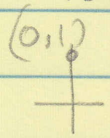
Ch10 Test

1. $\cos 75° \cos 15° + \sin 75° \sin 15°$ → cos formula; chg sign

$= \cos(75° - 15°) = \cos 60° = \frac{1}{2}$

1b. $\sin 75° \cos 15° + \cos 75° \sin 15°$ → sin formula, keep sign



$= \sin(75° + 15°) = \sin 90° = 1$

1c. $\cos(30° + x) \qquad + \cos(30° - x)$

$\cos 30° \cos x - \sin \cancel{30°} \sin x + \cos 30° \cos x + \sin \cancel{30°} \sin x$

$\frac{\sqrt{3}}{2} \cos x \qquad + \frac{\sqrt{3}}{2} \cos x = \sqrt{3} \cos x$

1d. $\sin(45° - x) \qquad - \sin(45° + x)$

$\sin 45° \cos x - \cos 45° \sin x - (\sin 45° \cos x + \cos 45° \sin x)$

$\sin \cancel{45°} \cos x - \cos 45° \sin x - \sin \cancel{45°} \cos x - \cos 45° \sin x$

$-\frac{\sqrt{2}}{2} \sin x \qquad -\frac{\sqrt{2}}{2} \sin x = -\sqrt{2} \sin$

2. $\cos 15° = \cos(45° - 30°)$

$= \cos 45° \cos 30° + \sin 45° \sin 30°$

$\frac{\sqrt{2}}{2} \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \frac{1}{2} = \frac{\sqrt{6}+\sqrt{2}}{4}$

3. $\tan\left(\frac{5\pi}{4} - \theta\right) = \frac{\tan \frac{5\pi}{4} - \tan\theta}{1 + \tan\frac{5\pi}{4} \tan\theta} = \frac{1 - (-\frac{1}{3})}{1 + (1)(-\frac{1}{3})}$

$\tan\theta = -\frac{1}{3}$

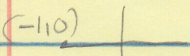
$= \frac{\frac{4}{3}}{\frac{2}{3}} = \frac{4}{3} \cdot \frac{3}{2} = 2$

4. $\tan\alpha = \frac{4}{3} \qquad \tan\beta = -\frac{1}{2}$

$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha \tan\beta} = \frac{{}^{2}_{2}\frac{4}{3} + \left(-\frac{1}{2}\right)^{3}_{3}}{1 - (\frac{4}{3})(-\frac{1}{2})} = \frac{\frac{8}{6} - \frac{3}{6}}{1 + \frac{4}{6}}$

$= \frac{5}{6} \div \frac{10}{6} = \frac{5}{6} \cdot \frac{6}{10} = \frac{1}{2}$

$$\tan(\pi-\beta) = \frac{\tan\pi - \tan\beta}{1+\tan\pi\tan\beta} = \frac{0-(-\frac{1}{2})}{1+(0)(-\frac{1}{2})} = \frac{\frac{1}{2}}{1} = \frac{1}{2}$$



$\tan\pi = \frac{0}{-1} = 0$

$\rightarrow \tan(\alpha+\beta) = \tan(\pi-\beta)$

5. $y = 2x+1$ $\qquad$ $y = 4-3x$

$m_1 = 2 = \tan\alpha$ $\qquad$ $m = -3 = \tan\beta$

$$\tan(\alpha-\beta) = \frac{\tan\alpha - \tan\beta}{1+\tan\alpha\tan\beta} = \frac{2 - -3}{1+(2)(-3)} = \frac{5}{-5} = -1$$

Since $\tan(\alpha-\beta)$ is negative $\rightarrow$ obtuse angle

$= 135^\circ$

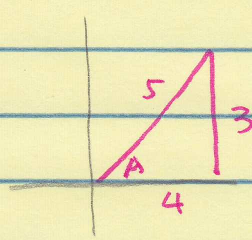
$$\tan(\beta-\alpha) = \frac{\tan\beta - \tan\alpha}{1+\tan\beta\tan\alpha} = \frac{-3-2}{1+(-3)(2)} = \frac{-5}{-5} = 1$$

since $\tan(\beta-\alpha)$ is pos $\rightarrow$ acute $\angle$

$= 45^\circ$

The 2 angles are supplementary

6 $\angle A$ is acute $\rightarrow$ QI



$\cos A = 4/5$

$\sin A = 3/5$

(a) $\sin A = 3/5$

(b) $\cos 2A = 2\cos^2 A - 1$

$= 2(4/5)^2 - 1$

$= 2\left(\frac{16}{25}\right) - 1 = \frac{32}{25} - \frac{25}{25} = \frac{7}{25}$

c $\sin 2A = 2\sin A\cos A$

$= 2(3/5)(4/5) = 24/25$

d. $\sin 4A = \sin 2(2A) = 2\sin 2A\cos 2A$

$= 2(24/25)(7/25) = 336/625$

7. $\dfrac{\sin 2x}{1-\cos 2x} = \dfrac{2\sin x\cos x}{1-(1-2\sin^2 x)} = \dfrac{2\sin x \cos x}{1-1+2\sin^2 x}$

