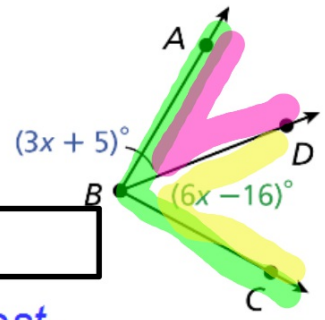
Write a justification for each step.



$NO = NM + MO$	*Segment Addition Post.*
$4x - 4 = 2x + (3x - 9)$	*Substitution Property of Equality*
$4x - 4 = 5x - 9$	*Simplify.*
$-4 = x - 9$	*Subtraction Property of Equality*
$5 = x$	*Addition Property of Equality*

Check It Out! Example 3

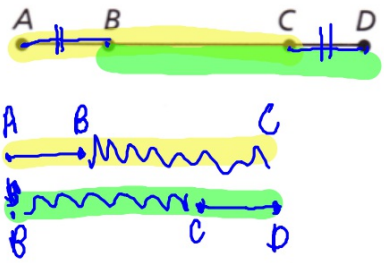
Write a justification for each step.



m∠ABC = 8x°

$m\angle ABC = m\angle ABD + m\angle DBC$	∠ Add. Post.
$8x° = (3x + 5)° + (6x - 16)°$	Subst. Prop. of Equality
$8x = 9x - 11$	Simplify.
$-x = -11$	Subtr. Prop. of Equality.
$x = 11$	Mult. Prop. of Equality. or DIV.

1. If A, B, C, and D are collinear, as shown in the figure, with $AC = BD$, then $AB = CD$. Complete the proof by writing the missing statements or reasons.

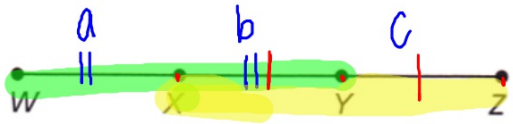


Given: $AC = BD$

Prove: $AB = CD$

Statements	Reasons
1. $AC = BD$	1. Given
2. $AC = AB + BC$; $BD = BC + CD$	2. Seg. add. post.
3. $AB + BC = BC + CD$	3. Substitution Property of Equality
4. $AB = CD$	4. Subtract POE

2. In the figure, X is the midpoint of $\overline{WY}$, and Y is th
midpoint of $\overline{XZ}$. Explain how to prove $WX = YZ$.



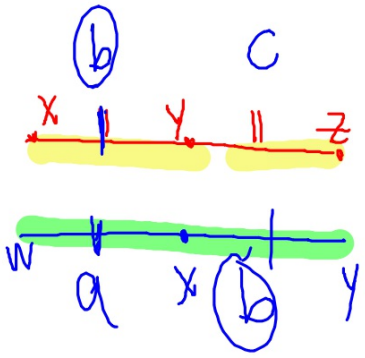
$\cong$

$a = b$

$b = c$

$a = c$ transit.

$(WX)\ (YZ)$



HW 2.5 Practice A #1-12
AP (p)
PS (p)

Additional Practice

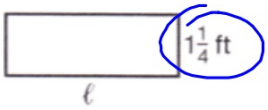
Solve each equation. Show all your steps and write a justification for each step

1. $\frac{1}{5}(a + 10) = -3$

Solve each equation. Show all your steps and write a justification for each step

2. $t + 6.5 = 3t - 1.3$

3. The formula for the perimeter P of a rectangle with length ℓ and width w is $P = 2(\ell + w)$. Find the length of the rectangle shown here if the perimeter is $9\frac{1}{2}$ feet. Solve the equation for ℓ and justify each step.



Given $P = 2(l + w)$
$w = 1\frac{1}{4}$ and $P = 9\frac{1}{2}$

SUBST. $9\frac{1}{2} = 2(l + 1\frac{1}{4})$

DISTRIB. $9.5 = 2l + 2.5$
$-2.5 \quad -2.5$

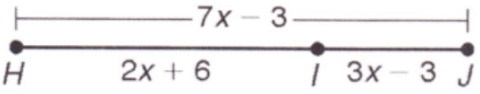
SUB. POE $7 = 2l$

div. POE $\frac{7}{2} = l$

$(3\frac{1}{2})$

1.25
2
2.5

Write a justification for each step.

4. 

$HJ = HI + IJ$	_____
$7x - 3 = (2x + 6) + (3x - 3)$	_____
$7x - 3 = 5x + 3$	_____
$7x = 5x + 6$	_____
$2x = 6$	_____
$x = 3$	_____

Identify the property that justifies each statement.

5. $m = n$, so $n = m$.

Identify the property that justifies each statement.

6. $\angle ABC \cong \angle ABC$

Identify the property that justifies each statement.

7. $\overline{KL} \cong \overline{LK}$

Identify the property that justifies each statement.

8. $p = q$ and $q = -1$, so $p = -1$.

Problem Solving

1. Because of a recent computer glitch, an airline mistakenly sold tickets for round-trip flights at a discounted price. The equation $n(p + t) = 3298.75$ relates the number of discounted tickets sold n, the price of each ticket p, and the tax per ticket t. What was the discounted price of each ticket if 1015 tickets were sold and the tax per ticket was \$1.39? Solve the equation for p. Justify each step.

Given $n(p+t) = 3298.75$

$n = 1015$ $t = 1.39$ Solve p

Subst $1015(p + 1.39) = 3$...

2. The equation $C = 7.25s + 15.95a$ describes the total cost of admission C to the aquarium. How many student tickets were sold if the total cost for the entire class and 6 adults was $298.70? Solve the equation for s. Justify each step.

s = number of student tickets
a = number of adult tickets
C = total cost of admission

Given $C = 7.25s + 15.95a$

Subtr. POE $C - 15.95a = 7.25s$

Div POE $\frac{C - 15.95a}{7.25} = s$

Substit. $\frac{298.7 - 15.95(6)}{7.25} = s$

Simplify $= s$

Refer to the figure. Choose the best answer.

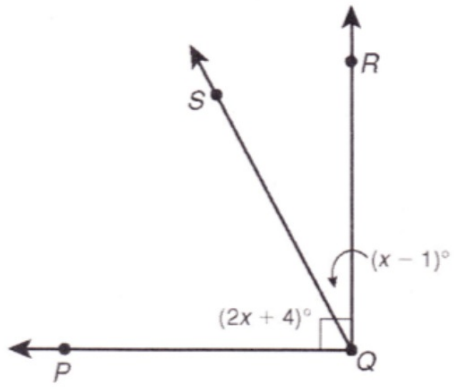
3. Which could be used to find the value of x?

   A Segment Addition Postulate

   B Angle Addition Postulate

   C Transitive Property of Congruence

   D Definition of supplementary angles



4. What is m∠*SQR*?

F 28°

G 29°

H 61°

J 62°