$= \dfrac{2\sin x\cos x}{2\sin^2 x} = \dfrac{\cos x}{\sin x} = \cot x$

b. $\underline{(1+\tan^2 y)}\,(\underline{\cos 2y}-1)$

$(\sec^2 y)\,(1-2\sin^2 y-1)$

$\dfrac{1}{\cos^2 y}\quad(-2\sin^2 y) = -2\dfrac{\sin^2 y}{\cos^2 y} = -2\tan^2 y$

c $\dfrac{\tan t}{\sec t+1} = \dfrac{\tan t}{\frac{1}{\cos t}+\frac{1}{1}\frac{\cos t}{\cos t}} = \dfrac{\frac{\sin t}{\cos t}}{\frac{1+\cos t}{\cos t}}$

$= \dfrac{\sin t}{\cos t} \div \dfrac{1+\cos t}{\cos t} = \dfrac{\sin t}{\cos t}\cdot\dfrac{\cos t}{1+\cos t}$

$= \dfrac{\sin t}{1+\cos t} = \tan\dfrac{t}{2}$

d. $\cos^2\frac{x}{2} - \sin^2\frac{x}{2}$ let $\theta = \frac{x}{2}$

$\cos^2\theta - \sin^2\theta = \cos 2\theta = \cos 2\left(\frac{x}{2}\right) = \cos x$

8 $2\cos^2\frac{\pi}{12} - 1$ let $\theta = \frac{\pi}{12}$

$2\cos^2\theta - 1 = \cos 2\theta = \cos 2\left(\frac{\pi}{12}\right) = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

b. $4\sin\frac{\pi}{6}\cos\frac{\pi}{6} \Longrightarrow$ let $\theta = \frac{\pi}{6}$

$4\sin\theta\cos\theta = 2\,(2\sin\theta\cos\theta) = 2(\sin 2\theta)$

$= 2\sin\left[2\left(\frac{\pi}{6}\right)\right] = 2\sin\frac{\pi}{3} = 2\cdot\frac{\sqrt{3}}{2} = \sqrt{3}$

Foil the left

a. $(1+\cot^2x)(1-\cos 2x)=2$

$1-\cos 2x+\cot^2x-\cot^2x\cos 2x$

$1-(1-2\sin^2x)+\cot^2x-\cot^2x\cdot(1-2\sin^2x)$

$\cancel{1}-\cancel{1}+2\sin^2x+\cancel{\cot^2x}-\cancel{\cot^2x}+2\cot^2x\sin^2x$

$2\sin^2x+2\cdot\frac{\cos^2x}{\cancel{\sin^2x}}\cdot\frac{\cancel{\sin^2x}}{1}$

$2\sin^2x+2\cos^2x$

$2(\sin^2x+\cos^2x)$

$2(1)=2$

b. $\dfrac{\sin\theta\sec\theta}{\tan\theta+\cot\theta}=\cos^2\theta-\cos 2\theta$

$$\frac{\sin\theta\cdot\frac{1}{\cos\theta}}{\frac{\sin\theta}{\sin\theta}\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}\frac{\cos\theta}{\cos\theta}}=\frac{\frac{\sin\theta}{\cos\theta}}{\frac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta}}$$

$=\dfrac{\sin\theta}{\cos\theta}\div\dfrac{1}{\sin\theta\cos\theta}$

$=\dfrac{\sin\theta}{\cancel{\cos\theta}}\cdot\dfrac{\sin\theta\cancel{\cos\theta}}{1}$

$=\sin^2\theta=$

$=\sin^2\theta+\cos^2\theta-\cos^2\theta$

$=\cos^2\theta-(\cos^2\theta-\sin^2\theta)$

$=\cos^2\theta-\cos 2\theta$

you can do this or you can simplify the right side to $=\sin^2\theta$

simplify the right side

$\cos^2\theta-\cos 2\theta=\cos^2\theta-(\cos^2\theta-\sin^2\theta)$

$=\cos^2\theta-\cos^2\theta+\sin^2\theta$

$=\sin^2\theta$

so $\dfrac{\sin\theta\sec\theta}{\tan\theta+\cot\theta}=\cos^2\theta-\cos 2\theta$

11. $\cos 2x = \cos x + 2$

$2\cos^2 x - 1 = \cos x + 2$

$2\cos^2 x - \cos x - 3 = 0$

$2\cos x \quad -3$

$\cos x \quad +1$

$(2\cos x - 3)(\cos x + 1) = 0$

$2\cos x - 3 = 0$

$\cos x = \frac{3}{2}$

not possible

$\cos x + 1 = 0$

$\cos x = -1 \rightarrow \underset{c,\ s}{(-1, 0)}$ π

$x = \pi + 2n\pi$

$n = 0 \rightarrow x = \boxed{\pi}$

$n = 1 \rightarrow x = 3\pi$ too big